The Cross-Section of Expected Stock Returns: Learning about Distress and Predictability in Heterogeneous Orchards

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We investigate the cross-sectional and term-structure of expected equity returns implications of distress events in connected networks. We study these implications in the context of an equilibrium asset pricing model with multiple Lucas (1978) trees subject to persistent distress events, where the agent has incomplete information about the state of an underlying common factor and learns from the events occurring to each tree. We focus on two new economic channels: cross-sectional learning and the cash-flow connectivity structure of the network. We find that these features helps to generate more realistic dispersion of cross-sectional expected returns relative to pure aggregate consumption risk models with complete information and disaster risk. Moreover, we find that this result is not obtained at the cost of explaining empirical equity premia and risk-free rate dynamics. Contrary to asset pricing models with learning in single-tree economies which produce a negative learning premium, we find that cross-sectional learning can generate a positive and unbounded (in risk aversion) risk premium. The model provides a simple setting to study the asset pricing implications of orchards in which the cash flow links among different trees are asymmetric and some trees are more exogenous than others in terms of their cash-flow dynamics. This allows to link reduced-form assumptions of cash-flow risk heterogeneity to the structural properties of the orchard. Finally, we study how cash-flow connectivity of a firm in the orchard is linked to the properties of expected equity returns, which has been recently studied in the context of the dividend strip curve. Sectors whose dividend process is exogenous in the orchard have negatively sloped term structures of dividend strips while the opposite holds for endogenous sectors.

I. Introduction

This paper studies the cross-sectional effects of incomplete information and learning in an equilibrium asset pricing model where several Lucas (1978) trees are subject to distress events. Rietz (1988), Barro (2009), Wachter (2010), and Gabaix (2009), among others, investigate the idea that the risk of rare disasters - abrupt, unpredictable output falls - helps to rationalize the equity premium and the interest rate puzzles discussed in Mehra and Prescott (1985). According to this strand of the literature the welfare implications of rare disasters can be significant; for example, Barro (2009) argues that the desire to hedge against chances of macroeconomic disasters is worth a significant portion of the GDP – as much as 20% each year, according to his calibration – while the welfare cost from usual economic fluctuations is much smaller and hardly able to explain the properties of expected returns – around 1.5% of GDP each year.

While successful on many dimensions, the disaster risk explanation faces a serious challenge when asked to explain also the cross-sectional characteristics of expected returns. Models that tend to yield high negative returns for the entire cross-section in response to distress events, that is, negative consumption shocks, may not generate a sufficient amount of cross-sectional dispersion of consumption risk relative to the observable cross-sectional variation of returns. For example, Julliard and Ghosh (2010) find that introducing a jump-risk rare disaster hypothesis in a C-CAPM model reduces the cross-sectional variance of the consumption betas by 34.4%. As a result, a C-CAPM with rare disaster risk explains poorly the risk premia of the 25 Fama and French portfolios. The goal of this paper is to investigate the trade-off between explaining the equity premium and the cross-section of expected returns in an economy with multiple assets in positive net supply (orchard). We explore a different notion of distress, a notion that emphasizes the role of the formation of expectations regarding fundamentals. While our distress events, which are states in which dividends discretely jump to a lower level, are specific to a particular sector, they are persistent and their intensities are related to the realization of a market-wide unobservable factor. This implies that even if a shock is initially confined to a single tree, it impacts the cross-section if it can be used for inference about the state of the common factor driving intensity probabilities; therefore, local distress events are pervasive in the cross-section. This depends on three characteristics: i) the persistence of the distress events, ii) the characteristics of connectivity of the orchard, that is, the mutual technological relation between sectors, and iii) the extent to which information about the latent factor is incomplete and agents learn about future distress probabilities in the cross-section from distress events in single sectors.

Our approach to model distress states is related but distinct from the concept of macroeconomic disasters, which has been investigated in the literature using transitory Poisson jump processes. The goal of this literature is to study the asset pricing implications of a 'peso problem', a remote but catastrophic event. In this context, an agent is concerned about the possibility of occurrence of a large event, rather than the aftermath of its occurrence, that is, its role to predict future economic fundamentals. While suited for this purpose, Poisson processes are less flexible to address our questions. Our notion of distress is less rare, less disastrous, but more persistent. In our approach we have in mind examples of distress events in which capital markets have tried to understand the extent to which localized shocks could have carried important information for the rest of the economy, therefore affecting the cross-section of expected returns.

