Dynamic Networks and Asset Pricing

Andrea Buraschi

The University of Chicago Booth School of Business
and
Imperial College London Business School

Paolo Porchia

IE Business School

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Abstract

We study an equilibrium model that maps the characteristics of the network connecting firm-specific risks to the cross-section of expected returns. Through the 'network' firms transfer a distress state to other firms' fundamentals in a directed and timely manner. We show that 'central' firms, active at transferring but immune to transferred distress, have lower P/D ratios and higher expected returns. We confirm this prediction using a newly proposed measure of network centrality, estimated on corporate earnings. We also argue that network centrality provides a natural explanation for the predictive power of the size and book-to-market firm characteristics.

I. Introduction

This paper studies the effects on the cross-section of expected returns of a dependence structure relating firm-specific cash-flow risks. We call such a dependence structure 'network'. An increasing literature investigates the role of interconnections between different firms and sectors, functioning as a potential propagation mechanism of idiosyncratic shocks throughout the economy. Acemoglou et al. (2011) use network structure to show the possibility that aggregate fluctuations may originate from microeconomic shocks to firms. Such a possibility is usually disregarded in standard macrofinance models, in light of a "diversification argument": as argued by Lucas (1977), among others, such microeconomic shocks would average out and thus have negligible aggregate effects, and little impact on asset prices. For instance, an investor holding a diversified portfolio of firms in the symmetric network of Figure 1a would not be exposed to firm specific shocks as the number of firms grows large.

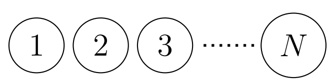

Figure 1a

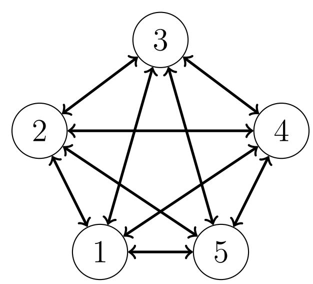

Figure 1b

In contrast, in the network of Figure 2a shocks to the central firm 1 propagate to the rest of the network: even if an investor were to hold an arbitrarily large number of stocks in the portfolio, she would still be exposed to shocks of firm 1. Similarly, in the network of Figure 2b she would be exposed to shocks of the $\overline{N}$ central firms. The centrality of a firm is then intuitively important for the cross-section of asset prices.

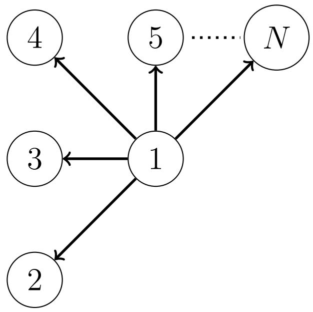

Figure 2a

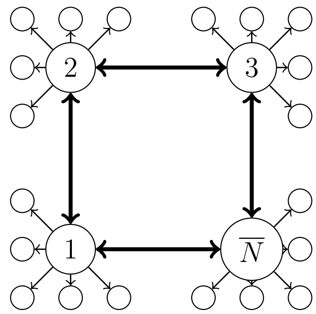

Figure 2b

The idea of the existence of an asymmetric network structure has been at the heart of much of policy decisions in the U.S. and Europe during the 2008 Credit Crisis, when several financial firms have been rescued to avoid adverse effects to the rest of the economy. Substantial regulatory effort is being devoted to understand the linkages across firms and sectors and limit excessive propagation of shocks. In this paper, we concentrate on the main asset pricing implication of such linkages, the cross-section of expected returns.

We study a Lucas economy with multiple trees, generating dividends which follow a Markov chain in continuous time, whose transition probabilities depend on other firms' dividend states in an heterogeneous manner. This allows to have a pairwise dependence structure, or network, in which firms propagate and/or absorb shocks differently. Firms' connectivity characteristics are linked to the cross-section of expected returns in an interesting and non apparent way. We show that the equilibrium risk premium is positively related to the extent that the firm is *actively* connected to the rest of the network, in that it can transfer its own shocks while being relatively insulated from others' (as firm 1 in Figure 2a). Fundamental shocks to such a firm lose soon their idiosyncratic nature and evolve into a systematic factor leading the business cycle. Almost tautologically, the marked exposition to their own risk makes active, or *central* firms highly pro-cyclical relative to passively connected ones (such as firm 2 in Figure 2a), justifying a greater risk premium. We propose a reduced-form univariate measure of network connectivity, or "Dynamic Centrality", with the purpose of capturing the degree of centrality of a firm and unveil the structural link between network characteristics and the cross-section of expected returns: the larger the dynamic centrality, the larger the expected return and the smaller the P/D ratio.

We use earnings data to estimate the characteristics of the network structure by maximum likelihood. We aggregate firms in portfolios by mimicking Fama and French (1992) methodology and find that in Fama-McBeth regressions, after controlling for market beta, size and book-to-market, the slope of the dynamic centrality characteristic is positive and significant at standard confidence levels. A simulation study confirms that this result is robust to estimation error. We build a centrality factor mimicking portfolio, long (short) the 25% of stocks with largest (smallest) dynamic centrality. Using the returns on such a portfolio, labeled *cmp* factor, to fit the centrality risk premium through standard APT regressions, we find nonnegligible and heterogeneous centrality prices of risk. For instance, on 10 book-to-market sorted portfolios, the centrality annual risk premium ranges from 0 to 15.3%. We also map firms' centrality to more direct characteristics that have been long related to the cross-section of returns, such as size and book-to-market. High book-to-market (value) stocks are unarguably more central than low book-to-market (growth) stocks, and the returns on a portfolio long the top and short the bottom quintile of the book-to-market distribution have a sizable and highly significant centrality premium component. We interpret this as a micro-foundation of the debated distress-related explanation for the value premium: value firms are systematic bad performers during economic downturns, thus mandating higher premia, because they are inherently central, and as such the catalysts of systematic distress. Moreover we find quite intuitive that low book-to-market firms appear less central in the network: 1) if their market equity value is mostly growth opportunities, a present distress shock has limited effect on other firms, at least in the short run. 2) Conversely, to put a growth opportunity in place, especially to finance it, they need external links, hence they are vulnerable to induced distress also in the short run. In terms of firm size, while in an input-output technological network a large firm is typically central , in a dynamic network linking firms' fundamentals it may not be: we find small firms significantly more central at the monthly horizons, while the opposite holds at quarterly horizons. This has an intuitive explanation : 1) large firms can absorb fundamental shocks more effectively in the short run, avoiding on average quick propagation to their peers, while tend to suffer the endurance of distress. 2) In the longer run, many small firms are of recent creation and less are survivals to a durable distress.

Related Literature. This paper is related to three streams of the literature. A first stream studies endowment economies with multiple dividend paying assets (orchards).
Cochrane, Longstaff, and Santa-Clara (2008), and Martin (2011) show that even if dividends have iid increments, simple market clearing can give rise to rich asset pricing implications. Santos and Veronesi (2009) study a multiple-trees economy where the SDF implied by nonlinear external habit formation preferences counterfactually generates higher expected returns for stocks with high price-dividend ratios – i.e. a 'growth premium' – if firms/trees are allowed to differ in their expected dividend growth, but not in their cash-flow risk,[1]. In these models, cross-sectional heterogeneity in asset prices is driven by the properties of trees' share sizes, which are responsible for systematic risk. Our main departure from this literature is in the emphasis on dynamic network connectivity, which allows us to explore causality among firms' fundamentals, rather than simple covariation. Lettau and Wachter (2007) advocate the importance of weak or positive covariance between the market price of risk and dividend shocks, in order to obtain a 'value premium'. Our contribution is to show both theoretically and empirically a structural channel independent of preferences that is consistent with some well studied features of the cross-section of expected returns.

A second stream of the literature studies the role of sectoral shocks in macro fluctuations; examples include Horvath (1998, 2000), Dupor (1999), Shea (2002), and Acemoglu,

[1] That is, in the covariance between consumption growth and their dividend growth

Carvalho, Ozdaglar, and Tahbaz-Salehi (2011). This literature focuses on shock propagation in static networks. We address the asset pricing implications of sectoral shocks in asymmetric networks, and to this end we introduce dynamics in the mechanism of shock propagation. Our network connectivity is also related to the role played by firm size distribution in Gabaix (2011), who shows that firm-level idiosyncratic shocks translate into aggregate fluctuations when the largest firms contribute disproportionally to aggregate output. While this could be the case in our network structure, we emphasize the importance of connectivity in networks of fundamentals.

A third strand of the literature studies the role of idiosyncratic risk in asset pricing. Ang, Hodrick, Xing, and Zhang (2006), show that idiosyncratic volatility risk is priced in the cross-section of expected stock returns, a regularity which is not subsumed by size, book-to-market, momentum, or liquidity effects.

The article is organized as follows. Section II describes the model. Section III derives security prices. Section IV studies risk premia, relating their cross-sectional behavior to network connectivity. Section V introduces a measure of network centrality. Sections VI is devoted to the empirical analysis, and Section VII concludes. Proofs are in the Appendix, and a separate Online Appendix collects derivations and additional results not reported in the paper for brevity.

II. The Economy

In an infinite-horizon, pure exchange Lucas economy a representative agent maximizes Constant Relative Risk Aversion utility of intertemporal consumption.

$$U_0 = \mathbb{E}\left[\int_0^\infty e^{-\delta s}\frac{C_s^{1-\gamma}}{1-\gamma}ds\right]. \tag{1}$$

γ and δ are the relative risk aversion coefficient and the subjective discount rate, respectively. The opportunity set of the investor consists of a locally risk-less security in zero net supply, with rate of return r_t (the interest rate), and N risky securities in positive net supply, each paying a stochastic dividend stream x_t^i, $i = 1, \ldots, N$. In equilibrium aggregate consumption is equal to the sum of the dividends: $C_t = \sum_{i=1}^N x_t^i$. We will often refer to trees as 'firms'. x_t^i is a two-state Markov chain in continuous time, with states $\overline{x}^i$ and $\underline{x}^i$, $\overline{x}^i > \underline{x}^i$. We label $\underline{x}^i$ the 'distress' state, and introduce the binary variable H_t^i, which takes value 1 if firm i is in distress state at time t, and 0 if it is not. Formally, security i's dividend evolves as:

$$\frac{dx_t^i}{x_t^i} = \frac{\underline{x}^i - \overline{x}^i}{\overline{x}^i}(1 - H_{t-}^i)dH_t^i - \frac{\overline{x}^i - \underline{x}^i}{\underline{x}^i}H_{t-}^i dH_t^i \tag{2}$$

An innovation $dH_t^i = 1$ denotes a distress event of firm i, while $dH_t^i = -1$ denotes a recovery event. λ_t^i (η_t^i) denotes the distress (recovery) intensity, that is, the probability of a negative (positive) dividend jump during next time instant, provided the tree is not (the tree is) in distress.

We refer to a network as a dependence structure that acts on firms' dividends on a finite time horizon. In particular, the idiosyncratic shocks dH_t^i are by definition instantaneously independent across trees: we introduce connectivity by assuming that the intensities depend on the state of distress of the other firms: $\lambda^i(\mathbf{H}_t)$, $\eta^i(\mathbf{H}_t)$.[2] Thus the likelihood of distress or recovery during the next small time interval is positively or negatively affected by the state of distress of directly connected trees. If intensities are constant, firm-specific risks are also unconditionally independent, and the economy is the disconnected network of Figure 1a. In general, the functional forms $\lambda^i(\,\cdot\,)$ and $\eta^i(\,\cdot\,)$ accommodate any asymmetric form of mutual influence among firms' distress. Thus the term 'network' is justified by the atomistic nature of the dependence structure, which is a pair-specific transfer mechanism of fundamental shocks. Our network connectivity has two distinctive features:

[2]When no confusion can arise, we adopt the notation $\mathbf{H}_t = (H_t^1, \ldots, H_t^N)$.

i) It is inherently dynamic, because it adopts the likelihood of future distress/recovery shocks, rather than immediate shocks, as transmission device, thus allowing a timely speed of propagation. This property is important in our context, which is concerned with the network determinants of the cross-section of returns, as it accounts for different predictability features at different time horizons. Indeed, a static network is recovered in our setting by assuming variations of $\lambda^i(\mathbf{H}_t)$ and $\eta^i(\mathbf{H}_t)$ (across different states) large enough to lead to almost immediate propagation.

ii) It emphasizes distress causality rather than simple distress covariance, as the state-dependence of the intensities allows to identify precisely the directionality of the shock transmission.

A highly central firm, such as Firm 1 in the example of Figure 3, is embedded in this context by: *i*) having its distress increase other firms' distress intensities: $\lambda^j(H_t^1 = 1) > \lambda^j(H_t^1 = 0)$, $j = 2, 3$. If $\lambda^j(H_t^1 = 1) = \infty$, Firm 1's distress propagates immediately. *ii*) Letting its distress intensity be insensitive to others' distress: $\lambda^1(H_t^j = 1) \approx \lambda^1(H_t^j = 0)$. Figure 3 reports as additional example a vertically integrated value chain, where shocks flow more quickly downstream: Firm 1 is upstream in terms of cash-flow shocks, while Firm 3 is downstream, so that $\lambda^2(H_t^1 = 1) > \lambda^1(H_t^2 = 1)$ and $\lambda^3(H_t^2 = 1) > \lambda^2(H_t^3 = 1)$.

Sector 1 Dynamic Centrality

Vertical Integration

III. Security prices

The network structure gives rise to interesting predictions for both price-dividend ratios and risk premia. We concentrate on the cross-sectional predictability of expected returns, with the purpose of deriving testable implications.[3]

Although the network structure can be general, it is possible to obtain closed-form solutions for security prices.

Proposition 1. *Let H denote a generic realization of $\mathbf{H}_t$, and $P^i(H)$ the price of the claim to the $i-$th endowment stream, x_t^i. We have:*

$$\frac{P^i(H)}{x_t^i} = \overline{1}_H' \mathbf{V}^i, \quad \textit{where} \quad \mathbf{V}^i = \left(\mathbf{a} - \mathbf{A}^H\right)^{-1} \mathbf{C}^i \tag{3}$$

$\overline{1}_H$ is a vector with 2^N entries – as many as the total number of states for $\mathbf{H}_t$ – with 1 in the entry corresponding to H and zero elsewhere. $\mathbf{a} = \delta I_{2^N}$, with I_{2^N} the 2^N-dimensional identity matrix. The Markov transition matrix of $\mathbf{H}_t$, $\mathbf{A}^{\mathbf{H}}$, and the vector $\mathbf{C}^i$ are reported in the Appendix.

The equilibrium price-dividend ratio satisfies a multidimensional stochastic Gordon growth formula, and depends on the state variable $\mathbf{H}_t$, the vector that tracks all trees' state of distress. In particular, $\mathbf{V}^i$ is the vector of P/D ratios conditional on each of the 2^N realizations of $\mathbf{H}_t$.[4] $\mathbf{V}^i$ is better understood by rewriting it as:

$$\mathbf{V}^i = \lim_{T\to\infty} \int_t^T \underbrace{\exp(\mathbf{A}^H(s-t))}_{1} \underbrace{e^{-\delta(s-t)}\mathbf{C}^i}_{2} ds \tag{4}$$

Term 2 is the conditional gross dividend growth of the tree discounted by the intertemporal

[3]Additional implications can be derived for the behavior of aggregate consumption, interest rates and of the market prices of risk. These properties are reported in the Online Appendix.

[4]$\mathbf{H}_t$ takes 2^N possible values, ranging from a combination where no tree/firm is in distress, $H_t^i = 0$, to one where all are in distress, $H_t^i = 1$, $i = 1, 2, \ldots, N$

marginal rate of (aggregate) consumption substitution, that is, by the equilibrium pricing kernel. Term 1 is the transition probability matrix of the state variable $\mathbf{H}_t$, from time t to s.[5] Therefore $\mathbf{V}^i$ amounts to the expectation of cumulative discounted dividend growth, in the infinite horizon limit, conditional on any possible initial distress state H. The influence of network connectivity on the cross-section of P/D ratios is embedded in Term 1, as present distress states affect the probabilities of future distress or recovery events. To gauge this effect, it is convenient to consider a disconnected economy first, and then add a network structure:

1) Suppose there is no connectivity, so that jump intensities do not depend on $\mathbf{H}$. Any distress event leads to an expected increase in dividend growth for the affected asset, hence consumption growth in equilibrium, because a recovery is eventually foreseen. In this case, low state prices (future marginal utility) imply a *reduced desire to invest* in any risky assets in order to substitute consumption intertemporally, so that all P/D-ratios drop. The higher the distress intensity λ and dividend share of the distressed tree, the more pronounced the negative spill-over effect, because the increment of expected consumption growth is maximal.

2) When trees are part of a network structure, those connected to the distressed one will experience an increase in their distress jump intensity. Fears of distress contagion jeopardize consumption recovery perspectives, hence the agent may want to invest in securities that can hedge the lower expected consumption growth. Pro-cyclical assets are bad in this role, thus their demand will drop. These are firms which are able to spread their own distress risk, but relatively immune from distress contagion: the 'exogenously connected' firms. An exogenous, or *central*, firm lays in distress when aggregate consumption is systematically low, because of its ability to cause generalized distress. Its dividends are highly correlated with aggregate consumption. The situation

[5] See the Appendix for details. $\exp(\cdot)$ in this expression in a matrix exponential.

is different for firms that are less central, hence more endogenous to the shock and have modest or negative correlation with future aggregate consumption: their demand will rise, or it will diminish less, and so their P/D-ratios.

It is intuitive that the degree of centrality (i.e. whether a firm is $N = 1$ or $N = 4$ in Figure 2a) of a firm determines its location in the cross-section of P/D ratios. Being a property of the dependence structure linking firms' fundamentals, centrality is not a directly observable characteristic. Our strategy is then to identify a reduced-form indicator that captures its distinctive property: exogeneity in the fundamental shock transmission. We first discuss the cross-section of model risk premia.

IV. Risk premia

It is convenient to decompose the equilibrium risk premium of the $i-$th security into a premium for recovery risk (μ^i_η) and a premium for distress risk (μ^i_λ). Our attention is focused on the latter, although the same qualitative intuition applies to the recovery premium.

Proposition 2. *Let μ^i_t denote the equilibrium risk premium of the $i-th$ security. We have:*

$$\begin{aligned} \mu^i_t &= \mu^i_\lambda + \mu^i_\eta && (5) \\ \mu^i_\lambda &= \sum_{j=1}^{N} (1 - H^j_t) \left[1 - \theta^j_t R^i(H^{-j})\right] \lambda^j_t. && (6) \\ \mu^i_\eta &= \sum_{j=1}^{N} H^j_t \left[1 - \theta^j_t R^i(H^{+j})\right] \eta^j_t. && (7) \end{aligned}$$

H^{-j} (H^{+j}) coincides with the current state H, except for firm j (not) in distress. θ^j_t is the market price of distress/recovery risk, reported in (22) of the Appendix.

$$R^i(H^{-j}) = \frac{P^i(H^{-j})}{P^i(H)}, \quad R^i(H^{+j}) = \frac{P^i(H^{+j})}{P^i(H)} \tag{8}$$

are the gross returns on security i triggered by a distress or, respectively, a recovery event of security j.[6] *Security prices $P^i(H^{-j})$ and $P^i(H)$, conditional to states H^{-j} and H, respectively, are explicitly available from (3) of Proposition 1.*

The distress risk premium (6) is easy to interpret. $\theta_t^j \lambda_t^j$ is the risk-neutral distress intensity of firm j. It is greater than the objective intensity λ^j: the market price of distress risk, $\theta_t^j > 1$, can thus be interpreted as the risk adjustment per unit of (instantaneous) probability that the agent requires as compensation for the risk of distress.[7] If the event materializes, security i responds with a gross returns $R^i(H^{-j})$. Thus the distress risk premium (6) is a weighted average of the risk adjusted returns on security i that would emerge if any tree had a distress, with the likelihoods of distress as weights. Again, the network determinants of the cross-section of risk premia are hard to capture at visual inspection, as they are embedded in the transition matrix of state variables. However expression (6) suggests that we can rely on our discussion of P/D ratios in Section III for a qualitative assessment. In particular, without network connectivity any distress shock triggers negative security returns (or gross returns smaller than one) for all the cross-section. The distress premium is at its highest. Network connectivity reshapes the cross-section consistently with the degree of centrality of each asset: central firms such as Firm 1 in Figure 2a have their price react relatively worse (with a smaller gross return) to a distress shock, for their inherent ineptitude to hedge the further dividend growth loss that network propagation of the distress generates. Central firms in a network structure will then have unconditionally higher risk premia.

A. Mapping firms' centrality to risk premia

To focus on the role of network structure, this discussion abstracts from dividend share size and assumes that dividend payments are homogeneous across firms. In a disconnected network such as Figure 1a, where trees' intensities are constant, the cross-section of equity

[6] We report them in expression (29) of the Appendix for completeness.

[7] It is smaller than one in case of recovery event. See the Online Appendix for details.

premia is then determined solely by the relative magnitude of distress intensities: the lower the latter the lower the risk premium, for finite number of firms N. The reason is that in this case the firm pays dividends in low aggregate consumption states. Moreover, as N becomes arbitrarily large, the market portfolio can diversify away firm-specific shocks, so that these will not bear any risk premium. In a 'star' network structure the situation is different. In Figure 2a, Firm 1 is 'central', because its distress jump increases all other distress intensities, but the converse is not true. Moreover, all other firms are disconnected among each other, thus unarguably 'noncentral'. Since Firm 1 is dominant, it is a source of systematic risk, because even for $N \to \infty$ the market portfolio is not able to diversify away its firm-specific risk. This result holds more generally: networks with a large cross-sectional dispersion in centrality do not satisfy the two fund separation property and firm-specific risk matters in equilibrium asset prices. Online Appendix C reports detailed results about the (failure of) asymptotic two-fund separation.

With their apparent distribution of firm centrality, 'star' networks are ideal candidates to formalize the link between centrality and the cross-section of returns. In particular, letting distress and recovery intensities coincide across noncentral firms, we can isolate centrality as the sole determinant of risk premia, because absent connectivity there would be no cross-sectional variation of expected returns at all. In this context, the next Proposition shows that Firm 1 has the highest risk premium when $N \to \infty$ and firms are not currently in distress, hence their risk premia are directly comparable.

Proposition 3. *Consider a 'Star' network economy where a distress of Firm 1, the central firm, increases the other distress intensities by a factor k. Assume that Assumption 1 and Assumption 2 in the Appendix are satisfied. There exists a k^*, dependent on firm characteristics, such that as N gets arbitrarily large and $k > k^*$, Firm 1 has a higher risk premium than any noncentral Firm N, conditional on any present state $\mathbf{H}_t$ where both firms 1 and N are not in distress.*

The intuition is that Firm 1's distress, by increasing other firms' chances of distress, leads the economy towards a low aggregate consumption state. Since firms accrue to distress when Firm 1 lays in it, the latter displays highly cyclical pay-outs. The only chance for the central firm of being less or equally exposed to the trough it creates, is (a) to have superior recovery ability and pay-off normal dividends while most are still trapped in distress and the discount factor (marginal utility) is large, or (b) to cause immediate distress propagation, in which case all firms have the same loading on this factor. Concerning the last observation, a key assumption of Proposition 3 is:[8]

$$k\lambda\left(1 - Nk\lambda\Delta - N\eta\Delta\right) > \left(1 - N\lambda\Delta - N\eta\Delta\right)\lambda \tag{9}$$

for small Δ. The left-hand-side of (9) denotes the approximate probability of distress and permanence in this state over the next small time interval, for a noncentral firm, when Firm 1 is in distress, and the economy size is large.[9] This has to be larger than its counterpart when Firm 1 is not in distress, for the latter to be riskier. Intuitively, an excessive strength of distress propagation – i.e. very large value of k – could not allow to distinguish significantly between central and noncentral firms, with regard to the correlation between their distress state and economic fundamentals (aggregate consumption, in our model). Indeed, with immediate propagation (infinite k) there cannot exist a state where the central firm is in distress and some other firm is not. On the other hand, insufficient propagation – i.e. $k < k^*$ – could also imply, for the opposite reason, economic fundamentals that are not significantly worse during Firm 1's distress compared to others'.

[8]See Assumption 2 in the Appendix. (9) reduces to expression (35).

[9]In which case the number of firms in distress or not is proportional to N.