To motivate our modeling approach, it is instructive to think at the 2007-2008 Credit Crisis. When, on September 21st 2007, Bear Stearns posted a 61 percent drop in net profits due to sub-prime related losses linked to the performance of its two hedge funds (Bear Sterns High-Grade Structured Credit Fund and Bear Sterns High-Grade Structured Credit Enhanced Leveraged Fund), investors were trying hard to understand the full implications of the news. On November 15th, it was revealed that Bear Stearns was writing down a further $1.2 billion in mortgage-related securities and would face its first loss in 83 years; Standard & Poor downgraded the company's credit rating from AA to A. Will the sub-prime crisis affect the broader banking sector? What would be the implications to other sectors, such as manufacturing and automotive? The collapse of the company was a prelude to the risk management meltdown of the Wall Street investment bank industry in September 2008. The degree of connectivity among different sectors quickly propagated the state of distress of the banking sector and turned the global financial crisis into one of the most severe recession since 1929. As events were unfolding, markets were pricing-in the effects of news on tree-specific shocks on the rest of the cross-section.[1]

In our paper, we address these aspects by studying the role of the interaction of two channels: (a) cross-sectional learning; (b) the cash-flow connectivity structure of the network. We consider a model in which a distress may occur with a probability that changes over time depending on the (hidden) state of a common stochastic factor. We ask to what extent distress event risk can be reconciled, at the same time, with realistic cross-sectional properties of expected returns, the aggregate equity risk premium and the risk-free rate puzzles. Since realizations of this stochastic factor are not directly observed by investors, but rather inferred from dividend realizations, that is from past distress and recoveries, events pertaining to a given tree provide information on the event risk of all the cross-section. This has immediate

[1] Other important examples of crisis in which real shocks were persistent and had significant cross-sectional implications include the railway sector crisis at the beginning of 18th century, the Great Depression, the Savings and Loans crisis in 1988-1993, the Internet bubble in 2000-2002.

cross-sectional implications that have the potential of reverting some previous findings. The intuition behind this ability is simple. Under complete information, distress risk implies non i.i.d. dividend growth, as following a distress events agents expect higher dividends, so that in equilibrium expected consumption growth is higher as recovery is foreseen. In this case, the agent is less willing to substitute present with future consumption by investing in a risky security with the ability to pay-off in bad consumption state. This implies a lower demand for equities, hence a negative shock on the value of all trees conditional on a distress event having occurred and higher risk premia. However, the negative (positive) return response is homogeneous across assets. This is a counterfactual prediction and it leads to a limited cross-sectional dispersion of consumption risk relative to the observed cross-sectional dispersion of expected returns (see Julliard and Ghosh (2008) for a detailed discussion). When information is incomplete, however, event shocks also act as signals for the common unobservable factor which drives event probabilities. Thus agents may interpret a distress (recovery) event as evidence in favor of high (low) likelihood of future distress for other trees, that is, she may revise upwards (downwards) her estimate of the distress correlation between dividends. If additional distress events are regarded as likely, expected consumption growth may fall (rise) after a distress (recovery). This implies that the assets which are more apt at hedging adverse future consumption states may experience an increase for their demand, hence a positive return. This depends on both cross-sectional learning and the cash flow connectivity structure of the network. This has important implications both for the aggregate market risk premium and the cross-section of expected returns.

The first contribution of this paper is to show that the additional layer of cross-sectional variability induced by learning is not obtained at the expense of the ability to generate high equity premia and low interest rates for moderate levels of risk aversion, even in a framework with time-separable preferences. This conclusion was not ex-ante obvious to us, since Veronesi (2000) shows that the equity premium component due to incomplete information about dividends' growth rate is negative. As a result, the equity premium arising in his economy is *bounded* above as a function of risk aversion: thus extreme levels of risk aversion do not necessarily generate large expected excess returns. The distinctive feature of our analysis is that while in Veronesi (2000) learning implies that negative dividend shocks always lead to lower expected consumption growth, in our economy recovery perspectives imply risk premia and returns volatility that are unbounded as a function of risk aversion. We also show that more information uncertainty, which arises when disaster and recovery events signal the state of the common latent factor less precisely, implies higher risk premia and volatilities. Both these two results are important since they help the model to reproduce a more realistic trade-off between expected stock return and volatility observed in the data.