V. An Indicator of Firm Centrality

The network literature has proposed several indicators to describe the connectivity structure of a static network. "Bonacich centrality "and "betweenness"are just a few examples.[10] We introduce a measure of dynamic centrality with the following properties: i) the measure captures centrality in the active connectivity, or distress causality sense, that we have discussed above as the most important network characteristic for expected returns. ii) The measure can be readily estimated and used in asset pricing tests.

Let $\mathcal{DC}_{ij}^{\tau}$ denote the τ-deferred cross-correlation of distress between firm i and firm j:

$$\mathcal{DC}_{ij}^{\tau} = \frac{P[H_{t+\tau}^{j} = 1, H_t^i = 1] - P[H_{t+\tau}^{j} = 1]P[H_t^i = 1]}{\sqrt{P[H_{t+\tau}^{j} = 1]P[H_t^i = 1](1 - P[H_{t+\tau}^{j} = 1])(1 - P[H_t^i = 1])}} \tag{10}$$

$\mathcal{DC}_{ij}^{\tau}$ is the unconditional correlation between the events that Firm i is in distress at time t and that Firm j is in distress τ periods afterwords.[11] The joint probability at the numerator can also be written as

$$P[H_{t+\tau}^{j} = 1, H_t^i = 1] = P[H_{t+\tau}^{j} = 1 | H_t^i = 1]P[H_t^i = 1] \tag{11}$$

$\mathcal{DC}_{ij}^{\tau}$ captures the distress causality of Firm i on Firm j, and it is naturally related to the statistical concept of exogeneity. In a two-firm economy, if $\mathcal{DC}_{ij}^{\tau} > \mathcal{DC}_{ji}^{\tau}$, Firm i demands a higher expected return, because its distress propagates to Firm j more systematically than the opposite. Firm i is 'actively' connected to the rest of the economy, and because of this property its distress status is strongly cyclical. It should be noted that although (10) is a pair-wise indicator, it takes into account the global network properties, as probabilities depend on the joint distribution of the vector $\mathbf{H}$. Moreover, since shocks in our economy

[10] We refer to the monograph Newman (2010) for an overview of the literature.

[11] Expressions for these probabilities are in Online Appendix A.

propagate dynamically, depending on the extent of amplification or absorption through the network structure, the $\mathcal{DC}_{ij}^{\tau}$ can vary sharply in the horizon dimension τ. The net deferred correlation

$$\mathcal{DC}_{ij}^{\tau} - \mathcal{DC}_{ji}^{\tau} \tag{12}$$

provides an indication of the mutual 'causality gap' between firms i over j. We define a stock's 'Dynamic Centrality' by aggregating at stock level all the net pairwise correlations :

$$\overline{\mathcal{DC}}_{i}^{\tau} = \sum_{j=1, j\neq i}^{N} \left(\mathcal{DC}_{ij}^{\tau} - \mathcal{DC}_{ji}^{\tau}\right) \tag{13}$$

A large $\overline{\mathcal{DC}}_{i}^{\tau}$ intuitively indicates a firm that is central in the network: its shocks are mainly transferred to other firms but it is less exposed to other firms' shocks. Consider for instance Figures 1a and 1b. In the former, since firms are disconnected and pairwise deferred correlations (10) are zero, Dynamic Centrality vanishes for all. The same is true for the latter if symmetry among firms is exact, and pairwise correlation measures (10) coincide. In Figure 2a, instead, Firm 1 has highest centrality, as $\mathcal{DC}_{1j}^{\tau} > 0$, while $\mathcal{DC}_{j1}^{\tau} = 0$.

VI. Empirical Analysis: Network Structure in Conditional Fama-McBeth Regressions

The theory links a firm characteristic, such as centrality in a network economy, to the cross-section of expected returns. In the previous section we have proposed an empirical measure of this characteristic. This Section explores the empirical relation between the cross-section of expected returns and Dynamic Centrality at the individual stock level.

A. Data and Portfolio Construction

We merge two main datasets. The first updates the Fama-French (1992) sample of portfolio returns double sorted according to market beta and size characteristics. We use these portfolios to populate the nodes in the network. The second dataset collects the earnings of each node (portfolio), which are used to estimate the network connectivity. We employ corporate earnings (Compustat EPSPX) rather than dividends in the estimation because the acknowledged smoothness of firms' pay-out policies may mask the propagation of fundamental shocks in the network. Further details are in the next Section.

We follow Fama and French (1992) and populate a set of 10×10 portfolios double sorted according to market beta and size (the "β-size" portfolios thereafter). The data consists of monthly stock returns on all firms listed on NYSE, AMEX and NASDAQ, with accounting data reported in the COMPUSTAT database from 1963 to 2007. Each year we discard the stocks in the first $5-th$ percentile of the size distribution, to avoid an excessive presence of small caps in the sample.[12] We form portfolios in June and then compute value-weighted returns from July to June of the next year. Betas of individual stocks are computed from a time series regression of excess returns on the market excess return, for 24 to 60 of the months preceding June of year t (included). The market equity value of individual stocks used in the size-sorting is recorded in June of year t. The monthly market returns and risk-free rate are from K. French.[13] As in Fama and French (1992), we use the time series of returns of a given portfolio to compute its beta, as the sum of the slopes in a time-series regression of excess returns on contemporaneous and 1-month lagged excess market returns. In the Fama-McBeth regressions to follow, each stock is assigned the beta of the portfolio to which it belongs in year t, and its book and market equity values available in December of year $t-1$. Thus portfolio construction and asset pricing tests are both designed to avoid any

[12]To this purpose, we use firms' market equity observed in June of the corresponding year.

[13]'F-F_Benchmark_Factors_Monthly' of K. French's website.

look-ahead (or contemporaneous) bias. Once stocks are assigned to portfolios, we also build a time series of earnings and dividends at the portfolio level, as detailed in Online Appendix B.

B. Estimation

We infer network connectivity from corporate earnings. To guarantee consistency with specification (2), we assume that earnings and dividend processes are connected by a similar network structure. We posit the following dynamics for earnings growth:

$$\frac{dE_t^i}{E_t^i} = u^i\, dU_t^i + d^i\, dL_t^i, \quad i = 1, \ldots, N. \tag{14}$$

u (d) denotes a positive (negative) shocks to earnings growth, with U_t^i (L_t^i) the corresponding Poisson counting process with intensity driven by the state-variable H_t^i:

$$\mathbb{E}\left[dU_t^i \middle| \mathcal{F}_t \right] = \left[\overline{\vartheta}_u^i (1 - H_t^i) + \underline{\vartheta}_u^i H_t^i \right] dt \tag{15}$$

$$\left(\mathbb{E}\left[dL_t^i \middle| \mathcal{F}_t \right] = \left[\overline{\vartheta}_d^i H_t^i + \underline{\vartheta}_d^i (1 - H_t^i) \right] dt \right) \tag{16}$$

Since $\overline{\vartheta}_j^i > \underline{\vartheta}_j^i$, $j = u, d$, a state of dividend distress implies smaller (larger) intensity of positive (negative) earnings growth.

We parameterize the network structure through the following linear specification for the state-dependent distress intensities of portfolios:

$$\lambda^i(\mathbf{H}_t) = \lambda_0^i + \lambda_1^i \sum_{j=1}^{N} c_j H_t^j \qquad i = 1, \ldots, N \tag{17}$$

where i is any of the beta-size sorted portfolios. Its distress intensity is λ_0^i when no firm is experiencing a distress, and it increases by $c_j \lambda_1^i$ upon distress of firm j. The recovery intensity η^i is assumed constant, hence unaffected by the network structure. Model (17) describes a multicentric network where all firms are connected, and the propensity of propagation, c, is firm-specific but not peer-specific. This specification is parsimonious enough to be estimated on the earnings dataset, and it is not restrictive, as we can think of c as the average propensity to distress propagation of firms living in a more heterogeneous network. We estimate model parameters with a Maximum Likelihood methodology where the distress state-variable $\mathbf{H}_t$ is integrated out using its stationary distribution. No asset price information is employed at this stage. Estimates of network parameters are used to compute the structural measure of dynamic centrality proposed in Section V. While the asset pricing tests below are conducted on a set of 10 × 10 beta-size sorted portfolios, the network is estimated on a set of 4 × 4 portfolios for feasibility of implementation.

C. Characteristics of Sorted Portfolios

Table I (Panel 1) reports the conditional ($\overline{\mathcal{DC}}_i^{\tau}$, $\tau = 1m$) Dynamic Centrality measures for each beta-size portfolio, arising from direct estimation, that is, plugging parameter estimates into expression (10).[14]

Insert Table I

According to this panel, the relation between centrality and both market beta and size is erratic and pronouncedly nonlinear, also due to the coarse stratification in the double sorting procedure. We explore the pattern by looking at the Dynamic Centrality of portfolios sorted into deciles of one characteristic at a time. Panel 2b reports the monthly ($\tau = 1$ month) and quarterly ($\tau = 3$ months) Dynamic Centrality of size-sorted deciles.[15] Bearing a technological network in mind, where a firm is the nexus of its input-output linkages, one would think of a larger cap firm as more central than a small cap. However, we find that

[14] Portfolio average returns are described in the Online Appendix.

[15] Since parameter estimates are not available on these portfolios, we obtain their centralities indirectly, by assigning to each stock the $\overline{\mathcal{DC}}_i^{\tau}$ of its beta-size sorted portfolio, and then computing an equally weighted average of the centralities in the same size decile: $\overline{\mathcal{DC}}_{ME-i}^{\tau}(t) = \sum_{j \in ME-i} \frac{\overline{\mathcal{DC}}_j^{\tau}}{n_t}$, where $ME - i$ is the $i-$th size decile, and n_t the number of stocks in sample in year t.

in a network connecting firms' fundamentals centrality has an important time dimension: at the monthly horizon, smallest caps appear more central than largest caps, while the opposite is true at quarterly horizon, with centrality differences between extreme deciles highly statistically significant in both cases. This result is intuitive: at short horizons, small firms are more suitable distress vehicles than larger firms, whose capital structure allows to avoid a quick transfer to connected firms, thus acting as temporary buffers to slow down distress propagation. At longer horizons instead, a small firm has more likely been recently created than survived to a durable distress, while any capital cushion of pre-existing large firms appears less adequate, which justifies the larger quarterly Dynamic Centrality of the latter group.

When we repeat the exercise on book-to-market sorted portfolios (Panel 2c), we find that highest book-to-market firms are undoubtedly more central than lowest ones at both monthly and quarterly horizons, as the difference of centralities is positive and highly statistically significant. A motivation of the value-premium known at least since Fama and French (1992), that value firms are consistent bad performers in periods of systematic downturns, is micro-founded by our model as a distress causality story: high book-to-market firms are the main catalysts of the systematic distress, because mostly prone to transfer a bad fundamental shock. We find intuitive that smaller centrality is associated to smaller book-to-market. If the market equity value embeds mainly growth perspectives, a firm's present distress is also less transferable to other firms, at least in the short run. Conversely, putting in place a growth project, especially financing it, requires tight links with other firms also in the short run, which implies that the firm is vulnerable to others' distress. According to this view the centrality gap between value and growth firms should be smaller at longer forecasting horizons, when part of growth projects are in place, so that the firm has developed 'outgoing connections' to transfer its distress. Indeed in Panel 2c the difference between quarterly centralities is 146% (in absolute terms) of the centrality of lowest book-to-market firms, in constrast with the 83% of the monthly horizon.

The next Section further explores this issue, but first it tests the relation between the centrality characteristic and the cross section of returns at the individual stock level.

D. Fama-MacBeth Regressions

We closely follow Fama and French (1992) and use the whole cross section of individual stock returns available on a given month. For monthly observations between July of year t and June of year $t+1$, we assign to a given stock its size (ME) and book-to-market (BE/ME) as reported on December of year $t-1$, and the post ranking beta of the portfolio to which it belongs: post-ranking betas are portfolio market betas obtained from a time series regression on the whole sample. The centrality matching procedure mimics the beta matching procedure, so that a stock is assigned the centrality measure of its portfolio. We use monthly Dynamic Centrality in this test. For each month, we run a cross-sectional regression of excess returns on market beta, size, book-to-market and Dynamic Centrality,[16] and report, in Panel 1 of Table II, time-series averages of slope coefficients and the corresponding t-statistics, obtained with time-series standard deviations of coefficients.

Insert Table II

After controlling for $\overline{\mathcal{DC}}^{\tau}$, the slopes of the size, book-to-market and beta characteristics are broadly consistent with those of Fama-French (1992). The slopes with respect to BE/ME and size are, respectively, positive and negative and both strongly significant across all specifications. The evidence of a positive market beta disappears after one controls for size. The slope of Dynamic Centrality is positive, meaning that more central stocks gain higher

[16]Namely, for each month s in year t, we consider the linear model:

$$R^i_{s+1} - r_{s+1} = \alpha_s + \theta^1_s \beta^i_t + \theta^2_s \log ME^i_t + \theta^3_s (BE/ME)^i_t + \theta^{\mathcal{DC}}_s \mathcal{DC}^i_t + \epsilon^i_{s+1}, \tag{18}$$

for all stocks i is sample in year t.

expected returns, and it is statistically significant at standard confidence levels. We have remarked that the positive relation between centrality and book-to-market suggests that the value premium may in part be a compensation for a pronounced tendency to transfer distress, which necessarily turns the firm into a cyclical performer. We further explore this interpretation by considering the 10 book-to-market sorted portfolios. To quantify the risk premium of these portfolios attributable to Dynamic Centrality, we denote by *cmp* a centrality factor,[17] defined as the return of a value-weighted portfolio long 25% of the stocks with largest centrality and short 25% with the smallest. We then regress portfolio excess returns on *cmp* and the Fama-French *smb* and market return factors:

$$R_t^i - r_t = \alpha^i + \beta_{mkt}^i (R_t^M - r_t) + \beta_{smb}^i smb_t + \beta_{cmp}^i cmp_t + \epsilon_t^i, \quad i = 1, \ldots, 10 \qquad (19)$$

Annual centrality premia, $\beta_{cmp}^i \overline{cmp}$,[18] range from a statistically insignificant 0.5% of the first book-to-market decile, to a significant 15.3% of the last. To test whether the value premium can also be interpreted as a centrality premium, we consider monthly returns of a portfolio long the last and short the first decile of the book-to-market distribution, and repeat regression (19) on them. Panel 2 of Table II reports the coefficients. The expected return of (most) value stocks in excess of (most) growth stocks is partly explained by a centrality premium component, with a highly significant and positive price of risk. Consistent with the cyclical nature of the premium, the market beta coefficient is positive and significant.

E. Robustness to Measurement Error

Since Dynamic Centrality is not directly observed, but obtained by plugging network parameter estimates in (13), there is the potential for statistical error to alter results.[19] To address

[17] *cmp*: ‘center minus periphery’.

[18] $\overline{cmp}$ is the sample average of the centrality factor.

[19] The error-in-variables problem for the CAPM beta in Fama-McBeth regressions is addressed in Shanken (1992).

this concern, we have simulated from the asymptotic distribution of parameter estimates, thus obtaining a distribution of centrality measures, and of corresponding slopes (and their t-stats) in the Fama-McBeth regression (18).[20] Table III reports summary statistics.

Insert Table III

The increasing relation between centrality and expected returns is robust to estimation error: there is less than 5% probability of obtaining a slope smaller than 0.24 (for a mean estimate of 0.54) and with a t-statistics smaller than 2.5.

VII. Conclusions

We study an economy where network links among firms' cash-flows generate cross-sectional predictability of returns. We interpret network connectivity as the ability to transfer a distress state to other firms' fundamentals in a directed and timely manner. Highly central firms, which actively determine the propagation of fundamental shocks in the economy, are pronouncedly cyclical and gain higher expected returns. We propose an easily implementable measure of network centrality, 'Dynamic Centrality', and use earnings data to estimate it. Consistent with the theoretical prediction, we find that central firms gain higher returns on average, and the positive price of risk of a centrality mimicking factor displays high cross-sectional heterogeneity. We also investigate how centrality relates to firm characteristics such as size and book-to-market. In this respect, we find that centrality helps to micro-found the distress-related nature of the value premium: part of the expected return of value stocks in excess of growth stocks is a centrality premium, earned to compensate for the pronounced ability of a firm to transfer distress, and contribute to turn it systematic.

[20]The Appendix contains further details of the procedure.

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Appendix: Proofs and Estimation Methodology

The notations OA.AX or OA.BX refer to equation number X reported in Online Appendix A or Online Appendix B, respectively. In what follows we drop functional arguments for the intensity processes when no confusion may arise, and denote them simply by λ_t and η_t.

Proof of Proposition 1. By market clearing, aggregate consumption reads $C_t \sum_{i=1}^{N} x_t^i$. Then, the consumption optimality condition of the representative agent implies the following equilibrium state price density, ξ_t:

$$\xi_t = e^{-\delta t} \left(\sum_{i=1}^{N} x_t^i \right)^{-\gamma} \tag{20}$$

As detailed in OA (expression (OA.A1) and thereafter), this implies the following expressions for the equilibrium interest rate (r_t) and market price of distress/recovery risk (θ_t):

$$\begin{aligned} r_t &= \delta - \sum_{i=1}^{N} \left\{ H_t^i \left[1 - \left(\frac{\overline{x}^i + \sum x_{t-}}{\underline{x}^i + \sum x_{t-}} \right)^{-\gamma} \right] \eta_t^i + (1 - H_t^i) \left[1 - \left(\frac{\underline{x}^i + \sum x_{t-}}{\overline{x}^i + \sum x_{t-}} \right)^{-\gamma} \right] \lambda_t^i \right\} && (21) \\ \theta_t^i &= H_t^i \left(\frac{\overline{x}^i + \sum x_{t-}}{\underline{x}^i + \sum x_{t-}} \right)^{-\gamma} + (1 - H_t^i) \left(\frac{\underline{x}^i + \sum x_{t-}}{\overline{x}^i + \sum x_{t-}} \right)^{-\gamma} \qquad i = 1, 2, \ldots N, && (22) \end{aligned}$$

where $\sum x_{t-}$ denotes the sum of trees' dividend excluding i at time $t-$.

Let H denote the current vector of distress (or not) state for the trees. Given the equilibrium state-price density ξ_t as in (20), the absence of arbitrage opportunities implies:

$$\begin{aligned} \frac{P^i(H)}{x_t^i} &= \frac{1}{\xi_t} \mathbb{E} \left[\int_t^\infty \xi_s \frac{x_s^i}{x_t^i} ds \middle| \mathcal{F}_t \right] \\ &= \frac{\widehat{P}^i(H)}{x_t^i \left(\sum_{j=1}^{N} x_t^j \right)^{-\gamma}}; \qquad \widehat{P}^i(H) = \mathbb{E} \left[\int_t^\infty e^{-\delta(s-t)} x_s^i \left(\sum_{j=1}^{N} x_s^j \right)^{-\gamma} ds \middle| \mathcal{F}_t \right] && (23) \end{aligned}$$

Online Appendix A (expression (OA.A9) and thereafter) derives an explicit form for the vector $\widehat{\mathbf{P}}^i = [\ldots, \widehat{P}^i(H), \ldots]'$, containing all 2^N realizations of $\widehat{P}^i(H)$:

$$\widehat{\mathbf{P}}^i = (\mathbf{a} - \mathbf{A^H})^{-1} \widetilde{\mathbf{C}}^i \tag{24}$$

$\mathbf{a}$ is a diagonal matrix with δ on the main diagonal. $\mathbf{A^H}$ is the transition matrix of the multidimensional

Markov chain $\mathbf{H}_t = (H_t^1, H_t^2, \ldots, H_t^N)'$. $\widetilde{\mathbf{C}}^i$ is the 2^N vector of dividends paid in each state, discounted by the marginal utility. According to (23), the vector of P/D ratios for all states H is then:

$$\mathbf{V}^i = (\mathbf{a} - \mathbf{A}^{\mathbf{H}})^{-1}\mathbf{C}^i; \qquad \mathbf{C}^i = \frac{\widetilde{\mathbf{C}}^i}{x_t^i \left(\sum_{j=1}^N x_t^j\right)^{-\gamma}} \tag{25}$$

□

Proof of Proposition 2. From (OA.A4), $\theta_t^i - 1$ is the market price of tree i's risk of dividend growth jumps, either distress or recoveries, depending on i's present state. $\lambda^i\theta_t^i$ and $\eta^i\theta_t^i$ are the risk-neutral intensities of distress and recovery. Let H denote the present realization of $\mathbf{H}_t$. To find the conditional risk premium of equity i, namely

$$\mu_t^i = \mathbb{E}\left[\left.\frac{dP^i(H)}{P^i(H)}\right| \mathcal{F}_t\right] + \frac{x_t^i}{P^i(H)} - r_t, \tag{26}$$

we apply Ito's lemma to the martingale $M_t^i = \xi_t P^i(H) + \int_0^t \xi_s x_s^i ds$, taking into account expression (OA.A4) for the state-price density. We obtain:

$$\begin{aligned} dM_t^i &= \xi_t x_t^i dt + \xi_t P^i(H) m_t^i dt - \xi_t P^i(H) r_t dt \\ &\quad - \sum_{j=1}^N H_t^j \left[\theta_t^j \xi_t P^i(H^{+j}) - \xi_t P^i(H)\right](-dH_t^j) + \sum_{j=1}^N (1 - H_t^j)\left[\theta_t^j \xi_t P^i(H^{-j}) - \xi_t P^i(H)\right] dH_t^j \end{aligned} \tag{27}$$

m_t^i denotes equity i's instantaneous expected return $\mathbb{E}[dP^i/P^i|\mathcal{F}_t]$. H^{-j} (H^{+j}) is the realization of $\mathbf{H}$ to which the present state H jumps if tree j has a distress (recovery). Dividing both sides of (27) by $\xi_t P^i(H)$, taking expectations and recalling that the martingale property of M_t^i implies that the drift component of (27) must vanish, we obtain:

$$\mu_t^i = m_t^i + \frac{x_t^i}{P^i(H)} - r_t = -\sum_{j=1}^N H_t^j \left[\theta_t^j \frac{P^i(H^{+j})}{P^i(H_t)} - 1\right]\eta_t^j - \sum_{j=1}^N (1 - H_t^j)\left[\theta_t^j \frac{P^i(H_t^{-j})}{P^i(H)} - 1\right]\lambda_t^j \tag{28}$$

The RHS of (28) coincides with the expression reported in the Proposition. The gross return on security i triggered, respectively, by a distress or a recovery of tree j, reads explicitly:

$$\frac{P^i(H^{-j})}{P^i(H)} = \frac{\bar{1}'_{H^{-j}}\mathbf{V}^i}{\bar{1}'_H \mathbf{V}^i}\frac{x^i(H_t^{-j})}{x^i(H_t)} \tag{29}$$

$$\frac{P^i(H^{+j})}{P^i(H)} = \frac{\bar{1}'_{H^{+j}}\mathbf{V}^i}{\bar{1}'_H \mathbf{V}^i}\frac{x^i(H_t^{+j})}{x^i(H_t)} \tag{30}$$

where the vector of conditional P/D ratios of security i, $\mathbf{V}^i$, is reported in expression (25), and the notation $x^i(\cdot)$ emphasizes the dependence on the state of the dividend x^i. $\square$

We only report the assumptions and a sketch of the proofs of Proposition 3, referring to Online Appendix A for the detailed derivations.

Assumptions and Sketch of the proof for Proposition 3.

Assumption 1. *The dividend processes are homogeneous across assets, and they are deterministic functions of the economy size N:*

$$x_t^i(H) = \begin{cases} \overline{f}(N) & \text{if } H_t^i = 0 \\ \underline{f}(N) & \text{if } H_t^i = 1 \end{cases} \qquad i = 1, \dots N \tag{31}$$

Moreover

$$\lim_{N\to\infty} \left(\frac{\sum_{j=1}^N x_t^j(H)}{\sum_{j=1}^N x_t^j(\mathbf{H}_t)} \right)^{-\gamma} \frac{x_t^i(H)}{x_t^i(\mathbf{H}_t)} = c(H, \mathbf{H}_t) \tag{32}$$

with $0 < c(H, \mathbf{H}_t) < \infty$, for all possible states H.

Assumption 2. *Let $\mathbf{H^1}$ denote the collection of states where firm 1 is in distress, and $\overline{\mathbf{H^1}}$ the states where it is not. Then:*

i) $\lambda^j(\mathbf{H}^1) = k\lambda$ *and* $\lambda^j(\overline{\mathbf{H}^1}) = \lambda$, $j = 2, 3, \dots, N$, *with* $k > 1$.

ii) $\eta^j(\overline{\mathbf{H}^1}) = \eta^j(\mathbf{H}^1) = \eta$.