The second contribution of the paper is to show a simple setting in which the cross-sectional learning induced by distress and recovery events generates a 'value premium' (i.e. high p/d ratio stocks have lower expected excess returns). Using a multiple-trees economy modeled with diffusive consumption share-processes, Santos and Veronesi (2009) show that habit preferences are consistent with the 'value premium' only when a fair amount of cash-flow risk heterogeneity is assumed. In our framework, even with standard time-separable utility, learning and cash-flow connectivity yields the cash-flow risk heterogeneity needed for a realistic value premium. Indeed, we show that risk premia are decreasing in the p/d ratio when agents' information set is incomplete, thus reversing the relationship arising in full-information, and that learning can induce significant cash-flow risk dispersion in the cross-section.

The third contribution of this paper is to propose a simple and parsimonious modeling setup that allows to study a second channel that affects the cross-sectional properties of asset prices: the cash-flow connectivity structure among different trees in the orchard. In a structural setting, we investigate orchards in which trees differ at a more fundamental level in terms of the in which the cash flows of a tree respond to shocks to other trees. We introduce the concept of economic 'exogeneity', which is defined in terms of the way the distress status of a tree depends on the occurrence of cash flow shocks to other trees. While the current literature use share dynamics in the context of vector diffusion processes to study the properties of orchards, we use Markov chain techniques that lends themselves more easily to our purpose. We find that the link between p/d ratios and expected returns (i.e., the 'value premium') is related to the cash-flow connectivity properties of trees in the orchard: the more a tree is 'exogenous' with respect to the rest of the orchard, the higher the value premium. We also find, however, that the relation between p/d ratios and expected returns can be non monotone and we characterize the properties when this may occur. This is important since it shows under which conditions p/d ratios are not sufficient statistics for inferring conditional expected returns.

Fourth, we find that the cross-sectional asset pricing properties of an orchard are linked to the shape of the term-structure of firms' equity premia. This is an important empirical property since it is connected to the dynamics of the cash-flow risks of different firms and/or sectors. Van Binsbergen, Brandt, and Koijen (2010) compare the empirical properties of expected returns in the dividend strips market with those implied by several leading asset pricing models. They show that both the Campbell and Cochrane (1999) external habit formation model and the Bansal and Yaron (2004) long-run risk model generate an upward sloping term structure of dividend strip curve (i.e. the risk premium on long-term dividend claims are higher than for short-term claim).[2] They also show that the Barro-Rietz rare disasters framework (Barro (2006)) as explored by Gabaix (2009) and Wachter (2010) expected returns are flat across maturities. These implications contrast with the empirical evidence of a downward sloping terms structure of expected returns. Lettau and Wachter (2007) propose a reduced-form model that is consistent with the empirical evidence. In our structural model we investigate the micro foundations that can give rise to properties of the stochastic discount factor that can be consistent with both the cross-section and term-structure of expected equity returns. We show that, even in the context of time-separable preferences, the slope ultimately depends on the cash-flow connectivity of firms in the network: if we allow for an asymmetric network structure, we obtain a hump-shaped or decreasing term structure of equity premia for those sectors that are actively connected to the rest of the orchard — meaning that their distress events propagate endemically; moreover, it is also shown that in equilibrium these stocks are those whose distress intensity is only weakly induced by shocks to the common factor. Since the distress status of these sectors is the most 'exogenous', that is, less dependent on shocks to the common factor and/or distress events of other firms, cross-sectional learning has the most pronounced impact on their cash-flow risk. It is interesting to notice that these term-structure properties are linked to the cross-section of expected returns since these sectors are also those with higher price-dividend ratio. The posterior probability embeds information on shocks to other firms in a way that decreases the conditional covariance at longer time-horizons between the cash-flows of this sector and aggregate consumption. The opposite happens for low p/d ratio ('value') stocks. Thus, the model produces novel testable restrictions that links the cash-flow connectivity of a firm in a network with the slope of the term structure of dividend strips and it allows learning to endogenously generate lower perceived cash-flow risk for sectors with higher price-to-fundamental ratio. This is important from a practical point of view since it links the observable shape of the term structure of dividend swaps at time t to expected returns based on cross-sectional long-short portfolios.