Intensity parameters depend on economy size N, in such a way that total distress and recovery risk are bounded as $N \to \infty$:

$$\begin{aligned} \lim\nolimits_{N\to\infty} N\lambda &= K^\lambda < \infty \\ \lim\nolimits_{N\to\infty} N\eta &= K^\eta < \infty \end{aligned} \tag{33}$$

which implies

$$\lim_{N\to\infty} \lambda = \lim_{N\to\infty} \eta = 0 \tag{34}$$

The centrality parameter k satisfies the condition:

$$1 - K^\lambda (k+1)\Delta - K^\eta \Delta > 0 \tag{35}$$

for small Δ.

Assumptions 1 simplifies the asymptotic behavior of dividend shares while insuring finite price-dividend ratios, thus allowing us to focus on distress connectivity. Assumption 2 introduces the structure of the network, where distress of the central firm (Firm 1) increases all others' distress intensities by a factor k. Condition (33) guarantees finite asymptotic asset prices and risk premia, while condition (35) is a balance condition which, as discussed in the text, guarantees that firm 1 is more exposed to its distress risk, by limiting the extent of distress propagation. The homogeneity assumption about dividends marginalizes the role of dividend share size and allows network connectivity be the sole determinant of risk premia. We outline the steps of the proof.

- The risk premium μ_t^i of any firm i can be expressed as:

$$-\left[\mu_t^i - \sum_{j=1}^{N} \tilde{\lambda}^j\right] P^i(\mathbf{H}_t) = \sum_{j=1}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-a(t_u - t)} \sum_{\mathbf{H}_{t_u}} Prob\left(\mathbf{H}_{t_u} | \mathbf{H}_t^{\pm j}\right) \mathcal{A}^i(\mathbf{H}_{t_u}) \tag{36}$$

  where $\mathcal{A}^i(H)$ denotes the entry of $\mathcal{A}^i = \mathbf{AC}^i$ corresponding to state H: the (marginal utility) discounted dividend paid in state H. $Prob\left(\mathbf{H}_{t_u} | \mathbf{H}_t^{\pm j}\right)$ is the probability of reaching $\mathbf{H}_{t_u}$ at time t_u conditional on state $\mathbf{H}_t^{\pm j}$ at time t. $\mathbf{H}_t^{\pm j}$ is the realization to which H_t moves from realization $\mathbf{H}_t$ (the initial state) after a distress $(-)$ of a recovery $(+)$ of firm j. $\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)$ is the risk neutral intensity of distress or recovery (depending on the current state $\mathbf{H}_t$) of firm j.

- Letting $\mathcal{R}^i$ denote the RHS of (36), we want to show that the assumptions above are sufficient for $\lim_{N\to\infty} \mathcal{R}^N - \mathcal{R}^1 \geq 0$, where N is any noncentral firm,[21] provided that neither N nor 1 are in distress in the initial state $\mathbf{H}_t$. In light of (36), this implies that $\lim_{N\to\infty} \mu_t^N - \mu_t^1 \leq 0$.

- To this end, note that

$$\mathcal{R}^N - \mathcal{R}^1 = \sum_{j=1}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-a(t_u - t)} \sum_{\mathbf{H}_{t_u}} Prob\left(\mathbf{H}_{t_u} | \mathbf{H}_t^{\pm j}\right) [\mathcal{A}^N(\mathbf{H}_{t_u}) - \mathcal{A}^1(\mathbf{H}_{t_u})] \tag{37}$$

  We consider all possible states $\mathbf{H}_{t_u}$ at future times t_u. According to Lemma 1 in the Online Appendix, the only nonzero terms in the last summation of (37) correspond to states $\mathbf{H}_{t_u}$ where firm 1 and N are not both in distress or both not in distress.

- Since in all nonzero terms the firms cannot have the same (distress) state, we can partition the last

[21] We use the same letter N to denote any noncentral firm and the number of firms in the economy. To which of the two we refer is implied by the context.

summation of (37) into pairwise sums of the form:

$$Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right)\left[\mathcal{A}^N(H^1) - \mathcal{A}^1(H^1)\right] + Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\left[\mathcal{A}^N(\overline{H}^1) - \mathcal{A}^1(\overline{H}^1)\right] \quad (38)$$

State H^1 and $\overline{H}^1$ coincide, with the exception that firm 1 (N) is in distress in the former (latter) and it is not in the latter (former).

- The purpose of Lemma 1 in the Online Appendix is to show that (38) is positive for $N \to \infty$ if the assumptions above hold. With the addition of for a few technical details, this fact proves the claim of the Proposition.

□

Estimation procedure. Index i refers henceforth to one of the $N = 16$ beta-size sorted portfolios.

The portfolio earnings process, denoted by E_t^i, is assumed to follow a two-state Markov chain, with states $(\overline{E}^i, \underline{E}^i)$, $\overline{E}^i > \underline{E}^i$, so that the following stochastic differential equation describes its dynamics:

$$dE_t^i = -(\overline{E}^i - \underline{E}^i)(1 - H_t^i)dH_t^i - (\overline{E}^i - \underline{E}^i)H_t^i dH_t^i \qquad i = 1, \dots, N \quad (39)$$

The distress/recovery indicator, H_t^i, is the same two-state continuous-time Markov chain driving the dividend process of firm i, x_t^i.[22] As mentioned in the text, we embed the network structure in the following state-dependent form of distress transition intensities:

$$\lambda_t^i = \lambda^i(H_t^1, \dots, H_t^N) = \lambda_0^i \left(1 + \sum_{j=1}^{N} c_j H_t^j\right), \qquad i = 1, \dots, N. \quad (40)$$

We denote by ϵ_t^i the observation error on the earnings of portfolio i at time t. E_t^i and its empirical counterpart, $\widehat{E}_t^i$, are then related by the measurement equation:

$$\widehat{E}_t^i = E_t^i + \epsilon_t^i \qquad i = 1, \dots, N \quad (41)$$

with $\epsilon_t^i \sim NID(0, \sigma_i)$. The observation errors are cross-sectionally independent, so that

$$(\epsilon_t^1, \epsilon_t^2, \dots, \epsilon_t^N) \sim \prod_{i=1}^{N} \phi(\epsilon_t^i | \sigma_i) \quad t = 1, \dots, T \quad (42)$$

[22] We remind in particular that in case of recovery jump we have $H_t^i dH_t^i = -1$.

where $\phi(\cdot|\sigma_i)$ is the Normal centered probability density function with variance σ_i^2, and $T = 177$ denotes the number of quarterly observations available, from January 1963 to December 2007. We estimate the parameters vector $\theta = (\lambda_0^i, \eta^i, \overline{E}^i, \underline{E}^i, c_i)'$, $i = 1, \ldots, N$, similarly to the Simulated Maximum Likelihood method of Brandt and Santa Clara (2002), namely we maximize the following unconditional likelihood criterion:

$$\theta^* = \arg\max_\theta \quad \mathbb{E}\left[\sum_{t=1}^{T}\sum_{i=1}^{N} \log\phi(\widehat{E}_t^i - E_t^i|\theta)\right] \tag{43}$$

The expectation is with respect to the stationary distribution of the Markov Chain that governs E_t^i in (39). We approximate it by Monte Carlo simulation as follows: i) given some parameter set θ, we simulate an earnings trajectory $E_t^i(\omega)$ of length $np \times T$, for large np, by sampling the next jump time τ_t^i after time t from an exponential distribution with the current intensity as parameter. Since all intensities change at $\min_i \tau_t^i$, we resample the next jump times at this instant. ii) We build an extended sample $\widehat{E}_t^i$ of length $np \times T$, by concatenating np copies of the original one, and finally set:

$$\mathbb{E}\left[\sum_{t=1}^{T}\sum_{i=1}^{N} \log\phi(\widehat{E}_t^i - E_t^i|\theta)\right] \approx \frac{1}{np}\sum_{t=1}^{np\times T}\sum_{i=1}^{N} \log\phi(\widehat{E}_t^i - E_t^i(\omega)|\theta). \tag{44}$$

□

Parameter standard errors are obtained with the standard asymptotic theory for ML estimators, hence we illustrate them in Online Appendix B for brevity.

Robustness of slope coefficient to $\mathcal{DC}$ estimation error. We compute Dynamic Centrality $\overline{\mathcal{DC}}_i^{1m.}$, as defined in (13), using the Maximum Likelihood estimate θ^* of the parameter vector (reported in Table V of the Online Appendix). We want to assess whether the main conclusions of the paper are affected by the estimation error of the centrality indicator. To this end, we test the robustness of the (average) slope of $\overline{\mathcal{DC}}_i^{1m.}$ in the Fama-McBeth regression, using a simulation procedure. We extract $N = 1500$ independent samples of the parameter set from its asymptotic distribution:

$$\hat{\theta}^i \quad = \quad \theta^* + \sqrt{\text{diag}[I^{-1}(\theta^*)]} \odot \epsilon^i \quad i = 1, \ldots, N \tag{45}$$

where the Fisher information matrix $I(\theta^*)$ is obtained as detailed in Online Appendix B, and ϵ^i is a standard Normal random vector. For each parameter sample $\hat{\theta}^i$ we obtain the corresponding firm Dynamic Centralities measures plugging $\hat{\theta}^i$ into (13), and repeat the Fama-McBeth regression, thus obtaining an empirical distribution of slopes and Fama-McBeth t-statistics implied by parameters' estimation error. Summary statistics of these distributions are in Table III.

Table I – Model (39)-(40) is estimated on the earnings of beta-size sorted portfolios, using the maximum likelihood procedure described in the Appendix. *Panel 1* reports the 1-month Dynamic Centrality measures (multiplied by a factor of 100) obtained from the Maximum Likelihood parameter estimates (reported in the Online Appendix). *Panel 2* reports Dynamic Centrality measures for extreme portfolio deciles of the size and then book-to-market distribution. Since parameter estimates are not available on these portfolios, we obtain their centralities indirectly, by assigning to each stock the $\overline{\mathcal{DC}}_i^{\tau}$ of its beta-size sorted portfolio, and then computing an equally weighted average of the centralities in the same size or book-to-market decile: $\overline{\mathcal{DC}}^{\tau}_{ME-i}(t) = \sum_{j \in ME-i} \frac{\overline{\mathcal{DC}}_j^{\tau}}{n_t}$, where $ME-i$ is the $i-$th size (or book-to-market) decile, and n_t the number of stocks in sample in year t.

Panel 1. 1-month Dynamic Centrality: beta-size sorted portfolios

	$\beta-1$	$\beta-2$	$\beta-3$	$\beta-4$
ME- 1	0.108	0.127	-0.003	0.001
ME- 2	-0.278	-0.047	-0.048	0.013
ME- 3	0.244	0.045	-0.061	-0.131
ME- 4	0.369	-0.228	0.096	-0.206

Panel 2. 1-month and 1-quarter Dynamic Centrality: size sorted and B/M sorted portfolios

	$ME-1$	$ME-10$	Δ	$B/M-1$	$B/M-10$	Δ
$\tau = 1\,m.$	0.0060	0.0013	-0.0047	-0.0028	0.0013	0.0041
	(85.76)	(4.17)	(-14.10)	(-8.03)	(7.22)	(10.42)
$\tau = 3\,m.(\times 1000)$	0.0995	0.297	0.198	-0.072	-0.011	0.060
	(65.15)	(12.76)	(8.58)	(-3.94)	(-1.40)	(3.03)

Table II – *Panel 1.* Fama-McBeth regressions results. The cross-sectional regression (18) of individual stock returns on beta, size, book-to-market, and dynamic centrality is run for each month in the sample, and time series averages of slopes are reported. Standard errors of slopes are in parenthesis. *Panel 2* reports results of the following regression: $R_t^{10} - R_t^1 = \alpha + \beta_{mkt}(R_t^M - r_t) + \beta_{smb} smb_t + \beta_{cmp} cmp_t + \epsilon_t$. On the left-hand-side: the returns of a value-weighted portfolio long the last decile of the book-to-market distribution and short the first. On the right-hand-side: the market excess return, the *smb* size factor, the centrality factor, defined as return of a value-weighted portfolio long the last and short the first quartile of the distribution of stocks with respect to $\mathcal{DC}^{1m.}$. We remind that each stock has the $\mathcal{DC}^{1m.}$ of the 4×4 beta-size sorted portfolio to which it belongs. T-statistics of the coefficients are in parenthesis.

Panel 1. Average Slopes of Fama-McBeth regressions
July 1963-June 2008

β	$\log(ME)$	BE/ME	$\overline{\mathcal{DC}}^{1m.}$	R^2
0.0021	-0.0023	0.0032	0.6205	5.02
(0.792)	(-5.79)	(7.765)	(4.77)	(10.25)

Panel 2. Long-short portfolio of BE/ME sorted deciles

α	β_{mkt}	β_{sml}	$\beta_{\mathcal{DC}}$	R^2
-0.003	0.335	-0.177	0.339	4.97%
(-0.42)	(1.99)	(-0.90)	(5.15)	

Table III – The simulation procedure described in the Appendix ('Robustness to estimation error'), produces an empirical distribution (implied by parameters estimation error) of $\mathcal{DC}^{1m.}$ slopes and their t-stats in the Fama-McBeth regression (18). The table reports summary statistics of these distributions.

Monte-Carlo slopes and t-statistics of Fama-McBeth regressions
adjusted for error-in-variables, July 1963-June 2008

	Slope $\overline{\mathcal{DC}}^{1m}$	$t-$stat
Mean	0.5381	4.3260
Median	0.5135	4.4179
Std	0.2124	1.0278
5-95%-*iles*	0.2408-0.9365	2.5837-5.9128

Online Appendix:
Dynamic Networks and Asset Pricing

This Online Appendix collects the proofs and the auxiliary results which are not included in the Appendix to the main text of the paper. It comprises an Online Appendix A, B, C, and D. The second is devoted to the estimation methodologies of the empirical Sections, the third to the failure of the asymptotic two-fund separation property, the fourth collects additional tables.

Propositions, lemmas, and equation numbers are prefixed with the letter that identifies the appendix. Numbers without prefix refer to propositions, Lemmas, or equations in the main text.

Online Appendix A

In what follows we drop functional arguments for the intensity processes when no confusion may arise, and denote them simply as λ_t and η_t.

The equilibrium interest rate and market prices of risk. According to the optimality conditions for the representative agent, the equilibrium state price density, ξ_t, is:

$$\xi_t = e^{-\delta t}\left(\sum_{i=1}^{N} x_t^i\right)^{-\gamma} \tag{OA.A1}$$

On the other hand, for any security price P_t^i adapted to $\mathcal{F}_t$, including the risk-less bond, the cum-dividend discounted price process $(P_t^i\xi_t+\int_0^t \xi_s x_s^i ds)$ is a martingale. Applying Ito's lemma to this expression, the martingale property implies that ξ_t must also obey:

$$\xi_t = \exp\left(-\int_0^t r_s ds - \int_0^t \sum_{i=1}^{N} \widehat{\lambda}_s^i(1-\theta_s^i)ds + \int_0^t \sum_{i=1}^{N} -\log(\theta_s^i)\text{sgn}(H_t^i)dH_s^i\right) \tag{OA.A2}$$

where $\text{sgn}(H_t^i) = -1$ if $H_t^i \leq 0$ and $\text{sgn}(H_t^i) = 1$ if $H_t^i > 0$. Furthermore, θ_t^i is the market price of event risk for tree i - distress risk, if tree i is in not in distress, i.e. $H_t^i = 0$, recovery risk if tree i is in distress, i.e. $H_t^i = 1$, and

$$\widehat{\lambda}_t^i = H_t^i\eta^i + (1-H_t^i)\lambda_t^i. \tag{OA.A3}$$

By applying Ito's lemma for jump processes to (OA.A2), we obtain:

$$d\xi_t = -\xi_t r_t dt + \xi_t\left[\sum_{i=1}^{N}(\theta_s^i - 1)(-\text{sgn}(H_t^i)dH_t^i - \widehat{\lambda}_t^i)\right] \tag{OA.A4}$$

By Ito's lemma for jump processes applied instead to (OA.A1), we obtain the alternative representation:

$$\begin{aligned} d\xi_t = -\delta\xi_t - \xi_t\sum_{i=1}^{N}\Bigg[(1-H_t^i)\frac{\left[(\underline{x}^i+\sum x_{t-})^{-\gamma}-(\overline{x}^i+\sum x_{t-})^{-\gamma}\right]}{(\overline{x}^i+\sum x_{t-})^{-\gamma}}\lambda_t^i+ \\ H_t^i\frac{\left[(\overline{x}^i+\sum x_{t-})^{-\gamma}-(\underline{x}^i+\sum x_{t-})^{-\gamma}\right]}{(\underline{x}^i+\sum x_{t-})^{-\gamma}}\eta_t^i\Bigg] + \xi_t\sum_{i=1}^{N}\Bigg[(1-H_t)\frac{\left[(\underline{x}^i+\sum x_{t-})^{-\gamma}-(\overline{x}^i+\sum x_{t-})^{-\gamma}\right]}{(\overline{x}^i+\sum x_{t-})^{-\gamma}}(dH_t^i-\lambda_t^i)+ \\ H_t^i\frac{\left[(\overline{x}^i+\sum x_{t-})^{-\gamma}-(\underline{x}^i+\sum x_{t-})^{-\gamma}\right]}{(\underline{x}^i+\sum x_{t-})^{-\gamma}}(-dH_t^i-\eta_t^i)\Bigg] \end{aligned} \tag{OA.A5}$$

$\sum x_{t-}$ denotes the sum of dividends across trees, excluding i, an instant before the jump of i takes place. Matching the coefficients of expression to (OA.A5) to those of expression (OA.A4), we obtain the equilibrium interest rate and market prices of risk:

$$r_t = \delta - \frac{1}{2}\gamma(\gamma+1)\sigma_Y^2 + \sum_{i=1}^{N}\left\{H_t^i\left[1-\left(\frac{\overline{x}^i+\sum x_{t-}}{\underline{x}^i+\sum x_{t-}}\right)^{-\gamma}\right]\eta_t^i + \right. \tag{OA.A6}$$

$$\left.(1-H_t^i)\left[1-\left(\frac{\underline{x}^i+\sum x_{t-}}{\overline{x}^i+\sum x_{t-}}\right)^{-\gamma}\right]\lambda_t^i\right\} \tag{OA.A7}$$

$$\theta_t^i = H_t^i\left(\frac{\overline{x}^i+\sum x_{t-}}{\underline{x}^i+\sum x_{t-}}\right)^{-\gamma} + (1-H_t^i)\left(\frac{\underline{x}^i+\sum x_{t-}}{\overline{x}^i+\sum x_{t-}}\right)^{-\gamma} \quad i=1,2,\ldots N \tag{OA.A8}$$

Detailed derivation of equilibrium asset prices. We need to determine the process $\widehat{P}^i(H)$ of (20) in the the Appendix. Since the process

$$\int_0^t e^{-\delta s}x_s^i\left(\sum_{j=1}^{N}x_s^i\right)^{-\gamma}ds + e^{-\delta t}\widehat{P}^i(H) \tag{OA.A9}$$

is an $\mathcal{F}_t-$martingale, it has no predictable component, therefore applying Ito's lemma to (OA.A9) and taking conditional expectations, the resulting expression must vanish. Using the notation $\widehat{\lambda}_t^i = H_t^i\eta^i + (1-H_t^i)\lambda_t^i$, we obtain:

$$\left(-\delta - \sum_{j=1}^{N}\widehat{\lambda}_t^j\right)\widehat{P}^i(H) + \sum_{j=1}^{N}\widehat{\lambda}_t^j\widehat{P}^i(H^{\pm j}) + x_t^i\left(\sum_{j=1}^{N}x_t^j\right)^{-\gamma} = 0 \tag{OA.A10}$$

The current distress (or not) state for the economy, H, moves to the state H^{+j} if tree j recovers from a distress, or to H^{-j} if it experiences a distress. Clearly equation (OA.A10) is solved jointly to the equations satisfied by the functions $\widehat{P}^i(H^{\pm j})$. We can write the resulting linear system of equations in vector form:

$$\left(\mathbf{a}-\mathbf{A^H}\right)\widehat{\mathbf{P}}^i - \widetilde{\mathbf{C}}^i = 0 \tag{OA.A11}$$

$\widehat{\mathbf{P}}^i = [\ldots,\widehat{P}^i(H),\ldots]'$ contains functions $\widehat{P}^i(\cdot)$ conditional on all 2^N possible states H.[1] Similarly $\widetilde{\mathbf{C}}^i = [\ldots,x^i(H)\left(\sum_{j=1}^{N}x^j(H)\right)^{-\gamma},\ldots]'$ contains all conditional (persistent) dividends discounted by the marginal utility of aggregate consumption. $\mathbf{a}$ is a $2^N\times 2^N$ diagonal matrix of δs. $\mathbf{A^H}$ is the Markov transition matrix of the system formed by all dividends' persistent components. From (OA.A11):

$$\widehat{\mathbf{P}}^i = \left(\mathbf{a}-\mathbf{A^H}\right)^{-1}\widetilde{\mathbf{C}}^i \tag{OA.A12}$$

Expressions of probabilities in distress correlations (10). We have denoted by $\mathbf{A^H}$ the transition matrix of the $N-$dimensional Markov chain $(H^1,H^2,\ldots,H^N)$. The 2^N vector of steady state probabilities implied by $\mathbf{A^H}$ solves:

$$\pi' = \pi'\exp\left(-\mathbf{A^H}\right) \tag{OA.A13}$$

We obtain π numerically by iterating equation (OA.A13) until a fixed point is reached within small tolerance. To

[1] Of course not all of them are mutually reachable, because at most one of the trees can fall in distress or recover at some time instant.

obtain the unconditional probabilities $P[H^j_{t+\tau}=1]$ ($=P[H^j_t=1]$) and $P[H^i_t=1]$ we sum the entries of π over all the states where j is in distress. The conditional probability $P[H^j_{t+\tau}=1|H^i_t=0]$ is given by the standard solution of the Chapman-Kolmogorov equations:

$$P[H^j_{t+\tau}=1|H^i_t=0]=\mathbf{I}_i'\exp\left(-\mathbf{A}^{\mathbf{H}}\tau\right)\mathbf{I}_j \tag{OA.A14}$$

$\mathbf{I}_j$ is a 2^N vector with ones for the combinations of $(H^1,H^2,\ldots,H^N)$ where tree j is in distress and zero elsewhere. $\mathbf{I}_i$ is similarly defined.

Proof of Proposition 3. Let $\mathbf{H}_t$ denote a state where firms 1 and N are not in distress ($H^1_t=H^N_t=0$). Let Δ be a small time interval. We denote by $\mathcal{P}^i_S(\Delta,\mathbf{H}_t)$ the price in state $\mathbf{H}_t$ of the claim to dividends of firm i paid until time $t+\Delta$, evaluated at time t: the dividend strip that expires in $t+\Delta$.

The price of the dividend strip can be found along the lines of the proof of Proposition 1. We have

$$\begin{aligned}
\mathcal{P}^i_S(\Delta,\mathbf{H}_t) &= \frac{1}{\left(\sum_{j=1}^N x^j_t\right)^{-\gamma}}\mathbb{E}\left[\int_t^{t+\Delta}e^{-\delta(s-t)}x^i_s\left(\sum_{j=1}^N x^i_s\right)^{-\gamma}ds\,\middle|\,\mathcal{F}_t\right] &\text{(OA.A15)}\\
&= \overline{\mathbf{1}}'(\mathbf{H}_t)\int_t^{t+\Delta}\exp(-(\mathbf{a}-\mathbf{A}^{\mathbf{H}})(s-t))ds\,\mathbf{C}^i &\text{(OA.A16)}
\end{aligned}$$

$\exp(\,\cdot\,)$ denotes the matrix exponential. $\overline{\mathbf{1}}(\mathbf{H}_t)$ is 2^N-dimensional column vector with 1 in the entry corresponding to state $\mathbf{H}_t$ and zeros otherwise. Since Δ is small, we can also write:

$$\begin{aligned}
\mathcal{P}^i_S(\Delta,\mathbf{H}_t) &\approx \overline{\mathbf{1}}'(\mathbf{H}_t)\exp(-(\mathbf{a}-\mathbf{A}^{\mathbf{H}})\Delta)\,\mathbf{C}^i\Delta &\text{(OA.A17)}\\
&\approx \overline{\mathbf{1}}'(\mathbf{H}_t)\mathbf{A}\,\mathbf{C}^i\Delta
\end{aligned}$$

where $\mathbf{A}=[I-(\mathbf{a}-\mathbf{A}^{\mathbf{H}})\Delta)]$. We can think of the stock price (an infinite maturity dividend strip) as an infinite sum of prices of forward-start dividend strips:

$$\begin{aligned}
P^i_S(\mathbf{H}_t) &= \frac{1}{\left(\sum_{u=1}^N x^u_t\right)^{-\gamma}}\mathbb{E}\left[\sum_{j=0}^{\infty}e^{-\delta(t_j-t)}\left(\sum_{u=1}^N x^u_{t_j}\right)^{-\gamma}\mathcal{P}^i_S(\Delta,\mathbf{H}_{t_j})\,\middle|\,\mathcal{F}_t\right]\\
&= \frac{1}{\left(\sum_{j=1}^N x^j_t\right)^{-\gamma}}\left[\sum_{j=0}^{\infty}\overline{\mathbf{1}}'(\mathbf{H}_t)\exp(-(\mathbf{a}-\mathbf{A}^{\mathbf{H}})(t_j-t))\int_{t_j}^{t_j+\Delta}\exp(-(\mathbf{a}-\mathbf{A}^{\mathbf{H}})(s-t_j))ds\,\mathbf{C}^i\right]\\
&\approx \frac{1}{\left(\sum_{j=1}^N x^j_t\right)^{-\gamma}}\left[\overline{\mathbf{1}}'(\mathbf{H}_t)\mathbf{A}\,\left(\mathbf{C}^i+\mathbf{A}\mathbf{C}^i+\mathbf{A}\mathbf{A}\mathbf{C}^i+\ldots\right)\Delta\right]
\end{aligned}$$

where $t_0=t$ and $t_j-t_{j-1}=\Delta$.

We use the notation $x^i(\mathbf{H}_t)$ to denote the dividend paid by firm i in the realization $\mathbf{H}_t$ of $\mathbf{H}$. As we are going to consider limiting behaviors for the number of firms N that grows unboundedly, we impose the following assumptions.

Assumption 1. *Dividends are homogeneous across assets, and they are deterministic functions of the economy* *size N:*

$$x_t^i(H) = \begin{cases} \overline{f}(N) & \text{if } H_t^i = 0 \\ \underline{f}(N) & \text{if } H_t^i = 1 \end{cases} \qquad i = 1, \dots N \tag{OA.A18}$$

Moreover

$$\lim_{N\to\infty} \left(\frac{\sum_{j=1}^N x_t^j(H)}{\sum_{j=1}^N x_t^j(\mathbf{H}_t)} \right)^{-\gamma} \frac{x_t^i(H)}{x_t^i(\mathbf{H}_t)} = c(H, \mathbf{H}_t) \tag{OA.A19}$$

with $0 < c(H, \mathbf{H}_t) < \infty$*, for all possible states* H*.*

We have emphasized the dependence of the limits on the particular state of aggregate distress for the economy. For simplicity we drop the dependence on economy size N from the $x_t^i(\cdot)$.

Assumption 2. *Let* $\mathbf{H^1}$ *denote the collection of states where firm 1 is in distress, and* $\overline{\mathbf{H^1}}$ *the states where it is not. Then:*

i) $\lambda^j(\mathbf{H}^1) = k\lambda$ *and* $\lambda^j(\overline{\mathbf{H}^1}) = \lambda$*,* $j = 2, 3, \dots, N$*, with* $k > 1$*.*

ii) $\eta^j(\overline{\mathbf{H}^1}) = \eta^j(\mathbf{H}^1) = \eta$*.*

Intensity parameters depend on economy size N*, in such a way that total distress and recovery risk are bounded as* $N \to \infty$*:*

$$\begin{aligned} \lim_{N\to\infty} N\lambda &= K^\lambda < \infty \\ \lim_{N\to\infty} N\eta &= K^\eta < \infty \end{aligned} \tag{OA.A20}$$

which implies

$$\lim_{N\to\infty} \lambda = \lim_{N\to\infty} \eta = 0 \tag{OA.A21}$$

The centrality parameter k *satisfies the condition:*

$$1 - K^\lambda(k+1)\Delta - K^\eta \Delta > 0 \tag{OA.A22}$$

for small Δ*.*
For simplicity we drop the dependence on N *from* λ *and* η*.*

Assumptions 1 simplifies the asymptotic behavior of dividend shares while insuring finite price-dividend ratios, thus allowing us to focus on distress connectivity. Assumption 2 serves two purposes: condition (OA.A20) guarantees finite asymptotic asset prices and risk premia, while condition (OA.A22) is a balance condition which, as discussed in the text, guarantees that firm 1 is more exposed to its distress risk, by limiting the extent of distress propagation. The dividend homogeneity assumption, which is formalized as

$$x^j(\mathbf{H}^1) = \overline{x} \qquad x^j(\overline{\mathbf{H}^1}) = \underline{x}, \quad j = 1, 2, 3, \dots, N, \tag{OA.A23}$$

marginalizes the role of dividend share size and allows network connectivity as sole determinant of risk premia.

The risk premium of the claim to the $i-$th firm is obtained from (5) of the text, after joining distress and recovery risk in a single expression:

$$\mu_t^i = \sum_{j=1}^N \tilde{\lambda}_N^j(\mathbf{H}_t) \left(1 - \theta_n^j(\mathbf{H}_t) \frac{P^i(\mathbf{H}_t^{\pm j})}{P^i(\mathbf{H}_t)} \right) \tag{OA.A24}$$

or

$$-\left[\mu_t^i - \sum_{j=1}^{N} \tilde{\lambda}^j\right] P^i(\mathbf{H}_t) = \sum_{j=1}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)P^i(\mathbf{H}_t^{\pm j}) \tag{OA.A25}$$

where $\tilde{\lambda}^i = H_t^i\eta^i + (1 - H_t^i)\lambda^i$. $\theta(\mathbf{H}_t)$ is the market price for the distress or recovery risk of firm j reported in (19). $P^i(\mathbf{H}_t^{\pm j})$ is the price to which security i jumps immediately after the distress or recovery of the j-th tree. Using expression (OA.A18) to represent $P^i(\mathbf{H}_t^{\pm j})$, it is convenient to restate the RHS of (OA.A25) as:

$$\mathcal{R}^i = \sum_{j=1}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \left[\overline{\mathbf{1}}'(\mathbf{H}_t^{\pm j}) \sum_{u=0}^{\infty} \exp(-(\mathbf{a} - \mathbf{A}^{\mathbf{H}})(t_u - t))\mathbf{A}\mathbf{C}^i\right] \tag{OA.A26}$$

Let $\mathcal{A}^i = \mathbf{A}\mathbf{C}^i$, with $\mathcal{A}^i(H)$ denoting the entries of $\mathcal{A}^i$ corresponding to state H. We also have

$$\overline{\mathbf{1}}'(\mathbf{H}_t^{\pm j}) \sum_{u=0}^{\infty} \exp(-(\mathbf{a} - \mathbf{A}^{\mathbf{H}})(t_u - t)) = \sum_{u=0}^{\infty} e^{-\delta(t_u - t)} Prob\left(\mathbf{H}_{t_u} | \, \mathbf{H}_t^{\pm j}\right) \tag{OA.A27}$$

where $Prob\,(\,\cdot\,)$ is the row vector of transition probabilities from time t to t_u conditional on state $\mathbf{H}_t^{\pm j}$ at time t.

We need the following two lemmas:

Lemma 1. *For any state H where firms 1 and N are both in distress or they are both not in distress, $\mathcal{A}^N(H) - \mathcal{A}^1(H) = 0$.*

Proof. When firms 1 and N are both in distress in state H we have:

$$\begin{aligned}\mathcal{A}^N(H) - \mathcal{A}^1(H) = &\sum_{j \in \mathcal{ND}(H)} k\lambda\Delta\left[C^N(H^{-j}) - C^1(H^{-j}) - \left(C^N(H) - C^1(H)\right)\right] \\ &- \sum_{\substack{j \in \mathcal{D}(H) \\ j \neq 1, N}} \eta\Delta\left[C^N(H) - C^1(H) - \left(C^N(H^{+j}) - C^1(H^{+j})\right)\right] + (1-\delta)(C^N(H) - C^1(H)) \\ &- \underbrace{\eta\Delta\left[C^N(H) - C^1(H) - \left(C^N(H^{+1}) - C^1(H^{+1})\right)\right]}_{1} - \underbrace{\eta\Delta\left[C^N(H) - C^1(H) - \left(C^N(H^{+N}) - C^1(H^{+N})\right)\right]}_{2}\end{aligned} \tag{OA.A28}$$

We have used the notation $C^i(H)$ to denote the entry of vector C^i that corresponds to state H. H^{+j} (H^{-j}) denotes the state reached from H when firm j recovers (has a distress). $\mathcal{D}(H)$ $(ND(H))$ denotes the collection of firm in (non) distress in state H^1. Using the homogeneous dividends assumption iii), we have $C^N(H) - C^1(H) = 0$, $C^N(H^{-j}) - C^1(H^{-j}) = 0$, $C^N(H^{+j}) - C^1(H^{+j}) = 0$, $j \neq 1, N$, while terms 1 and 2 in (OA.A28) are opposite, so that $\mathcal{A}^N(H) - \mathcal{A}^1(H) = 0$.

When firms 1 and N are both not in distress H we have:

$$\mathcal{A}^N(H) - \mathcal{A}^1(H) = \sum_{\substack{j\in\mathcal{ND}(H)\\ j\neq 1,N}} \lambda\Delta\left[C^N(H^{-j}) - C^1(H^{-j}) - \left(C^N(H) - C^1(H)\right)\right]$$
$$- \sum_{j\in\mathcal{D}(H)} \eta\Delta\left[C^N(H) - C^1(H) - \left(C^N(H^{+j}) - C^1(H^{+j})\right)\right] + (1-\delta)(C^N(H) - C^1(H))$$
$$+ \underbrace{\lambda\Delta\left[C^N(H^{-1}) - C^1(H^{-1}) - \left(C^N(H) - C^1(H)\right)\right]}_{1} + \underbrace{\lambda\Delta\left[C^N(H^{-N}) - C^1(H^{-N}) - \left(C^N(H) - C^1(H)\right)\right]}_{2} \tag{OA.A29}$$

Using the homogeneous dividends assumption *iii*), we have $C^N(H) - C^1(H) = 0$, $C^N(H^{-j}) - C^1(H^{-j}) = 0$, $C^N(H^{+j}) - C^1(H^{+j}) = 0$, $j \neq 1, N$, while terms 1 and 2 in (OA.A29) are opposite, so that $\mathcal{A}^N(H) - \mathcal{A}^1(H) = 0$.□

Lemma 2. *Consider two states, $H^1 \in \mathbf{H^1}$ and $\overline{H}^1 \in \overline{\mathbf{H^1}}$, identical in all components except 1 and N: in H^1 firm N is not in distress, while in $\overline{H}^1$ firm N is in distress. Conditional on some state H_t at time t where both firms are not in distress, then:*

$$\lim_{N\to\infty}\left[kProb\left(\mathbf{H}_{t_u} = H^1 \middle| H_t\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H^1}\middle| H_t\right)\right]\lambda\Delta \geq 0, \tag{OA.A30}$$

and there exists a $k^*(\lambda, \eta, k_h, k_l)$ such that, for $k > k^*$

$$\lim_{N\to\infty}\left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| H_t\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H^1}\middle| H_t\right)\right]\lambda\Delta \leq 0 \tag{OA.A31}$$

for any H^1, with $t_u \geq t$.

Furthermore:

$$Prob\left(\mathbf{H}_{t_u} = H^1 \middle| H_t\right)\left[\mathcal{A}^N(H^1) - \mathcal{A}^1(H^1)\right] - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1\middle| H_t\right)\left[\mathcal{A}^1(\overline{H}^1) - \mathcal{A}^N(\overline{H}^1)\right] =$$
$$\left[kProb\left(\mathbf{H}_{t_u} = H^1 \middle| H_t\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1\middle| H_t\right)\right]\lambda\Delta\left[\sum_{\substack{j\in\mathcal{ND}(H^1)\\ j\neq N}}\left(C^N(H^{1-j}) - C^1(H^{1-j}) - \left(C^N(H^1) - C^1(H^1)\right)\right)\right. \tag{OA.A32}$$

$$\left. - \left(C^N(H^1) - C^1(H^1)\right)\right] + \left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| H_t\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1\middle| H_t\right)\right]\left[(1-\delta)(C^N(H^1) - C^1(H^1)) + \right.$$
$$\left.\sum_{\substack{j\in\mathcal{D}(H^1)\\ j\neq 1}}\eta\Delta\left(\left(C^N(H^{1+j}) - C^1(H^{1+j})\right) - \left(C^N(H^1) - C^1(H^1)\right)\right) - \eta\Delta\left(C^N(H^1) - C^1(H^1)\right)\right] \tag{OA.A33}$$

Proof. For simplicity we adopt the following notation, limited to this proof: $p_t^{t_u}(H) = Prob\left(\mathbf{H}_{t_u} = H \middle| H_t\right)$. Similarly to the proof of Lemma 1, H^{+j} (H^{-j}) denotes the state reached from state H when firm j recovers (has a distress). In the same fashion, $H^{+j_1-j_2+j_3}$, for instance, denotes the state reached from state H after a recovery of firm j_1, then a distress of firm j_2, then a recovery of firm j_3. We decompose the time interval $t_u - t$ into N^Δ

subintervals of arbitrarily small length Δ, with N^{Δ} an arbitrarily large integer such that $\Delta N^{\Delta} = t_u - t$. We are going to use repeatedly the following facts:
Properties:

1. For small Δ and $H \in \mathbf{H^1}$:

$$
\begin{aligned}
Prob\left(\mathbf{H}_{t_u} = H \middle| \mathbf{H}_{t_u-\Delta} = H\right) &\approx e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))k\lambda]\Delta} && \text{(OA.A34)}\\
Prob\left(\mathbf{H}_{t_u} = H^{-j} \middle| \mathbf{H}_{t_u-\Delta} = H\right) &\approx 1 - e^{-k\lambda\Delta} && \text{(OA.A35)}\\
Prob\left(\mathbf{H}_{t_u} = H^{+j} \middle| \mathbf{H}_{t_u-\Delta} = H\right) &\approx 1 - e^{-\eta\Delta} && \text{(OA.A36)}\\
& && \text{(OA.A37)}
\end{aligned}
$$

   for some firm j. num($\mathcal{D}(H)$) (num($\mathcal{ND}(H)$)) is the number of firms that are (not) in distress in H. If $H \in \overline{\mathbf{H^1}}$ the expression $k\lambda$ is replaced by λ.

2 For any state H and $k > 1$:

$$
\lim_{N\to\infty} k e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))k\lambda]\Delta} \geq e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))\lambda]\Delta} \qquad \text{(OA.A38)}
$$

   because

$$
\begin{aligned}
&\lim_{N\to\infty} k e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))k\lambda]\Delta} - e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))\lambda]\Delta} \approx \lim_{N\to\infty} k\,[1 - (\text{num}(\mathcal{D}(H))\eta +\\
&\text{num}(\mathcal{ND}(H))k\lambda)\,\Delta] - [1 - (\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))\lambda)\,\Delta] = \lim_{N\to\infty} (k-1) - (k-1)\text{num}(\mathcal{D}(H))\eta\Delta\\
&- (k^2-1)\text{num}(\mathcal{ND}(H))\lambda\Delta \geq (k-1)\left[1 - K^{\eta}\Delta - (k+1)K^{\lambda}\Delta\right] > 0 \qquad \text{(OA.A39)}
\end{aligned}
$$

   for small Δ. The last equality in (OA.A39) follows from assumption (OA.A22).

3. num($\mathcal{ND}(H^1)$) = num($\mathcal{ND}(\overline{H}^1)$),[2] and similarly for the number of firms in distress, therefore we don't distinguish between these expressions.

4. Excluding firms 1 and N, the sets of (non) distressed trees in H^1 and $\overline{H}^1$ coincide.

We apply the backward Chapman-Kolmogorov equations to express $p_t^{t_u}(H^1)$ and $p_t^{t_u}(\overline{H^1})$ in terms of one-step transition probabilities $p_t^{t_u-\Delta}(\,\cdot\,)$. We can identify all states at the previous step from which H^1 and $\overline{H^1}$ can be reached, by the fact that in small time Δ at most one recovery or distress event can occur. We can thus write:

$$
\left[k p_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1})\right]\lambda\Delta \quad = \quad \Bigg[\underbrace{p_t^{t_u-\Delta}(H^{1+1})k\left(1-e^{-\lambda\Delta}\right) - p_t^{t_u-\Delta}(H^{1+1})\left(1-e^{-\lambda\Delta}\right)}_{1}
$$

[2]Similarly for any state reached after a sequence of common events, such as $H^{1+j_1-j_2+j_3}$ and $\overline{H}^{1+j_1-j_2+j_3}$, $j_1, j_2, j_3 \neq 1, N$.

$$+\underbrace{p_t^{t_u-\Delta}(H^{1-N})k\left(1-e^{-\eta\Delta}\right)-p_t^{t_u-\Delta}(H^{1-N})\left(1-e^{-\eta\Delta}\right)}_{2}$$

$$+p_t^{t_u-\Delta}(H^1)ke^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}$$

$$-p_t^{t_u-\Delta}(\overline{H^1})e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}$$

$$+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}}p_t^{t_u-\Delta}(H^{1+v})k\left(1-e^{-k\lambda\Delta}\right)-\sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}}p_t^{t_u-\Delta}(\overline{H}^{1+v})\left(1-e^{-\lambda\Delta}\right)$$

$$\left.+\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}}p_t^{t_u-\Delta}(H^{1-v})k\left(1-e^{-\eta\Delta}\right)-\sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}}p_t^{t_u-\Delta}(\overline{H}^{1-v})\left(1-e^{-\eta\Delta}\right)\right]\lambda\Delta \tag{OA.A40}$$

Since terms 1 and 2 in (OA.A40) are nonnegative because $k>1$, we can write:

$$\left[kp_t^{t_u}(H^1)-p_t^{t_u}(\overline{H^1})\right]\lambda\Delta=\left[p_t^{t_u-\Delta}(H^1)ke^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}\right.$$

$$-p_t^{t_u-\Delta}(\overline{H^1})e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}}p_t^{t_u-\Delta}(H^{1+v})k\left(1-e^{-k\lambda\Delta}\right)-\sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}}p_t^{t_u-\Delta}(\overline{H}^{1+v})\left(1-e^{-\lambda\Delta}\right)$$

$$\left.+\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}}p_t^{t_u-\Delta}(H^{1-v})k\left(1-e^{-\eta\Delta}\right)-\sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}}p_t^{t_u-\Delta}(\overline{H}^{1-v})\left(1-e^{-\eta\Delta}\right)\right]\lambda\Delta \tag{OA.A41}$$

$$=\underbrace{p_t^{t_u-2\Delta}(H^{1+1})\left(1-e^{-\lambda\Delta}\right)\left[ke^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}-e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}\right]\lambda\Delta}_{1}$$

$$+\underbrace{p_t^{t_u-2\Delta}(H^{1-N})\left(1-e^{-\eta\Delta}\right)\left[ke^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}-e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}\right]\lambda\Delta}_{2}$$

$$\underbrace{+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}}p_t^{t_u-2\Delta}(H^{1+v+1})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-k\lambda\Delta}\right)k\lambda\Delta-\sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}}p_t^{t_u-2\Delta}(H^{1+v+1})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-\lambda\Delta}\right)\lambda\Delta}_{3}$$

$$\underbrace{+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}}p_t^{t_u-2\Delta}(H^{1+v-N})\left(1-e^{-\eta\Delta}\right)\left(1-e^{-k\lambda\Delta}\right)k\lambda\Delta-\sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}}p_t^{t_u-2\Delta}(H^{1+v-N})\left(1-e^{-\eta\Delta}\right)\left(1-e^{-\lambda\Delta}\right)\lambda\Delta}_{4}$$

$$\underbrace{+\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}}p_t^{t_u-2\Delta}(H^{1-v+1})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-\eta\Delta}\right)k\lambda\Delta-\sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}}p_t^{t_u-2\Delta}(H^{1-v+1})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta}_{5}$$

$$\underbrace{+\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}}p_t^{t_u-2\Delta}(H^{1-v-N})\left(1-e^{-\eta\Delta}\right)\left(1-e^{-\eta\Delta}\right)k\lambda\Delta-\sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}}p_t^{t_u-2\Delta}(H^{1-v-N})\left(1-e^{-\eta\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta}_{6}$$

$$+ p_t^{t_u-2\Delta}(H^1)ke^{-\left[2\text{num}(\mathcal{D}(H^1))\eta+2\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}\lambda\Delta - p_t^{t_u-2\Delta}(\overline{H}^1)e^{-\left[2\text{num}(\mathcal{D}(H^1))\eta+2\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}\lambda\Delta$$

$$+ \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} p_t^{t_u-2\Delta}(H^{1+v})k\left[e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta} + e^{-\left[\text{num}(\mathcal{D}(H^{1+v}))\eta+\text{num}(\mathcal{ND}(H^{1+v}))k\lambda\right]\Delta}\right]\left(1-e^{-k\lambda\Delta}\right)\lambda\Delta$$

$$- \sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} p_t^{t_u-2\Delta}(\overline{H}^{1+v})\left[e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta} + e^{-\left[\text{num}(\mathcal{D}(H^{1+v}))\eta+\text{num}(\mathcal{ND}(H^{1+v}))\lambda\right]\Delta}\right]\left(1-e^{-\lambda\Delta}\right)\lambda\Delta$$

$$+ \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} p_t^{t_u-2\Delta}(H^{1-v})k\left[e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta} + e^{-\left[\text{num}(\mathcal{D}(H^{1-v}))\eta+\text{num}(\mathcal{ND}(H^{1-v}))k\lambda\right]\Delta}\right]\left(1-e^{-\eta\Delta}\right)\lambda\Delta$$

$$- \sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} p_t^{t_u-2\Delta}(\overline{H}^{1-v})\left[e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta} + e^{-\left[\text{num}(\mathcal{D}(H^{1-v}))\eta+\text{num}(\mathcal{ND}(H^{1-v}))\lambda\right]\Delta}\right]\left(1-e^{-\eta\Delta}\right)\lambda\Delta$$

$$+ \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \sum_{\substack{v_2\in\mathcal{D}(H^{1+v})\\ v_2\neq 1}} p_t^{t_u-2\Delta}(H^{1+v+v_2})k\left(1-e^{-k\lambda\Delta}\right)\left(1-e^{-k\lambda\Delta}\right)\lambda\Delta$$

$$- \sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} \sum_{\substack{v_2\in\mathcal{D}(\overline{H}^{1+v})\\ v_2\neq N}} p_t^{t_u-2\Delta}(\overline{H}^{1+v+v_2})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-\lambda\Delta}\right)\lambda\Delta \quad \text{(OA.A42)}$$

$$+ \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \sum_{\substack{v_2\in\mathcal{ND}(H^{1+v})\\ v_2\neq N}} p_t^{t_u-2\Delta}(H^{1+v-v_2})k\left(1-e^{-k\lambda\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta$$

$$- \sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} \sum_{\substack{v_2\in\mathcal{ND}(\overline{H}^{1+v})\\ v_2\neq 1}} p_t^{t_u-2\Delta}(\overline{H}^{1+v-v_2})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta$$

$$+ \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} \sum_{\substack{v_2\in\mathcal{D}(H^{1-v})\\ v_2\neq 1}} p_t^{t_u-2\Delta}(H^{1-v+v_2})k\left(1-e^{-k\lambda\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta$$

$$- \sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} \sum_{\substack{v_2\in\mathcal{D}(\overline{H}^{1-v})\\ v_2\neq N}} p_t^{t_u-2\Delta}(\overline{H}^{1-v+v_2})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta$$

$$+ \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} \sum_{\substack{v_2\in\mathcal{ND}(H^{1-v})\\ v_2\neq N}} p_t^{t_u-2\Delta}(H^{1-v-v_2})k\left(1-e^{-\eta\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta$$

$$- \sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} \sum_{\substack{v_2\in\mathcal{ND}(\overline{H}^{1-v})\\ v_2\neq 1}} p_t^{t_u-2\Delta}(\overline{H}^{1-v-v_2})\left(1-e^{-\eta\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta \quad \text{(OA.A43)}$$