We calibrate the model using data on financial distress from Moody and Standard and Poor's and dividend distributions (including share repurchases) on 12 US industries portfolios and investigate the marginal contributions of each of the two channels with respect to the previous four implications of the model. We find that both cross-sectional learning and the cash-flow connectivity can explain a portion of the cross-sectional variability of excess returns which, although still low in absolute terms, is dramatically higher than that obtained

[2] It should be noted that while this statement is correct for the original version of Bansal and Yaron (2004), it is possible to generalize their model to obtain a decreasing term structure by assuming an exogeneous negative correlation between dividend growth and expected dividend growth.

in traditional rare disasters models with complete information.

LITERATURE. Our paper is related to several strands of literature. An important area of financial economics studies economies with multiple positive net supply assets. In particular, Cochrane, Longstaff, and Santa-Clara (2007) and Martin (2009) show that expected returns depend on the share of aggregate endowment that each asset supplies, even if assets pay independent cash-flows, as dividend shocks impact aggregate consumption through market clearing, thus affecting equilibrium state-prices. While this channel for asset price comovement is also active in our economy and affects price dividend-ratios, we document the effect generated by cross-sectional learning and how distress events on a tree also impact the valuation of other trees due to the existence of a common unobservable latent factor, even in absence of cash flow news on the other trees. Martin (2009) considers trees whose dividends follow an i.i.d. Levy process, hence are also subject to distress events. As in Cochrane, Longstaff, and Santa-Clara (2007), market-clearing considerations imply that the share of aggregate consumption paid by each tree arises as a common factor: times when this share is smaller correspond to times of low absolute covariance between returns and equilibrium state prices, hence smaller risk premia and higher valuation ratios. With two trees dividend shares are perfectly correlated, therefore this time-series occurrence of 'value' and 'growth' effects – times of low price-dividend ratios correspond to times of high excess returns – also translates into a cross-sectional pattern. With more than two trees the link is substantially more complex since risk premia depend on the imperfectly correlated dividend shares of all trees. Instead of focusing on the role of the market clearing condition, we investigate a different property of multiple tree economies. We focus on the connectivity structure of orchards and study the extent to which the joint hypothesis of persistent distress events (i.e. non transitory jump processes) and cross-sectional learning can give rise to realistic cross-sectional properties of expected returns after large shocks. In Martin (2009) distress events have the ability to catalyze 'contagion' and 'flight-to-quality' phenomena. These effects depend on dividend share of the distressed firm: a small firm's distress typically implies 'flight-to-quality' towards larger firms. In our paper, dividend shock propagation goes beyond market share effects, and rather depends on the firms' mutual connectivity structure and on the idiosyncratic (i.i.d. jump) or systematic nature of distress events. We address this by modeling the correlation of firm's event risk with a common latent factor. Distress events in one sector are informative about the hidden state of the common factor, thus implying the possibility of propagation. The characteristics of the propagation depend on the connectivity of each sector in the orchard. As a result, this structure allows us to model the cross-sectional variability of expected returns in response to dividend shocks.

A second strand of the literature – notably Barro (2006), Wachter (2010) and Gabaix