The last equality in (OA.A43) follows by applying the Chapman-Kolmogorov equations to express $p_t^{t_u-\Delta}(\,\cdot\,)$ in terms of one-step backward transition probabilities $p_t^{t_u-2\Delta}(\,\cdot\,)$, and identifying all states from which next-period states can be reached, by the fact that in small time Δ at most one recovery or distress event can occur. Terms 1 and 2 in (OA.A43) are nonnegative because of Property 2. Terms 3, 4, 5, and 6 are nonnegative because $k > 1$. Therefore

$$\left[kp_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1})\right]\lambda\Delta \geq \text{last RHS in (OA.A43) excluding terms 1-6.} \quad \text{(OA.A44)}$$

Note that terms 1-6 in (OA.A43) derive from the fact that coupled states H^1 and $\overline{H}^1$ – or the states reached after a common sequence of distress and recoveries – have all elements in common except firm 1 and N, therefore in a time interval Δ can be reached from the same state, where either both 1 and N are in distress or both are not. By Property 2, and the fact that $k > 1$, these terms are nonnegative. Applying the Chapman-Kolmogorov equations to the RHS of (OA.A44) to condition on states at time $t_u - 3\Delta$, the resulting expression is then greater or equal than the same quantity that doesn't involve these terms. Iterating the procedure of backward induction until time $t_u - m\Delta$ and majorating the expression that neglects terms of the form 1-6 in (OA.A43), we can write:

$$
\begin{aligned}
&\Big[kp_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1})\Big]\lambda\Delta \geq \Big[p_t^{t_u-m\Delta}(H^1)k\mathcal{T}^0(m,H^1) - p_t^{t_u-m\Delta}(\overline{H}^1)\mathcal{T}^0(m,\overline{H}^1) \\
&+ \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}} p_t^{t_u-m\Delta}(H^{1+w_1v_1})k\mathcal{T}^1_{w_1}(m,H^1,v_1) - \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}} p_t^{t_u-m\Delta}(\overline{H}^{1+w_1v_1})\mathcal{T}^1_{w_1}(m,\overline{H}^1,v_1) \\
&\quad + \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(H^{1+w_1v_1})\\ v_2\neq l(w_2)}} p_t^{t_u-m\Delta}(H^{1+w_1v_1+w_2v_2})k\mathcal{T}^2_{w_1,w_2}(m,H^1,v_1,v_2) \\
&\quad - \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(\overline{H}^{1+w_1v_1})\\ v_2\neq \bar{l}(w_2)}} p_t^{t_u-m\Delta}(\overline{H}^{1+w_1v_1+w_2v_2})\mathcal{T}^2_{w_1,w_2}(m,\overline{H}^1,v_1,v_2)
\end{aligned}
$$

$$
\begin{aligned}
&+ \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(H^{1+w_1v_1})\\ v_2\neq l(w_2)}}\sum_{w_3=+1,-1}\sum_{\substack{v_3\in\mathcal{S}^{w_3}(H^{1+w_1v_1+w_2v_2})\\ v_3\neq l(w_3)}} p_t^{t_u-m\Delta}(H^{1+w_1v_1+w_2v_2+w_3v_3})k\times \\
&\qquad\qquad \times\mathcal{T}^3_{w_1,w_2,w_3}(m,H^1,v_1,v_2,v_3) \\
&- \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(\overline{H}^{1+w_1v_1})\\ v_2\neq \bar{l}(w_2)}}\sum_{w_3=+1,-1}\sum_{\substack{v_3\in\mathcal{S}^{w_3}(\overline{H}^{1+w_1v_1+w_2v_2})\\ v_3\neq \bar{l}(w_3)}} p_t^{t_u-m\Delta}(\overline{H}^{1+w_1v_1+w_2v_2+w_3v_3})\times \\
&\qquad\qquad \times\mathcal{T}^3_{w_1,w_2,w_3}(m,\overline{H}^1,v_1,v_2,v_3) \\
&\qquad\qquad \cdots\cdots \\
&\qquad\qquad \cdots\cdots \\
&+ \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(H^{1+w_1v_1})\\ v_2\neq l(w_2)}}\cdots\sum_{w_m=+1,-1}\sum_{\substack{v_m\in\mathcal{S}^{w_m}(H^{1+\sum_{h=1}^{m-1}w_hv_h})\\ v_m\neq l(w_m)}} p_t^{t_u-m\Delta}(H^{1+\sum_{h=1}^{m}w_hv_h})k\times \\
&\qquad\qquad \times\mathcal{T}^m_{w_1,\ldots,w_m}(m,H^1,v_1,\ldots,v_m) \\
&- \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(\overline{H}^{1+w_1v_1})\\ v_2\neq \bar{l}(w_2)}}\cdots\sum_{w_m=+1,-1}\sum_{\substack{v_m\in\mathcal{S}^{w_m}(\overline{H}^{1+\sum_{h=1}^{m-1}w_hv_h})\\ v_m\neq \bar{l}(w_m)}} p_t^{t_u-m\Delta}(\overline{H}^{1+\sum_{h=1}^{m}w_hv_h})\times \\
&\qquad\qquad \times\mathcal{T}^m_{w_1,\ldots,w_m}(m,\overline{H}^1,v_1,\ldots,v_m)\Big]\lambda\Delta, \quad \text{(OA.A45)}
\end{aligned}
$$

where

$$\mathcal{T}^0(m,H) = \begin{cases} 1 \text{ if } m=0 \\ \mathcal{T}^0(m-1,H)e^{-\left[\text{num}(\mathcal{D}(H))\eta+\text{num}(\mathcal{ND}(H))(k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1))\lambda\right]\Delta} & \text{otherwise} \end{cases} \quad \text{(OA.A46)}$$

$$\mathcal{T}^1_{w_1}(m,H,v_1) = \begin{cases} 0 \text{ if } m=0 \\ \mathcal{T}^0(m-1,H)\left(1-e^{-\tilde{\lambda}(w_1,H)\Delta}\right)+\mathcal{T}^1_{w_1}(m-1,H,v_1)\times \\ \times e^{-\left[\text{num}(\mathcal{D}(H^{+w_1v_1}))\eta+\text{num}(\mathcal{ND}(H^{+w_1v_1}))(k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1))\right]\Delta} & \text{otherwise} \end{cases} \quad \text{(OA.A47)}$$

$$\mathcal{T}^2_{w_1,w_2}(m,H,v_1,v_2) = \begin{cases} 0 \text{ if } m=0 \\ \mathcal{T}^1_{w_1}(m-1,H,v_1)\left(1-e^{-\tilde{\lambda}(w_2,H)\Delta}\right)+\mathcal{T}^2_{w_1,w_2}(m-1,H,v_1,v_2)\times \\ \times e^{-\left[\text{num}(\mathcal{D}(H^{+w_1v_1+w_2v_2}))\eta+\text{num}(\mathcal{ND}(H^{+w_1v_1+w_2v_2}))(k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1))\right]\Delta} & \text{otherwise} \end{cases} \quad \text{(OA.A48)}$$

$$\dots \quad \text{(OA.A49)}$$

$$\mathcal{T}^m_{w_1,\dots,w_m}(m,H,v_1,\dots,v_m) = \begin{cases} 0 \text{ if } m=0 \\ \mathcal{T}^{m-1}_{w_1,\dots,w_{m-1}}(m-1,H,v_1,\dots,v_{m-1})\left(1-e^{-\tilde{\lambda}(w_m,H)\Delta}\right) \\ +\mathcal{T}^m_{w_1,\dots,w_m}(m-1,H,v_1,\dots,v_m)e^{-\text{num}(\mathcal{D}(H^{+\sum_{h=1}^m w_hv_h}))\eta\Delta}\times \\ \times e^{-\text{num}(\mathcal{ND}(H^{+\sum_{h=1}^m w_hv_h}))(k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1))\Delta} & \text{otherwise} \end{cases} \quad \text{(OA.A50)}$$

$\mathbf{1}(\cdot)$ denotes the indicator function of an event. Furthermore:

$$\mathcal{S}^w(H) = \begin{cases} \mathcal{D}(H) & \text{if } w=+1 \\ \mathcal{ND}(H) & \text{if } w=-1 \end{cases} \quad \text{(OA.A51)}$$

$$l(w) = \begin{cases} 1 & \text{if } w=+1 \\ N & \text{if } w=-1 \end{cases} \quad \text{(OA.A52)}$$

$$\bar{l}(w) = \begin{cases} N & \text{if } w=+1 \\ 1 & \text{if } w=-1 \end{cases} \quad \text{(OA.A53)}$$

$$\tilde{\lambda}(w,H) = \begin{cases} k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1) & \text{if } w=+1 \\ \eta & \text{if } w=-1 \end{cases} \quad \text{(OA.A54)}$$

For $m=N^\Delta$, we have $t_u - m\Delta = t$, so that

$$p_t^{t_u-N^\Delta\Delta}(H) = \begin{cases} 1 \text{ if } \mathbf{H}_t \equiv H \\ 0 \text{ otherwise} \end{cases}, \quad \forall H \quad \text{(OA.A55)}$$

By assumption, in the initial state $\mathbf{H}_t$ both firms 1 and N are not in distress, while firm 1 is in distress in any state $H^{1+\sum_{h=1}^s w_hv_h}$, and firm N is in distress in any state $\overline{H}^{1+\sum_{h=1}^s w_hv_h}$, $s=1,\dots,N^\Delta$. This implies that taking $m=N^\Delta$ in expression (OA.A45), its RHS vanishes, and claim (OA.A30) follows.

To see that claim (OA.A31) holds, notice that

$$\lim_{k\to\infty} p_t^{t_u}(H^1) = 0, \quad \text{(OA.A56)}$$

because, according to (OA.A34)-(OA.A35),

$$\lim_{k\to\infty} Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_{t_u-\Delta} = H^1\right) = 0 \quad \text{(OA.A57)}$$

$$\lim_{k\to\infty} Prob\left(\mathbf{H}_{t_u} = H^{1-j} \middle| \mathbf{H}_{t_u-\Delta} = H^1\right) = 1 \quad \text{(OA.A58)}$$

In other words, as the propagation of distress approaches immediacy, the probability of a future state where firm 1 is in distress and some other firm is not approaches 0. (OA.A56) does not hold instead for states of the form $\overline{H^1}$, so that

$$\lim_{k\to\infty} p_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1}) < 0. \quad \text{(OA.A59)}$$

Then there must exist a $k^*(\lambda, \eta, k_h, k_l)$, also dependent on N, such that $p_t^{t_u}(H^1) < p_t^{t_u}(\overline{H^1})$ for $k > k^*(\lambda, \eta, k_h, k_l)$. We haven't been able to show that $p_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1})$ is monotonically decreasing in k for $k > 1$. We provide some supportive numerical evidence, on a finite economy with $N = 10$ firms. Table I reports the critical $k^*(\lambda, \eta, k_h, k_l)$ for different values of λ and η, and the percentage of violations of the condition $p_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1})$ for $k < k^*(\lambda, \eta, k_h, k_l)$, for all paired states (H^1,$\overline{H^1}$), and initial states $\mathbf{H}_t$. In all cases the percentages approach zero monotonically as $k \to k^*(\lambda, \eta, k_h, k_l)$.

Insert Table I

To see that claim (OA.A33) holds, we write:

$$p_t^{t_u}(H^1)\left[\mathcal{A}^N(H^1) - \mathcal{A}^1(H^1)\right] - p_t^{t_u}(\overline{H}^1)\left[\mathcal{A}^1(\overline{H}^1) - \mathcal{A}^N(\overline{H}^1)\right] = p_t^{t_u}(H^1)\left[\sum_{\substack{j\in\mathcal{ND}(H^1)\\ j\neq N}} k\lambda\Delta\left[C^N(H^{1-j}) - C^1(H^{1-j})\right.\right.$$
$$\left. - \left(C^N(H^1) - C^1(H^1)\right)\right] + \sum_{\substack{j\in\mathcal{D}(H^1)\\ j\neq 1}} \eta\Delta\left[\left(C^N(H^{1+j}) - C^1(H^{1+j})\right) - \left(C^N(H^1) - C^1(H^1)\right)\right]$$
$$\left. +(1-\delta)(C^N(H^1) - C^1(H^1)) - k\lambda\Delta\left(C^N(H^1) - C^1(H^1)\right) - \eta\Delta\left(C^N(H^1) - C^1(H^1)\right)\right] \quad \text{(OA.A60)}$$

$$- p_t^{t_u}(\overline{H}^1)\left[\sum_{\substack{j\in\mathcal{ND}(\overline{H}^1)\\ j\neq N}} \lambda\Delta\left[C^N(\overline{H}^{1-j}) - C^1(\overline{H}^{1-j})\right.\right.$$
$$\left. - \left(C^N(\overline{H}^1) - C^1(\overline{H}^1)\right)\right] + \sum_{\substack{j\in\mathcal{D}(\overline{H}^1)\\ j\neq 1}} \eta\Delta\left[C^N(\overline{H}^{1+j}) - C^1(\overline{H}^{1+j}) - \left(C^N(\overline{H}^1) - C^1(\overline{H}^1)\right)\right]$$
$$\left. +(1-\delta)(C^N(\overline{H}^1) - C^1(\overline{H}^1)) - \lambda\Delta\left(C^N(\overline{H}^1) - C^1(\overline{H}^1)\right) - \eta\Delta\left(C^N(\overline{H}^1) - C^1(\overline{H}^1)\right)\right] \quad \text{(OA.A61)}$$

Using the homogeneous dividends assumption *iii*), we have $C^N(H^1) - C^1(H^1) = C^1(\overline{H}^1) - C^N(\overline{H}^1)$, $C^N(H^{1-j}) - C^1(H^{1-j}) = C^1(\overline{H}^{1-j}) - C^N(\overline{H}^{1-j})$ if $j \neq 1, N$, $C^N(H^{1+j}) - C^1(H^{1+j}) = C^1(\overline{H}^{1+j}) - C^N(\overline{H}^{1+j})$, if $j \neq 1, N$. Moreover $\mathcal{ND}(H^1)$ excluding N coincides with $\mathcal{ND}(\overline{H}^1)$ excluding 1, and $\mathcal{D}(H^1)$ excluding 1 coincides with $\mathcal{D}(\overline{H}^1)$ excluding N. These facts allow to collect terms in (OA.A61) and obtain (OA.A33). □

Lemma 3. *The condition $k^*(\lambda, \eta, k_h, k_l) < k < k^{**}$, where k^{**} solves*

$$1 - K^\lambda (k+1)\Delta - K^\eta \Delta = 0 \quad \text{(OA.A62)}$$

is sufficient for

$$\lim_{N\to\infty} \mathcal{R}^N - \mathcal{R}^1 \geq 0 \quad \text{(OA.A63)}$$

to hold.

By virtue of (OA.A26) and (OA.A27):

$$\begin{aligned}
\mathcal{R}^N - \mathcal{R}^1 \;&=\; \sum_{j=1}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\left[\overline{\mathbf{1}}'(\mathbf{H}_t^{\pm j})\sum_{u=0}^{\infty}\exp(-(\mathbf{a}-\mathbf{A}^{\mathbf{H}})(t_u - t))\left(\mathcal{A}^N - \mathcal{A}^1\right)\right] \\
&=\; \left(\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND}\right) + \lambda^1(\mathbf{H}_t)\theta^1(\mathbf{H}_t)\sum_{u=0}^{\infty} e^{-\delta(t_u - t)}\left[\sum_{H^1} Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{-1}\right)\left(\mathcal{A}^N(H^1) - \mathcal{A}^1(H^1)\right)\right. \\
&\quad \left. + \sum_{\overline{H}^1} Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{-1}\right)\left(\mathcal{A}^N(\overline{H}^1) - \mathcal{A}^1(\overline{H}^1)\right)\right] \\
&\quad + \lambda^N(\mathbf{H}_t)\theta^N(\mathbf{H}_t)\sum_{u=0}^{\infty} e^{-\delta(t_u - t)}\left[\sum_{H^1} Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{-N}\right)\left(\mathcal{A}^N(H^1) - \mathcal{A}^1(H^1)\right)\right. \\
&\quad \left. + \sum_{\overline{H}^1} Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{-N}\right)\left(\mathcal{A}^N(\overline{H}^1) - \mathcal{A}^1(\overline{H}^1)\right)\right] \quad \text{(OA.A64)}
\end{aligned}$$

with

$$\begin{aligned}
\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} = \sum_{\substack{j=1 \\ j\neq 1,N}}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty} e^{-\delta(t_u - t)}\left(\sum_{H^1} Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right)\left(\mathcal{A}^N(H^1) - \mathcal{A}^1(H^1)\right)\right. \\
\left. + \sum_{\overline{H}^1} Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\left(\mathcal{A}^N(\overline{H}^1) - \mathcal{A}^1(\overline{H}^1)\right)\right) \quad \text{(OA.A65)}
\end{aligned}$$

Note that in the expression for $\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND}$, in states $\mathbf{H}_t^{\pm j}$ firms 1 and N are necessarily both not in distress, because in the initial state $\mathbf{H}_t$ they are not by assumption. Also note that the only relevant states at time t_u in expression (OA.A64), are necessarily the paired states of the form H^1 and $\overline{H}^1$ of Lemma 2: any state of (non) distress for the economy excluding firms 1 and N gives rise to four states; two of them are paired states H^1 and $\overline{H}^1$, and in the remaining two firm 1 and N are both in distress or both not in distress. In the latter case expression $\mathcal{A}^N(H) - \mathcal{A}^1(H)$ vanishes, according to Lemma 1. Using (OA.A33) of Lemma 2, expression (OA.A65) reads explicitly:

$$\begin{aligned}
\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} &= \sum_{\substack{j=1 \\ j\neq 1,N}}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty} e^{-\delta(t_u-t)}\sum_{H^1}\mathcal{U}(H^1,u) \\
\mathcal{U}(H^1,u) &= \Big\{\Big[kProb\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t\right)\Big]\lambda\Delta\times \\
&\left[\sum_{\substack{v\in\mathcal{ND}(H^1) \\ v\neq N}} \left(C^N(H^{1-v}) - C^1(H^{1-v}) - \left(C^N(H^1) - C^1(H^1)\right)\right) - \left(C^N(H^1) - C^1(H^1)\right)\right] \\
&+ \Big[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t\right)\Big]\Big[(1-\delta)(C^N(H^1) - C^1(H^1)) + \\
&\sum_{\substack{v\in\mathcal{D}(H^1) \\ v\neq 1}} \eta\Delta\left(\left(C^N(H^{1+v}) - C^1(H^{1+v})\right) - \left(C^N(H^1) - C^1(H^1)\right)\right) - \eta\Delta\left(C^N(H^1) - C^1(H^1)\right)\Big]\Big\}
\end{aligned} \tag{OA.A66}$$

Letting $N \to \infty$, we distinguish three possible cases concerning a given state H^1:

1. $\lim_{N\to\infty} \text{num}\left(\mathcal{ND}(H^1)\right) = \infty$, $\lim_{N\to\infty} \text{num}\left(\mathcal{D}(H^1)\right) = K$, for some finite integer K.

   Setting:

   $$K^1_n = \sum_{\substack{v\in\mathcal{ND}(H^1) \\ v\neq N}} \left(C^N(H^{1-v}) - C^1(H^{1-v}) - \left(C^N(H^1) - C^1(H^1)\right)\right), \tag{OA.A67}$$

   we have

   $$\sum_{\substack{v\in\mathcal{ND}(H^1) \\ v\neq N}} \left(C^N(H^{1-v}) - C^1(H^{1-v}) - \left(C^N(H^1) - C^1(H^1)\right)\right) - \left(C^N(H^1) - C^1(H^1)\right) = K^1(N) + o(K^1(N)) \geq 0 \tag{OA.A68}$$

   and

   $$(1-\delta)(C^N(H^1) - C^1(H^1)) + \sum_{\substack{v\in\mathcal{D}(H^1) \\ v\neq 1}} \left(\left(C^N(H^{1+v}) - C^1(H^{1+v})\right) - \left(C^N(H^1) - C^1(H^1)\right)\right) - \left(C^N(H^1) - C^1(H^1)\right) = o(K^1(N)) \tag{OA.A69}$$

   for N large. The sign of the RHSs in (OA.A68) derives from the fact that $C^N(H^{1-v}) - C^1(H^{1-v}) \geq C^N(H^1) - C^1(H^1)$ for $\gamma > 1$. Due to (OA.A30) and (OA.A31) we can conclude that $\lim_{N\to\infty}\mathcal{U}(H^1,u) \geq 0$.

2. $\lim_{N\to\infty} \text{num}\left(\mathcal{D}(H^1)\right) = \infty$, $\lim_{N\to\infty} \text{num}\left(\mathcal{ND}(H^1)\right) = K$, for some finite integer K.

Setting:

$$K_n^2 \quad = \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \left(C^N(H^1) - C^1(H^1) - \left(C^N(H^{1+v}) - C^1(H^{1+v})\right)\right), \tag{OA.A70}$$

we have

$$(1-\delta)(C^N(H^1) - C^1(H^1)) + \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \left(\left(C^N(H^{1+v}) - C^1(H^{1+v})\right) - \left(C^N(H^1) - C^1(H^1)\right)\right)$$
$$- \left(C^N(H^1) - C^1(H^1)\right) = -[K^2(N) + o(K^2(N))] \leq 0 \tag{OA.A71}$$

for N large, and

$$\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} \left(C^N(H^{1-v}) - C^1(H^{1-v}) - \left(C^N(H^1) - C^1(H^1)\right)\right) \; - \; \left(C^N(H^1) - C^1(H^1)\right) \quad = \quad o(K^2(N)) \tag{OA.A72}$$

The sign of the RHSs in (OA.A71) derives from the fact that $C^N(H^{1+v}) - C^1(H^{1+v}) \leq C^N(H^1) - C^1(H^1)$ for $\gamma > 1$. Since $K^2(N)$ is bounded $\forall N$ because of Assumption 1 and (OA.A21), claims (OA.A30) and (OA.A31) let us conclude that $\lim_{N\to\infty}\mathcal{U}(H^1, u) = 0$.

3. $\lim_{N\to\infty}\text{num}\left(\mathcal{D}(H^1)\right) = \infty$, $\lim_{N\to\infty}\text{num}\left(\mathcal{ND}(H^1)\right) = \infty$. By the reasoning as above: $\lim_{N\to\infty}\mathcal{U}(H^1, u) \geq 0$.

We then have

$$\lim_{N\to\infty} \mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} \geq 0 \tag{OA.A73}$$

It is clear from (OA.A64) and (OA.A73) that

$$\lim_{N\to\infty} \mathcal{R}^N - \mathcal{R}^1 = \mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} + o(\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND}), \tag{OA.A74}$$

because, by the assumption that in the initial state $\mathbf{H}_t$ firm 1 and N are not in distress, there are only two states $\mathbf{H}_t^{\pm j}$ where firm 1 or firm N is in distress, regardless of N. □

Using (OA.A63) we obtain:

$$\lim_{N\to\infty}\left[\mu_t^N - \sum_{j=1}^N \tilde{\lambda}^j\right] P^N(\mathbf{H}_t) = \lim_{N\to\infty} -\mathcal{R}^N \leq \lim_{N\to\infty} -\mathcal{R}^1 = \lim_{N\to\infty}\left[\mu_t^1 - \sum_{j=1}^N \tilde{\lambda}^j\right] P^1(\mathbf{H}_t) \tag{OA.A75}$$

so that

$$\lim_{N\to\infty}\left[\mu_t^N - \sum_{j=1}^N \tilde{\lambda}^j\right] \leq \left[\mu_t^1 - \sum_{j=1}^N \tilde{\lambda}^j\right]\frac{P^1(\mathbf{H}_t)}{P^N(\mathbf{H}_t)} \leq \lim_{N\to\infty}\left[\mu_t^1 - \sum_{j=1}^N \tilde{\lambda}^j_N\right] \tag{OA.A76}$$

The last inequality follows from the fact that $P^N(\mathbf{H}_t) > P^1(\mathbf{H}_t)$ for large N, which is a consequence of the reasoning above, once we notice that

$$P^N(\mathbf{H}_t) - P^1(\mathbf{H}_t) = \overline{\mathbf{1}}'(\mathbf{H}_t) \sum_{u=0}^{\infty} \exp(-(\mathbf{a} - \mathbf{A}^{\mathbf{H}})(t_u - t)) \left(\mathcal{A}^N - \mathcal{A}^1\right), \tag{OA.A77}$$

and that firms 1 and N are not in distress in $\mathbf{H}_t$. We can conclude that $\mu_t^1 \geq \mu_t^N$ for N large.
□

Online Appendix B

Dividend and Earnings portfolio series. Once stocks are assigned to portfolios, we collect annual data on their earnings-per-share before extraordinary items (Compustat EPSPX), and quarterly data on their cash-flow distributions, namely total dividends (Compustat DVT), repurchases of common and preferred stock (Compustat PRSTKC) and the redemption value of the preferred stock (Compustat PSTKRV), as well as the number of shares outstanding. As for beta, size, and book-to-market, we avoid including forward looking information by using only previous fiscal year cash-flow distributions. We follow the procedure outlined in Menzly, Santos, and Veronesi (2004), Hansen, Heaton, and Li (2002) and Bansal, Dittmar, and Lundblad (2002), to build the portfolio earnings and cash-flow series that take into account shares repurchases and redemptions, and that are consistent with value-weighted holdings of stocks in portfolios. Let $j = 1, \ldots, 100$ denote a given portfolio, Ω_t^j the collection of stocks in that portfolio at month t, and V_t^j its market value. Let t be the portfolio updating date (July of each year).