(2009) – focuses on single-endowment economies and show that disaster risk helps to explain realistic values for the equity premium. While our work is related to this literature, we focus on the cross-section of expected returns and explore a different type of distress risk. Our distress events are less large, more frequent but more persistent than macroeconomic disasters. Accordingly, a main departure from the literature is that we model distress (recovery) events as transitions of dividends to (out of) a persistent distress state. This allows us to retain the ability of events to solve the equity premium and the risk-free rate puzzle, while at the same time generating dividend growth and stock return predictability both in the time-series and in the cross-section. Two additional related works are by Gourio (2010) and Chen, Joslin, and Tran (2010). The first studies the effects of rare disasters in the context of a single-firm production economy with complete information. He finds that an increase in the risk of disaster leads to a collapse of investment and a recession, with no current or future change in productivity. Demand for precautionary savings increases, leading expected returns on safe assets to fall, while spreads on risky securities increase. More generally, he finds that model-implied variation in risk premia has an important effect on investment and output. Chen, Joslin, and Tran (2010) show that when agents differ in their beliefs about disasters – either regarding the chance of occurrence or the output loss upon occurrence, or both – the disaster risk premium may be significantly smaller than what predicted by belief homogeneity. This result relies on the complete-markets implementation of the equilibrium, according to which agents can invest in a continuum of continuously resettling insurance contracts, that pay-off in case a disaster of a given size occurs. In such a market structure, given the high market price of disaster risk, agents engage in significant risk-sharing, with each agent underwriting the insurance contract for the disaster events that she regards less likely or harmful. The higher the belief disagreement, the higher the incentive for risk-sharing and the lower the risk premium if the share of optimist in the economy is large. The focus of our paper is different as we document the cross-sectional implications of event risk in a multiple-tree economy, and our results do not hinge upon disaster insurability and complete-markets.

Our paper is related to a third strand of the literature that does not consider distress events or learning, but investigates the ability of different forms of C-CAPM to match the empirical characteristics of the cross-section of returns. Santos and Veronesi (2009) show that the SDF implied by nonlinear external habit formation preferences counterfactually generates higher expected returns for stocks with high price-dividend ratios – i.e. a 'growth premium' – , if firms/trees are allowed to differ only in terms of their expected dividend growth. They show that the 'value premium' can be obtained as long as one introduces heterogeneity in the firms' cash-flow risk, that is, in the covariance between consumption growth and their dividend growth. A related result is obtained in Lettau and Wachter (2007), who advocate the importance of weak or positive covariance between the market price of risk and dividend shocks, in order to obtain a 'value premium'. Our contribution is to show that cross-sectional learning can endogenously generate lower perceived (posterior) cash-flow risk for firms with high-price dividend ratio, and, at the same time, a lower covariance between their dividends and the equilibrium market price of risk.

In terms of empirical work, a truly vast literature has studied the cross-sectional properties of expected equity returns. A closely related empirical paper that find supporting evidence to our theoretical analysis is by Bianchi (2008). He studies equity returns in the context of distress events and suggests that distress events matter for the cross-section of returns because they may have significant influence on the way agents form expectations about economic fundamentals. He finds that the I-CAPM performs remarkably well in explaining Fama and French portfolios when the coefficients and the volatility of a VAR for the fundamentals are allowed to depend on a regime-switching latent factor.

The article is organized as follows. Section II describes the economy and the learning mechanism of the representative agent. In Section III we analyze the theoretical results of the model, solving in closed-form for price-dividend ratios and equity premia. In particular, we discuss the interaction of event risk with incomplete information and learning, and discuss their effects on equilibrium quantities. Sections IV to VII are devoted to the analysis of the cross-section of equilibrium expected returns arising in our economy. Section IV discusses the characteristics of the cross-sectional predictability that we can generate, while Section V contains an empirical analysis that investigates the implications of this predictability. Section VI analyzes expected returns in relation to the connectivity structure of the trees. Section VII is devoted to the term structure of assets' equity premia. Section VIII concludes. All proofs are in Appendix A, Appendix B contains details about the calibration procedure used in the empirical application, while Appendix C discusses the case where an infinite number of trees populate the economy.

II. The Economy

A. Preferences and Multivariate Endowment Structure

We consider a pure exchange Lucas economy populated by a single representative agent who maximizes isoelastic infinite horizon utility of intertemporal consumption, with Relative Risk Aversion coefficient γ and subjective discount rate δ:

$$U_0 = \mathbb{E}\left[\int_0^{\infty} e^{-\delta s}\frac{C_s^{1-\gamma}}{1-\gamma}ds\right]. \tag{1}$$

The opportunity set of the investor consists of a locally risk-less security in zero net supply, with rate of return r_t (the interest rate), and N risky securities in positive net supply, that pay the stochastic dividend stream $D_t = (D_t^1, D_t^2, \ldots D_t^N)'$ of a $N-$dimensional Lucas orchard.[3] Since the consumption good is not storable, the model is closed by noting that prices must adjust until aggregate consumption C_t coincides with the sum of the dividend processes $C_t = \sum_{i=1}^N D_t^i$. Single trees in the Lucas orchard can be, e.g., sectors or individual firms in a domestic economy, or countries in an international framework.[4] The main distinctive feature of our setting is that trees are subject to potential 'distress events', meaning that with some probability dividend levels can experience an abrupt discrete fall. At the same time, investors do not know the likelihood of such distress events: they can observe all endowment processes in the orchard and try to infer in a Bayesian fashion the probability of future distress from past distress observations. In what follows, we investigate the distinctive asset pricing implications of the interaction between distress risk and cross-sectional learning in the Lucas orchard.