- At time t, for each stock $i \in \Omega_t^j$ we find the number of shares θ_t^i that satisfies the value-weighting condition.
- During the quarter running from t to $t+1$, total cash-flows accruing to portfolio j are:

  $$D_{t,t+1}^j = \sum_{i \in \Omega_t^j} \theta_t^i \frac{DVT_{t+1}^i + PRSTKC_{t+1}^i - (PSTKRV_{t+1}^i - PSTKRV_t^i)}{N_t^i},$$

  where N_t^i denotes the number of firm's i shares outstanding at time t. The total earnings of the portfolio are:

  $$E_{t,t+1}^j = \sum_{i \in \Omega_t^j} \theta_t^i \, EPSPX_{t+1}^i,$$

- If repurchases or redemptions occur for stock i during the quarter, the number of shares held is updated in percentage of the total repurchase/redemption, excluding potential new issues:

  $$\theta_{t+1}^i = \theta_t^i \, \frac{N_t^i - [PRSTKC_{t+1}^i - (PSTKRV_{t+1}^i - PSTKRV_t^i)]/P_{t+1}^i}{N_t^i}.$$

  The numerator is the total number of shares outstanding at the beginning of next quarter before new issues.

- At $t+1$, the ex-dividend market value of portfolio j is $V_{t+1}^j = \sum_{i \in \Omega_{t+1}^j} \theta_{t+1}^i P_{t+1}^i$. The quarterly total return on the portfolio is $R_{t+1}^j = (V_{t+1}^j + D_{t,t+1}^j - V_t^j)/V_t^j$. Ω_{t+1}^j coincides with Ω_t^j, until date $t+4$, when the portfolio composition is updated and the procedure repeated.
- As in Menzly, Santos, and Veronesi (2004): we assume an initial investment in portfolio j, V_0^j, corresponding to the market capitalization of the portfolio per US capita: $V_0 = \sum_{i \in \Omega_0^j} N_0^i P_0^i / pop(0)$.[3] $pop(0)$ is the US population at time 0, June 1953; we assume that the consumption flow C_t is the per-capita US total consumption expenditure of non-durables plus services, as reported, already deflated and deseasoned, by National Income and Product Accounts (NIPA).

We deseason the cash-flow series using a four quarter trailing moving average. □

[3] Without loss of generality we have multiplied this figure by 100.

Characteristics of Average Portfolio Returns. We provide a brief description of the characteristics of portfolio average monthly returns, which are largely consistent with known stylized facts.

Insert Table II

Average returns, reported in Table II, display a strong decreasing pattern along the size dimension: as size increases from the first to the 10th decile, the average portfolio return decreases monotonically from 3.1% to 0.4% per month, with a t-statistics of -6.31 for this difference.[4] In contrast, it is hard to observe any pattern along the beta dimension: not only we do not observe a consistently increasing pattern, but high beta stocks do not appear to earn significantly larger returns at all, as the difference between the average return of the 10th and the 1st decile has a t-statistics of -0.53. We have also explored results of a stratification into beta and book-to-market portfolio deciles. Average returns are in Table IV: as expected, average returns are monotonically increasing in the book-to-market dimension, ranging from 0.05% of the first decile to 3.4% of the last, with a t-statistics of 7.1 for this difference. The value-premium puzzle is apparent from the table, where average returns are increasing in book-to-market not only unconditionally, but also within any beta portfolio decile. We have also confirmed the accuracy of the sorting procedure by looking at the portfolio betas obtained from time series regressions on the whole sample (the "post ranking betas"in the terminology of Fama-French (1992)): ex-post portfolio betas are consistent with the ex-ante betas of the ranked stocks (Table III in Online Appendix). Moreover, variation across the size dimension confirms the well known fact that beta is inversely correlated with size. When we sort in the book-to-market dimension (tables are available from the authors) we find that both value and growth stocks have larger post-ranking betas than median book-to-market deciles. Again, this nonmonotonicity of betas across book-equity/market-equity captures a well know tension in the CAPM.

Parameters' Standard Errors. Let θ^* denote the Maximum Likelihood estimator of θ, namely the parameter set achieving the maximum in (40) of the paper, and θ_0 the true parameter set. The asymptotic arguments developed for the Simulated Maximum Likelihood estimator of Brandt and Santa-Clara (2002) let us conclude that, as $T \to \infty$:

$$(\theta^* - \theta_0) \sim N(0, I^{-1}(\theta_0)), \quad I(\theta_0) = \mathbb{E}\left[-\sum_{t=1}^{T}\sum_{i=1}^{N} \frac{\partial^2 \log \phi(\widehat{E}_t^i - E_t^i|\theta)}{\partial\theta\partial\theta'} \right] \tag{OA.B1}$$

where I is the Fisher information matrix and $\frac{\partial^2 \log \phi}{\partial\theta\partial\theta'}$ denotes the Hessian matrix with respect to θ of the likelihood function. It is well known that $I(\theta_0)$ is also:

$$I(\theta_0) = \mathbb{E}\left[\sum_{t=1}^{T}\sum_{i=1}^{N} \frac{\partial \log \phi(\widehat{E}_t^i - E_t^i|\theta)}{\partial\theta} \frac{\partial \log \phi(\widehat{E}_t^i - E_t^i|\theta)}{\partial\theta}' \right] \tag{OA.B2}$$

where $\frac{\partial \log \phi}{\partial\theta}$ is the gradient of the log-likelihood. We approximate (OA.B2) by Monte-Carlo simulation and finite difference. For an initial state $H_0(i)$, we simulate $np = 3000$ trajectories of T quarters for $\mathbf{H}_t$ (hence portfolio

[4]Standard deviations of portfolio returns are not tabulated, but available upon request.

earnings), each starting at $H_0(i)$, and approximate the conditional Fisher information as:

$$\mathbb{E}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\frac{\partial\log\phi(\widehat{E}_t^i - E_t^i|\theta)}{\partial\theta}\frac{\partial\log\phi(\widehat{E}_t^i - E_t^i|\theta)}{\partial\theta}'\middle|\mathbf{H}_0 = H_0(i)\right] =$$

$$\frac{1}{np}\sum_{w=1}^{np}\sum_{t=1}^{T}\sum_{i=1}^{N}\frac{\log\phi(\widehat{E}_t^i - E_t^i(w)|\theta+\epsilon) - \log\phi(\widehat{E}_t^i - E_t^i(w)|\theta-\epsilon)}{2\epsilon}$$

$$\frac{\log\phi(\widehat{E}_t^i - E_t^i(w)|\theta+\epsilon) - \log\phi(\widehat{E}_t^i - E_t^i(w)|\theta-\epsilon)}{2\epsilon}' \quad \text{(OA.B3)}$$

where $E_t^i(w)$ denotes the $w-th$ simulated earnings path. We then compute the steady state distribution π of all 2^N $H_t(i)$ states , as in (OA.A13) and apply the law of iterated expectations to approximate the unconditional Fisher information matrix:

$$\mathbb{E}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\frac{\partial\log\phi(\widehat{E}_t^i - E_t^i|\theta)}{\partial\theta}\frac{\partial\log\phi(\widehat{E}_t^i - E_t^i|\theta)}{\partial\theta}'\right] =$$

$$\sum_{u=1}^{2^N}\pi_u\mathbb{E}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\frac{\partial\log\phi(\widehat{E}_t^i - E_t^i|\theta)}{\partial\theta}\frac{\partial\log\phi(\widehat{E}_t^i - E_t^i|\theta)}{\partial\theta}'\middle|\mathbf{H}_0 = H_0(u)\right] \quad \text{(OA.B4)}$$

In practice, we find that the parameter estimate θ^* implies that a subset $\mathbf{H^1}$ of states $H_0(i)$ accounts for more that $p^1 = 98\%$ of the total unconditional probability mass π. We consider only this subset in the approximation (OA.B4).

Standard errors of network parameters are in Table V.

Online Appendix C: The Failure of the Two-Fund Separation Property

The cross-sectional heterogeneity in the connectivity among different firms has immediate implications for the two-fund separation property of asset prices.

Proposition OA.C 1. *Consider a sequence of economies indexed by the total number of firms, N, where firms' characteristics satisfy Assumption 3 and Assumption 4 below.*

If the network is symmetric, as in Figure 1, two fund-separation holds as $N \to \infty$: assets' risk-premia have an exact one-factor representation. If the network is asymmetrically connected, two fund separation does not hold. In the 'Star' form of Figure 2a, three fund-separation holds as $N \to \infty$: firms' risk-premia are linear combinations of the central firm's (firm 1) risk premium and of an additional risk factor.

In a completely homogeneous economy, where also dividend jumps are the same across trees in the limit of $N \to \infty$, it is well known that expected returns are equal to the sum of two components: (a) the risk free rate and (b) the marginal contribution of the asset to the variance of the market portfolio. Idiosyncratic risk is not priced. The main reason is that for $N \to \infty$ the market portfolio can diversify away firm-specific shocks, so that these will not bear any risk premium. Indeed this is the case for the disconnected network structure described in Figure 1a. In Figure 1b instead, where network connections are identical, shocks are only systematic, in that they have perfect correlation with shocks to the market portfolio when N grows arbitrarily large. While two-fund separation holds in these cases, it does not hold for the 'Star' network in Figure 2a. The intuition is simple: since firm 1 is dominant in the network, even for $N \to \infty$ the market portfolio is not able to diversify away its firm-specific risk. This result holds more generally: networks with a large cross-sectional dispersion in centrality do not satisfy the two fund separation property and firm-specific risk matters in equilibrium asset prices. Figure 2b reports a typical clustered economy. There are $\overline{N}$ connected central firms, each with its own 'Star' subnetwork. Noncentral firms are disconnected among them and relate homogeneously to their 'Star'. The next Corollary generalizes Proposition OA.C 1 to this situation.

Corollary OA.C 1. *If the same assumptions of Proposition OA.C 1 hold, and the network is of the clustered 'Star' form of Figure 2b, with $\overline{N}$ 'Star' firms, $2\overline{N}+1-$fund separation holds as the number of noncentral firms in each subnetwork grows arbitrarily large and $\overline{N}$ remains finite.*

This result states that every central firm is a source of priced risk both because of its own idiosyncratic distress risk, and because of the idiosyncratic risk that is complementary to it. Noncentral firms are affected in distinct forms by their 'Star', even when 'Stars' are symmetrically connected, which creates independent forms of complementarity. Indeed, with additional assumptions – such as (9) in the one-star case – we could conjecture that the firm with the tightest links to its network – the most locally exogenous – has the larger expected return among the central firms, thus of all the economy, in light of Proposition 3. The intuition is that this firm suffers smaller consumption growth during its distress state, and the latter is the most systematic risk factor for the economy. Similarly, it is reasonable to expect that the firm most affected by its 'Star' has larger risk premium among noncentral firms, as its non idiosyncratic ('Star'-related) distress is shared by more subnet peers on average, which makes it more correlated with consumption risk.

Proof of Proposition OA.C 1. We use the same notation of the proof of Proposition 3. H will denote a generic realization of $\mathbf{H}$. Assumptions 1 and Assumption 2 of Proposition 3 are replaced by the following:

Assumption 3. *Dividends are deterministic functions of the economy size N, and asymptotically homogeneous:*

$$x_t^i(H) = \begin{cases} \overline{f}^i(N) & \text{if } H_t^i = 0 \\ \underline{f}^i(N) & \text{if } H_t^i = 1 \end{cases} \qquad i = 1, \ldots N \tag{OA.C1}$$

with $\lim_{N\to\infty} x_t^i(H) = \lim_{N\to\infty} x_t^j(H)$, $\forall i, j$. *Moreover*

$$\lim_{N\to\infty} \left(\frac{\sum_{j=1}^N x_t^j(H)}{\sum_{j=1}^N x_t^j(\mathbf{H}_t)} \right)^{-\gamma} \frac{x_t^i(H)}{x_t^i(\mathbf{H}_t)} = c(H, \mathbf{H}_t) \tag{OA.C2}$$

with $0 < c^i(H, \mathbf{H}_t) < \infty$, *for all possible states H.*

Assumption 4. *For a given economy size N, intensities $\lambda^i(H)$ and $\eta^j(H)$, $j = 1, \ldots, N$ are independent conditionally on the state H, and they are realizations of common (across firms) distributions $F^\lambda(N, H)$ and $F^\eta(N, H)$. These distributions are such that the following condition holds*

$$\begin{aligned} \lim_{N\to\infty} \textstyle\sum_{v=1}^N \lambda^v(H) &= K^\lambda(H) < \infty \\ \lim_{N\to\infty} \textstyle\sum_{v=1}^N \eta^v(H) &= K^\eta(H) < \infty \end{aligned} \tag{OA.C3}$$

which implies

$$\lim_{N\to\infty} \lambda^i(H) = \lim_{N\to\infty} \eta^i(H) = 0 \qquad \forall H,\ i = 1, \ldots, N \tag{OA.C4}$$

For simplicity we drop the dependence on N from the $\lambda^i(\cdot)$, $\eta(\cdot)$ and $x_t^i(\cdot)$.

Consider an initial state $\mathbf{H}_t$ and an economy size N. Following the lines of the proof of Proposition 3, we redefine $\mathcal{R}^i$ as

$$\begin{aligned} \mathcal{R}^i &= \sum_{j=1}^N \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \left[\mathbf{\overline{1}}'(\mathbf{H}_t^{\pm j}) \sum_{u=0}^\infty \exp(-(\mathbf{a} - \mathbf{A^H})(t_u - t))\mathbf{A}\mathbf{c}^i \right] \\ &= \mathcal{R}_{ND}^i + \lambda^i(\mathbf{H}_t)\theta^i(\mathbf{H}_t) \sum_{u=0}^\infty e^{-\delta(t_u - t)} \left[\sum_H Prob\left(H_{t_u} = H | \, \mathbf{H}_t^{-i} \right) \mathcal{A}^i(H) \right] \end{aligned} \tag{OA.C5}$$

The reason to partition $\mathcal{R}^i$ in (OA.C5) is to isolate the only term where firm i is in distress in $\mathbf{H}_t^{\pm j}$. We have set

$$\mathcal{R}_{ND}^i = \sum_{\substack{j=1 \\ j\neq i}}^N \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^\infty e^{-\delta(t_u - t)} \left(\sum_H Prob\left(H_{t_u} = H | \, \mathbf{H}_t^{\pm j} \right) \mathcal{A}^i(H) \right) \tag{OA.C6}$$

$\mathbf{c}^i = \mathbf{C}^i / x_t^i(\mathbf{H}_t)$ is the vector of dividends paid by firm i in each possible state H, discounted by the marginal rate of intertemporal substitution, and scaled by the current dividend $x_t^i(\mathbf{H}_t)$. We denote by $c^i(H)$ the entry of vector $\mathbf{c}^i$ corresponding to state H. We have also set $\mathcal{A}^i = \mathbf{A}\mathbf{c}^i$, with $\mathcal{A}^i(H)$ the entry of this vector corresponding to state H. We use the familiar representation for the risk premium of the $i-$th equity security:[5]

$$\left[\mu_t^i - \sum_{j=1}^N \tilde{\lambda}^j \right] \frac{P^i(\mathbf{H}_t)}{x_t^i(\mathbf{H}_t)} \quad = \quad -\mathcal{R}^i \tag{OA.C7}$$

[5]Remind that $\tilde{\lambda}^i = H_t^i \eta^i + (1 - H_t^i)\lambda^i$

In particular, we consider the limit $\lim_{N\to\infty} \mathcal{R}^{i_1} - \mathcal{R}^{i_2}$, for any pair of firms i_1 and i_2.

For convenience, the two symmetric networks of Figure 1, disconnected and fully connected, are considered first and last respectively, while the 'Star' network of Figure 2 is considered in between.

'Disconnected' Network of Figure 1a.
If firms are not connected, the distribution of firms' intensity parameters is independent of the state H, so that $\lambda^i(\mathbf{H}_t) = \lambda^i$ and $\eta^i(\mathbf{H}_t) = \eta^i$, $i = 1, \ldots, N$, are independent and with identical distributions $F^\lambda(N)$ and $F^\eta(N)$, respectively.

Lemma 1 holds in this context, therefore we need only consider paired states H^{i_1} and $\overline{H}^{i_1}$, having all firms' (distress or not) states in common, except for firms i_1 and i_2: the former is in distress in H^{i_1} but not in $\overline{H}^{i_1}$. The converse holds for i_2.

As in (OA.A66) of Proposition 3, taking into account that $k = 1$ and the asymptotic homogeneity of dividends in Assumption 4, we have, for N large:

$$
\begin{aligned}
\mathcal{R}^{i_2}_{ND} - \mathcal{R}^{i_1}_{ND} &= \sum_{\substack{j=1\\ j\neq i_1,i_2}}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-\delta(t_u - t)} \sum_{H^{i_1}} \mathcal{U}(H^{i_1}, u) \\
\mathcal{U}(H^{i_1}, u) &= \left[Prob\left(\mathbf{H}_{t_u} = H^{i_1} \,\middle|\, \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \,\middle|\, \mathbf{H}_t^{\pm j}\right)\right] \mathcal{B}^1(H^{i_1}) \\
&\quad + \left[Prob\left(\mathbf{H}_{t_u} = H^{i_1} \,\middle|\, \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \,\middle|\, \mathbf{H}_t^{\pm j}\right)\right] \mathcal{B}^2(H^{i_1}) \qquad \text{(OA.C8)} \\
\mathcal{B}^1(H^{i_1}) &= \left[\underbrace{\sum_{\substack{v\in\mathcal{ND}(H^{i_1})\\ v\neq i_2}} \lambda^v \Delta \left(c^{i_2}(H^{i_1 - v}) - c^{i_1}(H^{i_1 - v}) - \left(c^{i_2}(H^{i_1}) - c^{i_1}(H^{i_1})\right)\right) -}_{1} \right. \\
&\quad \left. \underbrace{\left(\lambda^{i_2} - \lambda^{i_1}\right)\left(c^{i_2}(H^{i_1}) - c^{i_1}(H^{i_1})\right)\Delta}_{2} \right] \\
\mathcal{B}^2(H^{i_1}) &= \underbrace{(1-\delta)(c^{i_2}(H^{i_1}) - c^{i_1}(H^{i_1})) - \Delta\left(\eta^{i_1} c^{i_2}(H^{i_1}) - \eta^{i_2} c^{i_1}(H^{i_1})\right)}_{3} \\
&\quad \underbrace{+ \sum_{\substack{v\in\mathcal{D}(H^{i_1})\\ v\neq i_1}} \eta^v \Delta \left[\left(c^{i_2}(H^{i_1+v}) - c^{i_1}(H^{i_1+v})\right) - \left(c^{i_2}(H^{i_1}) - c^{i_1}(H^{i_1})\right)\right]}_{4}
\end{aligned}
$$

As $N \to \infty$, given a generic H^{i_1}, we have either $\lim_{N\to\infty} \mathcal{ND}(H^{i_1}) = K$, for some integer $K < \infty$, or $\lim_{N\to\infty} \mathcal{ND}(H^{i_1}) = \infty$. In the former case, $\lim_{N\to\infty} \mathcal{B}^1(H^{i_1}) = 0$ because of Assumption 1 and (OA.C4). In the latter, $\mathcal{B}^1(H^{i_1})$ is an infinite sum of independent random variables, because of Assumption 1. We assume that F^λ and $(\overline{f}(N), \underline{f}(N))$ are such that the Lindberg condition – see Durret (1995) – is satisfied, which is not restrictive in light of (OA.C4) and (OA.A19). The Lindberg-Feller theorem then mandates that $\lim_{N\to\infty} \mathcal{B}^1(H^{i_1}) = \epsilon_1$, where $\epsilon_1 \sim N(\mu_1, \sigma_1)$.[6] Similarly, we either have $\mathcal{B}^2(H^{i_1}) \approx 0$ or $\mathcal{B}^2(H^{i_1}) \approx \epsilon_2 \sim N(\mu_2, \sigma_2)$ for N large. We now

[6]Mean and variance parameters do not play a specific role, hence we leave them unspecified.

show that

$$\lim_{N\to\infty} Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\mathbf{H}_t^{\pm j}\right)=0 \tag{OA.C9}$$

We refer to the proof of Lemma 2 above, where we set $k=1$, because the network is disconnected, as assign its own λ^j (η^j) to the distress (recovery) event of firm j, instead of having homogeneous parameters. Terms 1 and 2 in (OA.A40) become:

$$\underbrace{p_t^{t_u-\Delta}(H^{i_1+i_1})\left(1-e^{-\lambda^{i_1}\Delta}\right)-p_t^{t_u-\Delta}(H^{i_1+i_1})\left(1-e^{-\lambda^{i_2}\Delta}\right)}_{1} \tag{OA.C10}$$
$$\underbrace{p_t^{t_u-\Delta}(H^{i_1-i_2})\left(1-e^{-\eta^{i_2}\Delta}\right)-p_t^{t_u-\Delta}(H^{i_1-i_2})\left(1-e^{-\eta^{i_1}\Delta}\right)}_{2}$$

For N large, (OA.C4) implies $\lambda^{i_2}\approx\lambda^{i_1}\approx\eta^{i_2}\approx\eta^{i_1}\approx 0$, so that term 1 $\approx$ term 2 ≈ 0. Terms 1 and 2 of (OA.A43) become

$$\underbrace{p_t^{t_u-2\Delta}(H^{i_1+i_1})\left[\left(1-e^{-\lambda^{i_1}\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\lambda^v\right]\Delta}\right.} \tag{OA.C11}$$

$$\underbrace{\left.-\left(1-e^{-\lambda^{i_2}\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\lambda^v\right]\Delta}\right]\lambda\Delta}_{1} \tag{OA.C12}$$

$$+\underbrace{p_t^{t_u-2\Delta}(H^{i_1-i_2})\left[\left(1-e^{-\eta^{i_2}\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\lambda^v\right]\Delta}\right.} \tag{OA.C13}$$

$$\underbrace{\left.-\left(1-e^{-\eta^{i_1}\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\lambda^v\right]\Delta}\right]\lambda\Delta}_{2} \tag{OA.C14}$$

As $N\to\infty$, the summations at the exponentials in square brackets converge to the same limit. Term 3 in (OA.A43) becomes

$$\lim_{N\to\infty}\left[\left(1-e^{-\lambda^{i_1}\Delta}\right)-\left(1-e^{-\lambda^{i_2}\Delta}\right)\right]\sum_{\substack{v\in\mathcal{D}(H^{i_1})\\ v\neq 1}} p_t^{t_u-2\Delta}(H^{i_1+v+i_1})\left(1-e^{-\lambda^v\Delta}\right)=0, \tag{OA.C15}$$

because $\lambda^{i_1}\approx\lambda^{i_2}$ and Assumption 4 guarantees that the summation converges to a bounded limit. The same reasoning applies to terms 4-6 in expression (OA.A43), and to terms of this type that arise from further backward substitutions (see the proof of Lemma 2). The rest of the proof is unchanged. Since $p_t^t(H)=0$ for any H of the type H^{i_1} and $\overline{H}^{i_1}$, by the assumption that i_1 and i_2 are not in distress in $\mathbf{H}_t^{\pm j}$, the limit (OA.C9) follows. In light of (OA.C8) we have

$$\lim_{N\to\infty}\mathcal{R}_{ND}^{i_2}-\mathcal{R}_{ND}^{i_1}=0, \tag{OA.C16}$$

so that

$$\lim_{N\to\infty}\mathcal{R}^{i_2}-\mathcal{R}^{i_1} = \sum_{j=i_1,i_2}\lambda^j\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}\sum_{H^{i_1}}\left\{\left[Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\mathbf{H}_t^{-j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\mathbf{H}_t^{-j}\right)\right]\times\right.$$
$$\left.\times\mathcal{B}^1(H^{i_1})+\left[Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\mathbf{H}_t^{-j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\mathbf{H}_t^{-j}\right)\right]\mathcal{B}^2(H^{i_1})\right\} \tag{OA.C17}$$

We now show that

$$\lim_{N\to\infty} Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\mathbf{H}_t^{-i_1}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\mathbf{H}_t^{-i_1}\right)$$
$$=-\lim_{N\to\infty} Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\mathbf{H}_t^{-i_2}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\mathbf{H}_t^{-i_2}\right) \quad \text{(OA.C18)}$$

We proceed as with the proof of (OA.C9), starting from (OA.A40) – after the proper modifications: $k=1$ and firm specific intensities – and canceling terms 1 and 2, then canceling terms 1-6 in (OA.A43), until we arrive at

$$p_t^{t_u}(H^{i_1})-p_t^{t_u}(\overline{H^{i_1}})\approx \text{RHS of (OA.A45)} \quad \text{(OA.C19)}$$

for N large. $\mathbf{H}_t^{-i_1}$ is of the form H^{i_1}, while $\mathbf{H}_t^{-i_2}$ is of the form $\overline{H}^{i_1}$, therefore letting $m=N^{\Delta}$ on the RHS of (OA.A45) it must be

$$(*)\quad Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\mathbf{H}_t^{-i_1}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\mathbf{H}_t^{-i_1}\right)=\mathcal{T}^0(m,H^{i_1})$$
$$(**)\quad Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\mathbf{H}_t^{-i_2}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\mathbf{H}_t^{-i_2}\right)=-\mathcal{T}^0(m,\overline{H}^{i_1})$$

or

$$(*)=\sum_{w_1=+1,-1}\ \sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^{i_1})\\ v_1\neq l(w_1)}}\mathcal{T}^1_{w_1}(m,H^{i_1},v_1)$$

$$(**)=-\sum_{w_1=+1,-1}\ \sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^{i_1})\\ v_1\neq \bar{l}(w_1)}}\mathcal{T}^1_{w_1}(m,\overline{H}^{i_1},v_1)$$

or

$$(*)=\sum_{w_1=+1,-1}\ \sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^{i_1})\\ v_1\neq l(w_1)}}\ \sum_{w_2=+1,-1}\ \sum_{\substack{v_2\in\mathcal{S}^{w_2}(H^{i_1+w_1v_1})\\ v_2\neq l(w_2)}}\mathcal{T}^2_{w_1,w_2}(m,H^{i_1},v_1,v_2)$$
$$(**)=-\sum_{w_1=+1,-1}\ \sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^{i_1})\\ v_1\neq \bar{l}(w_1)}}\ \sum_{w_2=+1,-1}\ \sum_{\substack{v_2\in\mathcal{S}^{w_2}(\overline{H}^{i_1+w_1v_1})\\ v_2\neq \bar{l}(w_2)}}\mathcal{T}^2_{w_1,w_2}(m,\overline{H}^{i_1},v_1,v_2)$$

or

...