B. Specification of Disasters and Recoveries from Disasters

We specify the vector of $N-$dimensional endowment processes simply as $D_t = Y_t x_t$, the product of an aggregate dividend factor Y_t, which is common across all trees and follows a geometric Brownian motion:

$$\frac{dY_t}{Y_t} = \mu_Y dt + \sigma_Y dZ_t, \tag{2}$$

and $x_t = (x_t^1, x_t^2, \ldots, x_t^N)' \in \mathbf{R}_+^N$, which is a positive $N-$dimensional stochastic process independent of Y_t. Y_t models the smooth common component of dividends in the orchard. The process x_t, instead, allows us to introduce in a tractable way the possibility of individual distress events in the economy. We model x_t as a set of N two-states continuous-time Markov chains.

Assumption 1 *Process x_t is a collection of N two-state Markov chains x_t^i, $i = 1, 2, \ldots, N$ with possible states $\overline{x}^i$ and $\underline{x}^i$, and transition matrix:*

$$\Lambda_t^i = \begin{pmatrix} -\lambda_t^i & \lambda_t^i \\ \eta_t^i & -\eta_t^i \end{pmatrix}. \tag{3}$$

[3] Appendix C discusses how our results, in particular those concerning equilibrium risk premia, would be affected by the presence of an infinite number of trees in the orchard.

[4] In this case distress events can be interpreted as persistent macroeconomic disasters.

Since we focus on the implications of distress events, we interpret (and later calibrate) $\overline{x}^i$ as the 'normal' state of the $i-$th component, and $\underline{x}^i$ the 'distress' state of the $i-$th component. A distress event for tree i occurs when the $i-$th process x_t^i has a transition from the normal state $\overline{x}^i$ to the distress state $\underline{x}^i$. Similarly, a recovery for tree i occurs in case of a transition from the distress to the normal $i-$th state. We model events as transitions to persistent states – rather than temporary shocks – where the persistence of the distress (normal) state is determined by the recovery (distress) intensity η_t^i, (λ_t^i). The persistence of *distress* states is important in our context and a point of departure of our paper from the rare *disaster* literature. This literature focuses on the ex-ante implications of catastrophic peso events and it typically models economic disasters as transitory Poisson-type jumps. Conversely, we are interested in the cross-sectional propagation and asset pricing implications of localized distress events in a connected network. Thus, their long-run persistence plays a key role.

We allow distress and recovery intensities to be time-varying and state-dependent. This achieves two objectives. First, it allows different firms to have different unconditional and conditional dividend growth trajectories after a disaster. Second, it allows serial cross-dependence across different sectors of the orchard. To achieve this we assume that the time-variation of (λ_t^i, η_t^i) depends (deterministically) on a common latent (unobservable) factor z_t, which follows a two-state Markov chain. Thus, the intensity processes $\lambda_t := (\lambda_t^1, \lambda_t^2, \ldots, \lambda_t^N)'$ and $\eta_t := (\eta_t^1, \eta_t^2, \ldots, \eta_t^N)$ follow themselves a two-state continuous-time Markov chain. This property is crucial for the asset pricing implications of the model since the dependence of the intensities of different trees on a common factor opens the possibility for cross-sectional learning to have asset pricing implications.

Assumption 2 *The intensity process (λ_t, η_t) follows a two-state Markov chain with upper state $(\overline{\lambda}, \overline{\eta})$ and lower state $(\underline{\lambda}, \underline{\eta})$, respectively, where $\overline{\lambda} \leq \underline{\lambda}$ and $\overline{\eta} \geq \underline{\eta}$. The transition of (λ_t, η_t) between states is governed by the intensity matrix:*

$$I = \begin{pmatrix} -k_h & k_h \\ k_l & -k_l \end{pmatrix}. \tag{4}$$

Depending on the model setting, the matrix I can be assumed to be either constant or a function of x_t. We will use this latter feature later in the paper to study orchards with asymmetric features, where distress events that occurred in one tree affect the properties of other trees.