... (OA.C20)

Notice that any set of the form $\mathcal{S}^{w_1}(H^{i_1})$, excluding $l(w_1)$, coincides with $\mathcal{S}^{w_1}(\overline{H}^{i_1})$ excluding $\bar{l}(w_1)$. Notice also that $\lim_{N\to\infty}\mathcal{T}^0(m,H^{i_1})=\lim_{N\to\infty}\mathcal{T}^0(m,\overline{H}^{i_1})$, $\lim_{N\to\infty}\mathcal{T}^1_{w_1}(m,H^{i_1},v_1)=\lim_{N\to\infty}\mathcal{T}^1_{w_1}(m,\overline{H}^{i_1},v_1)$, and so on: considering expressions (OA.A46)-(OA.A50), we notice that terms of the form $(1-\exp(-\omega^i\Delta))$, $\omega=\lambda,\eta$ become independent of the specific firm i because of (OA.C4). We can conclude that (OA.C18) holds. Considering expression (OA.C17), it is clear that (OA.C18) and the fact that $\lambda^{i_1}\approx\lambda^{i_2}$ for N large (because of (OA.A21)) imply that

$$\lim_{N\to\infty}\mathcal{R}^{i_2}-\mathcal{R}^{i_1}=0. \quad \text{(OA.C21)}$$

Thus

$$\lim_{N\to\infty}\left[\mu_t^{i_1}-\sum_{j=1}^{N}\tilde{\lambda}^j\right]\frac{P^{i_1}(\mathbf{H}_t)}{x_t^{i_1}(\mathbf{H}_t)}=\lim_{N\to\infty}\left[\mu_t^{i_2}-\sum_{j=1}^{N}\tilde{\lambda}^j\right]\frac{P^{i_2}(\mathbf{H}_t)}{x_t^{i_2}(\mathbf{H}_t)} \tag{OA.C22}$$

Since any two risk premia can be expressed asymptotically as a linear combination of each other, an exact one factor asymptotic structure holds for the expected returns of firms not currently in distress.

'Star' Network of Figure 2.
Firms' intensity parameters are independent only conditionally on a given state of aggregate (non) distress H. Thus parameters on the same row of the transition matrix $\mathbf{A}^H$ are mutually independent, but parameters on different rows are correlated. To model the 'star' network of Figure 2, let $\mathbf{H^1}$ denote the states where firm 1 (the central firm) is in distress, and $\overline{\mathbf{H^1}}$ the states where it is not. Then:

$$\begin{aligned}\lambda^i(\mathbf{H^1}) &= k\lambda^i, \quad k>1, \quad i=2,\ldots,N && \text{(OA.C23)}\\ \lambda^i(\overline{\mathbf{H^1}}) &= \lambda^i\sim F^{\lambda}(N), \quad i=1,\ldots,N && \text{(OA.C24)}\\ \eta(\mathbf{H^1}) &= \eta(\overline{\mathbf{H^1}})=\eta^i\sim F^{\eta}(N), \quad i=1,\ldots,N && \text{(OA.C25)}\end{aligned}$$

Let N denote a generic noncentral firm. Then, from the previous case:

$$\begin{aligned}\lim_{N\to\infty}\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1 &= \lim_{N\to\infty}\sum_{\substack{j=1\\ j\neq 1,N}}^{N}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}\sum_{H^1}\mathcal{U}(H^1,u)\\ \mathcal{U}(H^1,u) &= \left[kProb\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{\pm j}\right)\right]\epsilon^1(H^1)\\ &\quad+\left[Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\,\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\,\mathbf{H}_t^{\pm j}\right)\right]\epsilon^2(H^1)\\ \epsilon^1(H^1) &= \begin{cases}0 & \text{if } \lim_{N\to\infty}\mathcal{ND}(H^1)=K<\infty\\ \epsilon^1\sim N(\mu_1,\sigma_1) & \text{if } \lim_{N\to\infty}\mathcal{ND}(H^1)=\infty\end{cases}\\ \epsilon^2(H^1) &= \begin{cases}0 & \text{if } \lim_{N\to\infty}\mathcal{D}(H^1)=K<\infty\\ \epsilon^2\sim N(\mu_2,\sigma_2) & \text{if } \lim_{N\to\infty}\mathcal{D}(H^1)=\infty\end{cases} && \text{(OA.C26)}\end{aligned}$$

Because of Assumption 4, Lemma 2 in the proof of Proposition 3 holds in the present context. Let

$$\begin{aligned}\epsilon_D^1 &= \lambda^N(\mathbf{H}_t)\theta^N(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}\left[\sum_{H}Prob\left(H_{t_u}=H\middle|\,\mathbf{H}_t^{-N}\right)\mathcal{A}^N(H)\right]\\ &\quad-\lim_{N\to\infty}\lambda^1(\mathbf{H}_t)\theta^1(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}\left[\sum_{H}Prob\left(H_{t_u}=H\middle|\,\mathbf{H}_t^{-1}\right)\mathcal{A}^1(H)\right] && \text{(OA.C27)}\end{aligned}$$

Since N was an arbitrary noncentral firm, and all noncentral firms are identical, the random variable ϵ_D^1 does not depend on N. The same reasoning applies to $\lim_{N\to\infty}\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1$. Then

$$\mathcal{R}^N-\mathcal{R}^1=\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1+\epsilon_D^1=\varepsilon^1 \tag{OA.C28}$$

for N large, and

$$\lim_{N\to\infty}\left[\mu_t^1-\sum_{j=1}^N\tilde{\lambda}^j\right]\frac{P^1(\mathbf{H}_t)}{x_t^1(\mathbf{H}_t)}=\lim_{N\to\infty}\left[\mu_t^N-\sum_{j=1}^N\tilde{\lambda}^j\right]\frac{P^N(\mathbf{H}_t)}{x_t^N(\mathbf{H}_t)}+\varepsilon^1 \tag{OA.C29}$$

where ε^1 depends only on firm 1. Expression (OA.C29) shows that for the 'Star' network of Figure 2, an asymptotic three-fund separation holds.

Symmetrically Connected Network of Figure 1b.

In this case all firms are connected among each other, but the effect of a distress event on the rest of the firms does not depend on the specific firm that experiences the distress. All firms are 'central' in an homogeneous way. Without loss of generality, we model this network as follows:

$$\lambda^i(H) = \tilde{k}(H)\lambda^i, \quad i=1,2,\ldots,N \tag{OA.C30}$$

$$\lambda^i(H^{\mathrm{nd}}) = \lambda^i \sim F^{\lambda}(N), \quad i=1,\ldots,N \tag{OA.C31}$$

$$\eta(H) = \eta(H^{\mathrm{nd}})=\eta^i\sim F^{\eta}(N), \quad i=1,\ldots,N \tag{OA.C32}$$

$$\tilde{k}(H) = \begin{cases} k^{\mathrm{num}(\mathcal{D}(H))} & \text{if} \quad \mathrm{num}(\mathcal{D}(H))\le N^D \\ k^{N^D} & \text{otherwise}\end{cases} \qquad k>1, \tag{OA.C33}$$

If no firm is in distress (state H^{nd}), firm distress intensities are iid. If some firm is in distress, these intensities are compounded as many times as firms in distress at a common gross rate k, up to a maximum number N^D. As we are considering limiting behaviors as $N\to\infty$, and distress propagation needs to occur at bounded intensity, the boundedness assumption is necessary.

For two arbitrary firms 1 and N, since in the generic state H^1 and its paired $\overline{H}^1$ the same number of firms are in distress, expression (OA.C26) becomes:

$$\lim_{N\to\infty}\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1 = \lim_{N\to\infty}\sum_{\substack{j=1\\ j\neq 1,N}}^N(1+\tilde{k}(\mathbf{H}_t)\mathbf{1}(H_t^j=0))\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}\sum_{H^1}\mathcal{U}(H^1,u)$$

$$\begin{aligned}
\mathcal{U}(H^1,u) &= \tilde{k}(H^1)\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\epsilon^1(H^1)\\
&\quad+\left[Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\mathbf{H}_t^{\pm j}\right)\right]\epsilon^2(H^1)\\
\epsilon^1(H^1) &= \begin{cases}0 & \text{if} \quad \lim_{N\to\infty}\mathcal{ND}(H^1)=K<\infty\\ \epsilon^1\sim N(\mu_1,\sigma_1) & \text{if} \quad \lim_{N\to\infty}\mathcal{ND}(H^1)=\infty\end{cases}\\
\epsilon^2(H^1) &= \begin{cases}0 & \text{if} \quad \lim_{N\to\infty}\mathcal{D}(H^1)=K<\infty\\ \epsilon^2\sim N(\mu_2,\sigma_2) & \text{if} \quad \lim_{N\to\infty}\mathcal{D}(H^1)=\infty\end{cases},
\end{aligned} \tag{OA.C34}$$

We have

$$\lim_{N\to\infty}Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)=0 \qquad j=1,\ldots,N \tag{OA.C35}$$

We refer to the proof of Lemma 2. Terms 1 and 2 of (OA.A40) become

$$\underbrace{p_t^{t_u-\Delta}(H^{1+1})\left(1-e^{-\tilde{k}(H^{1+1})\lambda^1\Delta}\right)-p_t^{t_u-\Delta}(H^{1+1})\left(1-e^{-\tilde{k}(H^{1+1})\lambda^N\Delta}\right)}_{1} \tag{OA.C36}$$

$$\underbrace{p_t^{t_u-\Delta}(H^{1-N})\left(1-e^{-\eta^N\Delta}\right)-p_t^{t_u-\Delta}(H^{1-N})\left(1-e^{-\eta^1\Delta}\right)}_{2}$$

For N large, (OA.C4) implies $\lambda^{i_2}\approx\lambda^{i_1}\approx\eta^{i_2}\approx\eta^{i_1}\approx 0$, which together with the boundedness assumption (OA.C33) implies term 1 $\approx$ term 2 ≈ 0. Terms 1 and 2 of (OA.A43) become

$$\underbrace{p_t^{t_u-2\Delta}(H^{1+1})\left[\left(1-e^{-\tilde{k}(H^1)\lambda^1\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\tilde{k}(H^1)\lambda^v\right]\Delta}\right.} \tag{OA.C37}$$

$$\underbrace{\left.-\left(1-e^{-\tilde{k}(H^1)\lambda^N\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\tilde{k}(H^1)\lambda^v\right]\Delta}\right]\lambda\Delta}_{1} \tag{OA.C38}$$

$$+\underbrace{p_t^{t_u-2\Delta}(H^{1-N})\left[\left(1-e^{-\eta^N\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\tilde{k}(H^1)\lambda^v\right]\Delta}\right.} \tag{OA.C39}$$

$$\underbrace{\left.-\left(1-e^{-\eta^1\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\tilde{k}(H^1)\lambda^v\right]\Delta}\right]\lambda\Delta}_{2} \tag{OA.C40}$$

In light of (OA.C4) and (OA.C33), terms 1 and 2 also vanish for N large. The same reasoning applies to terms 1-6 in expression (OA.A43), and to the term of this type that arise from further backward substitutions (see the proof of Lemma 2). The rest of the proof is unchanged. Since $p_t^t(H)=0$ for any H of the type H^1 and $\overline{H}^1$, by the assumption that firm 1 and N are not in distress in $\mathbf{H}_t^{\pm j}$, the limit (OA.C35) follows, and

$$\lim_{N\to\infty}\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1=0, \tag{OA.C41}$$

so that

$$\begin{aligned}\lim_{N\to\infty}\mathcal{R}^N-\mathcal{R}^1 \;=\; &\sum_{j=1,N}\tilde{k}(\mathbf{H}_t)\lambda^j\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}\sum_{H^1}\left\{\tilde{k}(H^1)\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{-j}\right)\right.\right. && \text{(OA.C42)}\\ &\left.-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{-j}\right)\right]\mathcal{B}^1(H^1)\\ &\left.+\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{-j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{-j}\right)\right]\mathcal{B}^2(H^1)\right\} && \text{(OA.C43)}\end{aligned}$$

We now show that

$$\begin{aligned}&\lim_{N\to\infty}Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\mathbf{H}_t^{-1}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{-1}\right)\\ &\qquad=-\lim_{N\to\infty}Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{-N}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{-N}\right)\end{aligned} \tag{OA.C44}$$

We proceed as with the proof of (OA.C9), starting from (OA.A40) – after the proper modifications: $k=1$ outside of exponentials, firm specific intensities of distress $\lambda^i\tilde{k}(H_t)$ and recovery η^i – and canceling terms 1 and 2, then canceling terms 1-6 in(OA.A43), until we arrive at

$$p_t^{t_u}(H^{i_1}) - p_t^{t_u}(\overline{H^{i_1}}) \approx \text{RHS of (OA.A45)} \tag{OA.C45}$$

for N large. Term $\mathcal{T}^0(m,H)$ becomes

$$\mathcal{T}^0(m,H) \;=\; \begin{cases} 1 \text{ if } m=0 \\ \mathcal{T}^0(m-1,H)e^{-\left[\sum_{v\in\mathcal{D}(H)}\eta^v+\sum_{v\in\mathcal{ND}(H)}\tilde{k}(H)\lambda^v\right]\Delta} & \text{otherwise} \end{cases} \tag{OA.C46}$$

and similarly for higher order terms in (OA.A46)-(OA.A50). $\mathbf{H}_t^{-1}$ is of the form H^1, while $\mathbf{H}_t^{-N}$ is of the form $\overline{H}^1$, therefore letting $m = N^{\Delta}$ on the RHS of (OA.A45) it must be

$$\begin{aligned}
&(*) \quad Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{-1}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{-1}\right) = \mathcal{T}^0(m,H^1)\\
&(**) \quad Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{-N}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{-N}\right) = -\mathcal{T}^0(m,\overline{H}^1)\\
&\text{or}\\
&(*) = \sum_{w_1=+1,-1}\ \sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}} \mathcal{T}^1_{w_1}(m,H^1,v_1)\\
&(**) = -\sum_{w_1=+1,-1}\ \sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}} \mathcal{T}^1_{w_1}(m,\overline{H}^1,v_1)\\
&\text{or}\\
&(*) = \sum_{w_1=+1,-1}\ \sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}}\ \sum_{w_2=+1,-1}\ \sum_{\substack{v_2\in\mathcal{S}^{w_2}(H^{1+w_1v_1})\\ v_2\neq l(w_2)}} \mathcal{T}^2_{w_1,w_2}(m,H^1,v_1,v_2)\\
&(**) = -\sum_{w_1=+1,-1}\ \sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}}\ \sum_{w_2=+1,-1}\ \sum_{\substack{v_2\in\mathcal{S}^{w_2}(\overline{H}^{1+w_1v_1})\\ v_2\neq \bar{l}(w_2)}} \mathcal{T}^2_{w_1,w_2}(m,\overline{H}^1,v_1,v_2)\\
&\text{or}\\
&\dots\\
&\dots
\end{aligned} \tag{OA.C47}$$

Notice that any set of the form $\mathcal{S}^{w_1}(H^1)$, excluding $l(w_1)$, coincides with $\mathcal{S}^{w_1}(\overline{H}^1)$ excluding $\bar{l}(w_1)$. Notice also that $\lim_{N\to\infty}\mathcal{T}^0(m,H^1) = \lim_{N\to\infty}\mathcal{T}^0(m,\overline{H}^1)$, $\lim_{N\to\infty}\mathcal{T}^1_{w_1}(m,H^1,v_1) = \lim_{N\to\infty}\mathcal{T}^1_1(m,\overline{H}^1,v_1)$, and so on: considering expressions (OA.A46)-(OA.A50), we notice that terms of the form $(1-\exp(-\omega^i\Delta))$, $\omega = \tilde{k}(H)\lambda,\eta$ become independent of the specific firm i because of (OA.C4) and (OA.C33). We can conclude that (OA.C44) holds. Considering expression (OA.C17), it is clear that (OA.C44) and the fact that $\tilde{k}(\mathbf{H}_t)\lambda^1 \approx \tilde{k}(\mathbf{H}_t)\lambda^2$ for N large (because of (OA.C4)) imply that $\lim_{N\to\infty}\mathcal{R}^N \approx \lim_{N\to\infty}\mathcal{R}^1$, and that an exact conditional one factor asymptotic structure holds for expected returns of the assets that are not currently in distress.

Proof of Corollary OA.C 1. We use the notation of the proof of Proposition 3. We model a clustered economy with $\overline{N}$ central firms as follows: $\mathcal{G}$ is the collection of central firms; each firm $j \in \mathcal{G}$ is central for his own subnetwork, which is organized as in Figure 2. $\mathcal{NG}_j$ denotes the collection of noncentral firms in the subnetwork of j, with $(\mathcal{NG}_{j_1} \bigcap \mathcal{NG}_{j_2}) = \varnothing$, $j_1, j_2 \in \mathcal{G}$. Central firms are all symmetrically interconnected for simplicity, as in Figure 1b. We summarize this description as follows:

$$\lambda^i(H) = \tilde{k}^i(H)\lambda^i, \quad i = 1, 2, \ldots, N \tag{OA.C48}$$
$$\lambda^i = \sim F^{\lambda}(N), \quad i = 1, \ldots, N \tag{OA.C49}$$
$$\eta(H) = \eta^i \sim F^{\eta}(N), \quad i = 1, \ldots, N \tag{OA.C50}$$
$$\tilde{k}^i(H) = \begin{cases} \prod_{v \in \mathcal{D}_c(H)} k_0 & \text{if} \quad i \in \mathcal{G} \\ k_j & \text{if} \quad i \in \mathcal{NG}_j \end{cases} \quad k_0, k_j > 1,\ j \in \mathcal{G}. \tag{OA.C51}$$

The number of central firms is independent of the economy size N. $\mathcal{D}_c(H)$ denotes the set of central firms that are in distress in state H. Therefore each central distress event compounds the distress risk of other central firms at some homogeneous rate k_0. Noncentral firms are affected only by the distress of their 'Star'. Assumption 3 and Assumption 4 hold.

To identify an asymptotic factor structure of expected returns, we relate the risk premia of any two firms of all possible types. As in Proposition OA.C 1 and Proposition 3, when we consider the risk premia of firms i and j, it is convenient to decompose the space of possible states H into: *i*) paired states $(H^i, \overline{H}^i)$ – i (j) is in distress in the former (latter), but not in the latter (former), while the state of all other firms coincide –; *ii*) states H where both i and j are not in distress; *iii*) states where they are both in distress. The following Lemma allows to concentrate only on cases *i*) and *ii*).

Lemma 4. *If firms i and j are both in distress in state H, then $\mathcal{A}^i(H) - \mathcal{A}^j(H) = 0$. If both are not in distress in H, then*

$$\mathcal{A}^i(H) - \mathcal{A}^j(H) = \left(\tilde{k}^i(H)\lambda^i - \tilde{k}^j(H)\lambda^j\right) \Delta \left(C^i(H^{-i}) - C^j(H^{-i})\right) \tag{OA.C52}$$

Proof. If i and j are both in distress, we have:

$$\begin{aligned} \mathcal{A}^i(H) - \mathcal{A}^j(H) = & \sum_{v \in \mathcal{ND}(H)} \tilde{k}^v(H)\lambda^v \Delta \left[c^i(H^{-v}) - c^j(H^{-v}) - \left(c^i(H) - c^j(H)\right)\right] \\ & - \sum_{\substack{v \in \mathcal{D}(H) \\ v \neq i,j}} \eta\Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{+v}) - c^j(H^{+v})\right)\right] + (1-\delta)(c^i(H) - c^j(H)) \\ & - \underbrace{\eta\Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{+i}) - c^j(H^{+i})\right)\right]}_{1} - \underbrace{\eta\Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{+j}) - c^j(H^{+j})\right)\right]}_{2} \end{aligned} \tag{OA.C53}$$

$c^i(H) - c^j(H) = 0$, $c^i(H^{-v}) - c^j(H^{-v}) = 0$, $c^i(H^{+v}) - c^j(H^{+v}) = 0$, $v \neq i, j$, while terms 1 and 2 in (OA.C53) are opposite, so that $\mathcal{A}^i(H) - \mathcal{A}^j(H) = 0$. If i and j are both not in distress, we have:

$$\begin{aligned} \mathcal{A}^i(H) - \mathcal{A}^j(H) = & \sum_{\substack{v \in \mathcal{ND}(H) \\ v \neq i,j}} \tilde{k}^v(H)\lambda^v \Delta \left[c^i(H^{-v}) - c^j(H^{-v}) - \left(c^i(H) - c^j(H)\right)\right] \\ & - \sum_{v \in \mathcal{D}(H)} \eta\Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{+v}) - c^j(H^{+v})\right)\right] + (1-\delta)(c^i(H) - c^j(H)) \\ & - \underbrace{\lambda^i \tilde{k}^i(H)\Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{-i}) - c^j(H^{-i})\right)\right]}_{1} - \lambda^j \tilde{k}^j(H) \underbrace{\eta\Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{-j}) - c^j(H^{-j})\right)\right]}_{2} \end{aligned} \tag{OA.C54}$$

Again $c^i(H) - c^j(H) = 0$, $c^i(H^{-v}) - c^j(H^{-v}) = 0$, $c^i(H^{+v}) - c^j(H^{+v}) = 0$, $v \neq i, j$. (OA.C52) follows adding terms 1 and 2 and taking into account that $c^i(H^{-i}) - c^j(H^{-i}) = -(c^i(H^{-j}) - c^j(H^{-j}))$. $\square$

For ease of notation, we call firms 1 and N regardless of their type.