According to Assumption 2, the vector of intensities (λ_t, η_t) can be either in a high or low state in the infinitesimal time interval $[t, t + \Delta]$. This means that there is a probability $k_h\Delta$ that both distress and recovery intensities jump from their high state values

$\overline{\lambda} = (\overline{\lambda}^1, \overline{\lambda}^2, \ldots, \overline{\lambda}^N)'$ and $\overline{\eta} = (\overline{\eta}^1, \overline{\eta}^2, \ldots, \overline{\eta}^N)'$ to their low state values $\underline{\lambda} = (\underline{\lambda}^1, \underline{\lambda}^2, \ldots, \underline{\lambda}^N)'$ and $\underline{\eta} = (\underline{\eta}^1, \underline{\eta}^2, \ldots, \underline{\eta}^N)'$. Similarly, there is a probability $k_l\Delta$ that distress and recovery intensities jump from their low state to their high state values. We interpret the high state as a state of good overall economic conditions. This assumption implies that even if the jump process $\Delta x_t := x_t - x_{t-}$ has independent components, the stochastic intensities of these jumps are driven by a common factor. This assumption means that the observation of a shock on one tree can have immediate cross-sectional implications in an economy with incomplete information and learning: a shock can be informative about the state of the common factor. Notice therefore that the cross-sectional effect exists even when the tree's dividend share is small: it depends on the informational content of the shock, which is related to the sensitivity of the intensity to the common factor and on the tree's connectivity to the rest of the orchard. Thus, the effect we study differs from the previous orchard literature (see Cochrane, Longstaff, and Santa-Clara (2007), Santos and Veronesi (2009), and Martin (2009)), in which shocks to a tree can affect the rest of the orchard through the role of the market clearing condition, so that the effect depends on the size of the dividend share of the tree.

Remark. If λ^i and η^i were constant, distress probabilities would also be constant and there would not be serial dependence across different sectors of the orchard. This would make it difficult to investigate the propagation of shocks in the connected network. In the rare disaster context, Watcher (2009) and Gabaix (2009) emphasize the importance of time-varying disaster probabilities for matching the empirical regularities of asset prices. In our model, persistence of distress events and time-varying distress and recovery probabilities imply a time-varying expected dividend growth and volatility, leading to a setting in which distress events and non i.i.d. dividend growth naturally coexist. The role of time-varying conditional moments of consumption in asset pricing is emphasized, for instance, in Bansal and Yaron (2004). In our model, time variation in the conditional distribution of firm dividends follows from the time-varying probability of distress and recovery events across firms, and from the Markov chain mechanism governing the event risks x_t.

Orchard Quality: Modeling Asymmetric Feed-backs Effects. It is realistic to assume that different economic sectors or trees are interconnected, to the extent that future dividend streams supplied by a tree are not independent of the dividends paid by other trees. It would be desirable, therefore, to model situations in which 'distress' conditions of a few sectors could propagate endemically economy-wide. It is realistic to imagine, however, that different sectors perform specific functions, so that the quality and/or magnitudes of such transmissions depend on the specific identity of the sector which suffered the initial shock.

The previous specification lends itself to investigate, in a parsimonious way, such settings in which shocks are transmitted through asymmetric feed-back effects. This can be achieved assuming that k_h and k_l depend on the occurrence of distress events among sectors (trees) of the orchard, $k_h = k_h(x_t)$ and $k_l = k_l(x_t)$, that is, assuming that the time-varying distress-recovery intensities imply a probability of a transition between good and bad economic states that depends itself on the occurrence of distress events across sectors.