Firm 1 is central, Firm N is not

Let H_d denote a generic state where firms 1 and N are both in distress. Adapting (OA.A66) of Proposition 3 to the network characteristics reported in (OA.C48), keeping in mind the asymptotic homogeneity of dividends, and taking Lemma 4 into account:

$$\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} = \sum_{\substack{v=j \\ j\neq 1,N}}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty} e^{-\delta(t_u - t)}\left(\sum_{H^1}\mathcal{U}^1(H^1,u) + \sum_{H_d}\mathcal{U}^2(H_d,u)\right) \tag{OA.C55}$$

$$\begin{aligned}
\mathcal{U}(H^1,u) = & \underbrace{\sum_{\substack{u\in\mathcal{G} \\ u\neq 1}}\sum_{\substack{v\in\mathcal{ND}(H^1) \\ v\in\mathcal{NG}_u \\ v\neq N}} \tilde{k}^v(H^1)\lambda^v \left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]\Delta\left(c^N(H^{1-v}) - c^1(H^{1-v}) - \right.}_{} \\
& \underbrace{\left.\left(c^N(H^1) - c^1(H^1)\right)\right)}_{1} + \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1) \\ v\in\mathcal{NG}_1 \\ v\neq N}}\left[k_1 Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]\lambda^v\Delta\left(c^N(H^{1-v}) - c^1(H^{1-v})\right.}_{} \\
& \underbrace{\left.-\left(c^N(H^1) - c^1(H^1)\right)\right)}_{2} - \underbrace{\left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]}_{} \times \\
& \times \underbrace{\left(\lambda^N\tilde{k}^N(H^1) - \lambda^1\tilde{k}^1(H^1)\right)\left(c^N(H^1) - c^1(H^1)\right)\Delta}_{3} \\
& + \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1) \\ v\in\mathcal{G} \\ v\neq 1}} \tilde{k}^v(H^1)\lambda^v\left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]\Delta\left(c^N(H^{1-v}) - c^1(H^{1-v}) - \right.}_{} \\
& \underbrace{\left.\left(c^N(H^1) - c^1(H^1)\right)\right)}_{4} + \underbrace{\left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]}_{} \times \\
& \times \underbrace{\left[(1-\delta)(c^N(H^1) - c^1(H^1)) - \Delta\left(\eta^N c^N(H^1) - \eta^1 c^1(H^1)\right)\right.}_{} \\
& \underbrace{\left.+ \sum_{\substack{v\in\mathcal{D}(H^1) \\ v\neq 1}}\eta^v\Delta\left(\left(c^N(H^{1+v}) - c^1(H^{1+v})\right) - \left(c^N(H^1) - c^1(H^1)\right)\right)\right]}_{5}
\end{aligned}$$

$$\mathcal{U}^2(H_d,u) = Prob\left(\mathbf{H}_{t_u} = H_d \middle| \mathbf{H}_t^{\pm j}\right)\left(\tilde{k}^N(H_d)\lambda^N - \tilde{k}^1(H_d)\lambda^1\right)\Delta\left(c^N(H_d^{-N}) - c^1(H_d^{-N})\right)$$

We note that $\tilde{k}^i(H) < \infty$, $\forall N$, $i = 1, \ldots, N$, because the number of central firms, $\overline{N}$, is bounded by assumption. Notice that the characteristics of the specific subnetwork of firm N enter only in term 3, which is o(term 2) as $N \to \infty$. By Assumptions 3 and 4, and the fact that $Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right)$ and $Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right)$ converge to a deterministic limit as $N \to \infty$, terms 1, 2, 4, 5 are sums of independent random variables. We also assume that the Linderg-Feller conditon holds. The Central Limit Theorem then guarantees asymptotic convergence of the sum of these terms to a random variables which only depends on firm 1 characteristics, because of the asymptotic behavior of term 3. Thus

$$\lim_{N\to\infty} \sum_{H^1} \mathcal{U}^1(H^1, u) = \epsilon^1_{u,j}(1) \tag{OA.C56}$$

Due to Assumptions 1 and 4, $\sum_{H_d} \mathcal{U}^2(H_d, u)$ is a sum of independent random variables. By the Central Limit Theorem argument already applied in Proposition OA.C 1:

$$\lim_{N\to\infty} \sum_{H_d} \mathcal{U}^2(H_d, u) = \epsilon^2_{u,j}(1, N) \sim N(\mu^2_{u,j}, \sigma^2_{u,j}) \tag{OA.C57}$$

The random variable $\epsilon^2_{u,j}(1, N)$ in general takes positive and negative values with nonzero probability. Its distribution depends on the centrality parameters ks. Thus, by means of (OA.C55), and again the Central Limit Theorem argument

$$\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} \approx \sum_{\substack{v=j \\ j\neq 1,N}}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-\delta(t_u - t)} \left(\epsilon^1_{u,j}(1) + \epsilon^2_{u,j}(1, N)\right) \tag{OA.C58}$$

for N large. (OA.C58) then implies that

$$\lim_{N\to\infty} \mathcal{R}^N - \mathcal{R}^1 = \lim_{N\to\infty} \mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} + \sum_{j=1,N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-\delta(t_u - t)} \left(\sum_{H^1} \mathcal{U}^1(H^1, u) + \sum_{H_d} \mathcal{U}^2(H_d, u)\right) \tag{OA.C59}$$

Due to Assumption 4 $\tilde{\lambda}^1 \approx \tilde{\lambda}^N \approx 0$ for N large, while $\left(\sum_{H^1} \mathcal{U}^1(H^1, u) + \sum_{H_d} \mathcal{U}^2(H_d, u)\right)$ converge to a random variable that is bounded $P-$a.s, by virtue of Assumptions 3 and 4. We conclude that

$$\lim_{N\to\infty} \mathcal{R}^N - \mathcal{R}^1 = \sum_{\substack{v=j \\ j\neq 1,N}}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-\delta(t_u - t)} \left(\epsilon^1_{u,j}(1) + \epsilon^2_{u,j}(1, N)\right) \tag{OA.C60}$$

Firm 1 is central, Firm N is central (OA.C55) becomes

$$\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} = \sum_{\substack{v=j \\ j\neq 1,N}}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-\delta(t_u - t)} \left(\sum_{H^1} \mathcal{U}^1(H^1, u) + \sum_{H_d} \mathcal{U}^2(H_d, u)\right) \tag{OA.C61}$$

$$\mathcal{U}(H^1,u) = \underbrace{\sum_{\substack{u\in\mathcal{G}\\u\neq 1,N}} \sum_{\substack{v\in\mathcal{ND}(H^1)\\v\in\mathcal{NG}_u}} \tilde{k}^v(H^1)\lambda^v \left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\Delta\left(c^N(H^{1-v})-c^1(H^{1-v})-\right.}$$

$$\underbrace{\left.\left(c^N(H^1)-c^1(H^1)\right)\right)}_{1} + \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\v\in\mathcal{NG}_1}} \left[k_1 Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\lambda^v\Delta\left(c^N(H^{1-v})-c^1(H^{1-v})\right.}$$

$$\underbrace{\left.-\left(c^N(H^1)-c^1(H^1)\right)\right)}_{2} + \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\v\in\mathcal{NG}_N}} \left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - k_N Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\lambda^v\Delta\left(c^N(H^{1-v})-c^1(H^{1-v})\right.}$$

$$\underbrace{\left.-\left(c^N(H^1)-c^1(H^1)\right)\right)}_{3} - \underbrace{\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\left(\lambda^N\tilde{k}^N(H^1)-\lambda^1\tilde{k}^1(H^1)\right)\times}$$

$$\underbrace{\times\left(c^N(H^1)-c^1(H^1)\right)\Delta}_{4} + \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\v\in\mathcal{G}\\v\neq 1,N}} \tilde{k}^v(H^1)\lambda^v\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\Delta\times}$$

$$\underbrace{\times\left(c^N(H^{1-v})-c^1(H^{1-v})-\left(c^N(H^1)-c^1(H^1)\right)\right)}_{5} + \underbrace{\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\times}$$

$$\times\underbrace{\left[(1-\delta)(c^N(H^1)-c^1(H^1)) - \Delta\left(\eta^N c^N(H^1)-\eta^1 c^1(H^1)\right)\right.}$$

$$\underbrace{\left. + \sum_{\substack{v\in\mathcal{D}(H^1)\\v\neq 1}} \eta^v\Delta\left(\left(c^N(H^{1+v})-c^1(H^{1+v})\right)-\left(c^N(H^1)-c^1(H^1)\right)\right)\right]}_{6}$$

$$\mathcal{U}^2(H_d,u) = Prob\left(\mathbf{H}_{t_u}=H_d\middle|\mathbf{H}_t^{\pm j}\right)\left(\tilde{k}^N(H_d)\lambda^N - \tilde{k}^1(H_d)\lambda^1\right)\Delta\left(c^N(H_d^{-N})-c^1(H_d^{-N})\right)$$

Due to terms 2 and 3, Assumptions 3 and 4 allow to apply the Central Limit Theorem, which guarantees convergence of $\mathcal{U}^1(H^1,u)$ to a random variable – or more correctly, to a sum of random variables – that depends on the characteristics of the central firms 1 and N. Taking into account that $\lambda^1 \approx \lambda^N \approx 0$ for N large, we have :

$$\begin{aligned} \lim_{N\to\infty}\mathcal{R}^N-\mathcal{R}^1 &= \lim_{N\to\infty}\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND} + \sum_{j=1,N}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}\sum_{H^1}\mathcal{U}^1(H^1,u) \\ &= \lim_{N\to\infty}\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND} = \sum_{\substack{v=j\\j\neq 1,N}}^{N}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}\epsilon^3_{u,j}(1,N) \end{aligned} \quad \text{(OA.C62)}$$

Firm 1 is not central, Firm N is not central (OA.C55) becomes

$$\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} = \sum_{\substack{v=j \\ j\neq 1,N}}^{N} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty} e^{-\delta(t_u - t)}\left(\sum_{H^1}\mathcal{U}^1(H^1,u) + \sum_{H_d}\mathcal{U}^2(H_d,u)\right) \quad \text{(OA.C63)}$$

$$\mathcal{U}(H^1,u) = \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} \tilde{k}^v(H^1)\lambda^v \left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]\Delta\left(c^N(H^{1-v}) - c^1(H^{1-v}) - \right.}$$

$$\underbrace{\left.\left(c^N(H^1) - c^1(H^1)\right)\right)}_{1} \underbrace{- \left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]\left(\lambda^N\tilde{k}^N(H^1) - \lambda^1\tilde{k}^1(H^1)\right)\left(c^N(H^1) - c^1(H^1)\right)\Delta}_{2}$$

$$+\underbrace{\left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]\left[(1-\delta)(c^N(H^1) - c^1(H^1)) - \Delta\left(\eta^N c^N(H^1) - \eta^1 c^1(H^1)\right)\right.}$$

$$\underbrace{\left. + \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \eta^v \Delta\left(\left(c^N(H^{1+v}) - c^1(H^{1+v})\right) - \left(c^N(H^1) - c^1(H^1)\right)\right)\right]}_{3}$$

$$\mathcal{U}^2(H_d,u) = Prob\left(\mathbf{H}_{t_u} = H_d \middle| \mathbf{H}_t^{\pm j}\right)\left(\tilde{k}^N(H_d)\lambda^N - \tilde{k}^1(H_d)\lambda^1\right)\Delta\left(c^N(H_d^{-N}) - c^1(H_d^{-N})\right)$$

Notice that term 2 is o(term 1) for $N \to \infty$, while applying previous arguments, terms 1 and 3 converge to a random variable that doesn't depend on the firm 1 and N's subnetwork. On the other hand:

$$\lim_{N\to\infty}\sum_{H_d}\mathcal{U}^2(H_d,u) = \epsilon^4_{u,j}(1,N) \sim N(\mu^4_{u,j}, \sigma^4_{u,j}) \quad \text{(OA.C64)}$$

where $\epsilon^4_{u,j}(1,N)$ in nonnegative $P-a.s.$ if $k_N \leq k_1$. Summarizing:

$$\begin{aligned}
\lim_{N\to\infty}\mathcal{R}^N - \mathcal{R}^1 &= \lim_{N\to\infty}\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} + \sum_{j=1,N}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}\left(\sum_{H^1}\mathcal{U}^1(H^1,u) + \sum_{H_d}\mathcal{U}^2(H_d,u)\right) \\
&= \lim_{N\to\infty}\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} = \sum_{\substack{v=j\\ j\neq 1,N}}^{N}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-\delta(t_u-t)}, \epsilon^4_{u,j}(1,N) \quad \text{(OA.C65)}
\end{aligned}$$

after taking into account that, due to Assumption 4, $\tilde{\lambda}^1 \approx \tilde{\lambda}^N \approx 0$ for N large, while $\left(\sum_{H^1}\mathcal{U}^1(H^1,u) + \sum_{H_d}\mathcal{U}^2(H_d,u)\right)$ converge to a random variable that is bounded $P-$a.s.

Substituting (OA.C60), then (OA.C62), then (OA.C65) into expression (OA.C7) for the risk premium, we realize that the risk premium of firm i, can be expressed as a linear combination of the risk premium of firm j and of an additional random variable $\varepsilon(i,j)$

$$\lim_{N\to\infty}\left[\mu^i_t - \sum_{v=1}^{N}\tilde{\lambda}^v\right]\frac{P^i(\mathbf{H}_t)}{x^i_t(\mathbf{H}_t)} = \lim_{N\to\infty}\left[\mu^j_t - \sum_{v=1}^{N}\tilde{\lambda}^v\right]\frac{P^j(\mathbf{H}_t)}{x^j_t(\mathbf{H}_t)} + \varepsilon(i,j) \quad \text{(OA.C66)}$$

Table I – For the 'Star' network of Proposition OA.C 1, the table reports the critical k^* for different parameter combinations. It also reports the percentage of states for which condition $p_t^{t_u}(H^1) < p_t^{t_u}(\overline{H^1})$ is violated when $k^i < k^*$, where k^i is 1.05 for $i = 1$, k^* for $i = 6$, and is linearly increasing in i. $t_u - t = 1$ year.

(λ, η)	k^*	% of violations				
		k^1	k^2	k^3	k^4	k^5
(0.40 , 0.50)	22.02	61.72	22.05	6.58	1.29	0.28
(0.43 , 0.50)	19.96	57.77	19.55	5.41	1.29	0.28
(0.47 , 0.50)	18.23	54.30	17.17	5.03	1.08	0.27
(0.50 , 0.50)	16.74	49.52	14.98	4.48	1.08	0.27
(0.50 , 0.40)	15.72	45.34	14.17	4.48	1.14	0.27
(0.50 , 0.43)	16.15	47.42	14.42	4.48	1.02	0.27
(0.50 , 0.47)	16.61	49.41	15.39	4.47	1.08	0.27
(0.50 , 0.50)	17.13	50.75	15.41	4.48	1.08	0.27

$\varepsilon(i, j)$ depends on firm i's and j's type: on the specific firms if they are central, on their subnetwork if they are not central. Since any two $\varepsilon(i_1, j_1)$ and $\varepsilon(i_2, j_2)$ are imperfectly correlated, there are $\overline{N}$ central firms and $\overline{N}$ subnetworks, any risk premium can be expressed as a linear combination of $2\overline{N}$ other risk premia, and a $2\overline{N} + 1$-factor representation holds.

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Online Appendix D: Additional Tables

Table II – Average monthly returns of portfolio sorted according to market beta and size.

Average monthly returns, July 1963-June 2007: market beta-size deciles

	β avg	$ME-1$	$ME-2$	$ME-3$	$ME-4$	
ME avg		0.03114	0.02247	0.01776	0.01401	
$\beta-1$	0.01436	0.03331	0.02256	0.01543	0.01300	
$\beta-2$	0.01334	0.03352	0.01842	0.01480	0.01214	
$\beta-3$	0.01257	0.02857	0.01600	0.01608	0.01408	
$\beta-4$	0.01391	0.02626	0.02870	0.01894	0.01492	
$\beta-5$	0.01419	0.02959	0.02133	0.01792	0.01221	
$\beta-6$	0.01515	0.04108	0.02340	0.01917	0.01877	
$\beta-7$	0.01398	0.03423	0.02160	0.01717	0.01580	
$\beta-8$	0.01363	0.02404	0.02410	0.02657	0.01327	
$\beta-9$	0.01323	0.03041	0.02483	0.01593	0.01135	
$\beta-10$	0.01236	0.03038	0.02381	0.01560	0.01454	
	$ME-5$	$ME-6$	$ME-7$	$ME-8$	$ME-9$	$ME-10$
ME avg	0.01117	0.01089	0.00921	0.00855	0.00714	0.00440
$\beta-1$	0.01143	0.01129	0.01123	0.01126	0.00938	0.00468
$\beta-2$	0.01031	0.01068	0.00956	0.00829	0.00885	0.00685
$\beta-3$	0.00932	0.01245	0.00861	0.00784	0.00639	0.00642
$\beta-4$	0.01322	0.00872	0.00918	0.00725	0.00795	0.00391
$\beta-5$	0.01432	0.01506	0.00957	0.00932	0.00725	0.00529
$\beta-6$	0.01071	0.01025	0.00914	0.00738	0.00824	0.00338
$\beta-7$	0.01241	0.01073	0.00853	0.00821	0.00767	0.00343
$\beta-8$	0.01017	0.00856	0.01007	0.00922	0.00584	0.00444
$\beta-9$	0.01158	0.01258	0.01024	0.00793	0.00472	0.00275
$\beta-10$	0.00822	0.00860	0.00599	0.00880	0.00511	0.00287

Table III – Post-ranking betas of portfolios sorted according to market beta and book-equity/market-equity.

Post-ranking betas, July 1963-June 2007: market beta- size deciles

	β avg	$B/M-1$	$B/M-2$	$B/M-3$	$B/M-4$	
B/M avg		1.13112	1.11346	1.08451	1.16000	
$\beta-1$	0.81325	1.00302	0.73155	0.78403	0.74576	
$\beta-2$	0.83423	0.81439	0.84641	0.78948	0.82510	
$\beta-3$	0.89889	0.92320	0.93405	0.83764	0.90155	
$\beta-4$	1.00131	0.96969	0.96846	1.09347	1.06513	
$\beta-5$	1.07845	1.21609	1.01639	1.07719	1.14484	
$\beta-6$	1.10929	1.20066	1.15471	1.07706	1.02531	
$\beta-7$	1.16892	1.12261	1.08995	1.25656	1.32119	
$\beta-8$	1.23686	1.20130	1.22913	1.03735	1.33045	
$\beta-9$	1.39660	1.30480	1.45803	1.32287	1.44631	
$\beta-10$	1.65830	1.55541	1.70595	1.56945	1.79437	
	$B/M-5$	$B/M-6$	$B/M-7$	$B/M-8$	$B/M-9$	$B/M-10$
B/M avg	1.16694	1.16423	1.14553	1.14887	1.09106	0.99038
$\beta-1$	0.87035	0.85323	0.75812	0.84795	0.75506	0.78344
$\beta-2$	0.83912	0.93062	0.73784	0.89570	0.83267	0.83100
$\beta-3$	0.88902	0.98963	0.99362	0.88632	0.96475	0.66913
$\beta-4$	0.99454	1.01981	1.03369	0.98158	1.00596	0.88083
$\beta-5$	1.10714	1.12278	1.03598	1.09060	1.04324	0.93030
$\beta-6$	1.09279	1.18042	1.15718	1.11595	1.09944	0.98937
$\beta-7$	1.21830	1.18384	1.21744	1.12192	1.14475	1.01262
$\beta-8$	1.39047	1.30383	1.31240	1.26455	1.17489	1.12419
$\beta-9$	1.50720	1.42414	1.53859	1.38541	1.32540	1.25329
$\beta-10$	1.76048	1.63396	1.67049	1.89873	1.56448	1.42965

Table IV

– Average monthly returns of portfolio sorted according to market beta and book-equity/market-equity.

Average monthly returns, July 1963-June 2007: market beta- b/m deciles

	β avg	$B/M-1$	$B/M-2$	$B/M-3$	$B/M-4$	
B/M avg		0.00056	0.00303	0.00486	0.00612	
$\beta-1$	0.01177	0.00200	0.00116	0.00535	0.00372	
$\beta-2$	0.01044	0.00284	0.00590	0.00553	0.00430	
$\beta-3$	0.01065	-0.00021	0.00582	0.00595	0.00679	
$\beta-4$	0.01006	0.00094	0.00066	0.00060	0.00557	
$\beta-5$	0.01021	-0.00101	0.00465	0.00533	0.00688	
$\beta-6$	0.00848	0.00059	0.00190	0.00625	0.00513	
$\beta-7$	0.00942	-0.00117	0.00114	0.00348	0.00825	
$\beta-8$	0.00959	0.00059	0.00528	0.00371	0.00676	
$\beta-9$	0.01120	0.00157	0.00256	0.00577	0.00710	
$\beta-10$	0.01055	-0.00047	0.00124	0.00666	0.00669	
	$B/M-5$	$B/M-6$	$B/M-7$	$B/M-8$	$B/M-9$	$B/M-10$
B/M avg	0.00723	0.00815	0.01092	0.01274	0.01480	0.03396
$\beta-1$	0.00723	0.00973	0.00885	0.01613	0.01396	0.04959
$\beta-2$	0.00582	0.00694	0.01034	0.01421	0.01467	0.03384
$\beta-3$	0.00610	0.00704	0.00783	0.01144	0.01706	0.03869
$\beta-4$	0.00612	0.00946	0.01177	0.01390	0.01295	0.03866
$\beta-5$	0.00660	0.01035	0.01280	0.00987	0.01213	0.03452
$\beta-6$	0.00771	0.00521	0.01125	0.00724	0.01135	0.02813
$\beta-7$	0.00683	0.00664	0.00913	0.01116	0.01610	0.03270
$\beta-8$	0.00579	0.00743	0.01096	0.00959	0.01690	0.02888
$\beta-9$	0.01077	0.01029	0.01347	0.01940	0.01384	0.02727
$\beta-10$	0.00937	0.00844	0.01279	0.01447	0.01901	0.02731

Table V – Parameter estimates. Model (36)-(37) is estimated on the earnings of beta-size sorted portfolios, using the maximum likelihood procedure described in the Appendix. The panel reports estimated parameters λ_0, η and c for each portfolio, and their standard errors in parenthesis.

Parameter Estimates

	$ME - \beta$	1-1	1-2	1-3	1-4	2-1	2-2	2-3	2-4
λ_0		0.4732	0.6860	0.3545	0.9640	0.3158	0.7741	0.6917	0.7557
		(0.0807)	(0.1856)	(0.1141)	(0.2554)	(0.0419)	(0.1893)	(0.1975)	(0.2076)
η		0.9304	0.4327	0.1474	0.4909	0.8869	0.3863	0.3157	0.7873
		(0.1522)	(0.1159)	(0.0433)	(0.1369)	(0.1107)	(0.0931)	(0.0873)	(0.2022)
c		7.0176	10.0707	5.1481	5.2788	2.0002	2.2135	2.5144	5.7365
		(1.7015)	(2.6212)	(1.5894)	(1.5202)	(0.5415)	(0.6425)	(0.6932)	(1.5461)
	$ME - \beta$	3-1	3-2	3-3	3-4	4-1	4-2	4-3	4-4
λ_0		0.5654	0.2790	0.2756	0.4094	0.2476	0.2587	0.2743	0.4069
		(0.0726)	(0.0537)	(0.0654)	(0.0713)	(0.0366)	(0.0435)	(0.0549)	(0.0874)
η		0.7698	0.0371	0.0631	0.7273	0.7595	0.3604	0.1098	0.6325
		(0.0956)	(0.0069)	(0.0136)	(0.1302)	(0.1091)	(0.0621)	(0.0212)	(0.1221)
c		10.3987	7.1317	3.0810	3.0196	8.4203	1.9123	7.5036	1.4001
		1.7390	(1.4823)	(0.7178)	(0.6556)	(1.6114)	(0.4024)	(1.4361)	(0.2882)