Example. As an illustration, consider an economy with three sectors: Housing, Banking, and Manufacturing (sectors 1, 2, and 3, respectively). Suppose that we have reasons to believe that these three sectors fulfill different functions. For instance, we may assume that the Housing sector has a very sensitive connection to the Banking sector (through the supply-side credit channel, because of mortgage and financial securitization links), and less so with the Manufacturing sector. On the other hand, the Banking sector is connected to all other sectors because of its role of credit provider:

$$\textit{Housing} \underset{\Longrightarrow}{\Longleftarrow} \textit{Banking} \Longrightarrow \textit{Manufacturing} \tag{5}$$

Given constants k_h, k_l, a_1, a_2, b_1,$b_2 \geq 0$, and $x_t = (x_t^1, x_t^2, x_t^3)$ a simple form of a time-varying matrix $I = I(x_t)$ in equation (4) can be used to model this situation:

$$\begin{aligned} k_h(x_t) &= k_h[1 + a_1\mathbf{1}(x_t^1 = \underline{x}^1) + a_2\mathbf{1}(x_t^2 = \underline{x}^2)] \\ k_l(x_t) &= k_l[1 - b_1\mathbf{1}(x_t^1 = \underline{x}^1) - b_2\ \mathbf{1}(x_t^2 = \underline{x}^2)] \end{aligned} \tag{6}$$

where $\mathbf{1}(A) = 1$ if event A is true and zero otherwise. k_h is the probability that the whole economy switches to a 'bad' state if the Banking and Housing sectors are not in distress. If the Housing sector (Housing and Banking sectors) is (are) in distress, this probability is higher and equal to $k_h(1+a_1)(k_h(1+a_1+a_2))$. For instance, by assuming $a_2 > a_1$ the probability of an overall transition to a bad economic state increases more when the (more interconnected) Banking sector is in distress than when the Housing sector is in distress. Thus, equation (6) allows, in a parsimonious way, to capture the main feed-backs and economic intuitions implied by assumption (5) about the structure of the economy.

The dependence structure of the trees can be further enriched, accounting for a more direct and less systematic form of distress contagion. Namely, we can assume that event intensities are themselves functions of the number and type of distress events across trees: $\lambda_t = \lambda_t(x_t)$. For instance, the characteristics of the economy outlined in (5) can be directly replicated assuming that upon distress of the Banking sector, i.e. $\mathbf{1}(x_t^2 = \underline{x}^2) = 1$, the parameters λ of remaining sectors increase by a given percentage, while upon distress of the Housing sector, the λ of the Banking sector alone increases by a given percentage.

The aspects defining the quality of the orchard are obviously very important with regards to the implications in terms of cross-sectional predictability. In Section VI, we will use both these elements that allow to model asymmetric network structures, when we analyze the cross-section of returns arising in our economy.

C. *Learning About Distress: Perceived Distress Contagion*

Distress events and recoveries from distress are typically not frequent, even though they are less rare than macroeconomic disasters. Thus, their probability might be difficult to estimate from historical information. We assume that investors observe both x_t and Y_t, but do not observe (λ_t, η_t), which control the intensity of the event process. Agents infer these parameters using the information set $\mathcal{F}_t^{x,Y}$ generated by continuous-time observations of the components of the multivariate dividend process D_t. Let $(\widehat{\lambda}_t, \widehat{\eta}_t)$ be agents' Bayesian estimates of (λ_t, η_t), given the available information $\mathcal{F}_t^{x,Y}$:

$$(\widehat{\lambda}_t, \widehat{\eta}_t) = \mathbb{E}_t[(\lambda_t, \eta_t)|\mathcal{F}_t^{x,Y}] = p_t^h(\overline{\lambda}, \overline{\eta}) + (1 - p_t^h)(\underline{\lambda}, \underline{\eta}) \tag{7}$$

where

$$p_t^h = \mathbb{P}[(\lambda_t, \eta_t) = (\overline{\lambda}, \overline{\eta})|\mathcal{F}_t^{x,Y}]. \tag{8}$$

Observations of distress events and recoveries from distress provide useful information on whether the conditional intensity of distress and recovery events should be closer to either $(\overline{\lambda}, \overline{\eta})$ or $(\underline{\lambda}, \underline{\eta})$. Intuitively, because intensities (λ_t, η_t) depend on a common latent factor, observations of a distress or recovery on one of the trees have cross-sectional implications for the distress intensities of all other trees, even if true distress intensities of those trees did not change. In our setting, learning has the potential to generate a form of 'perceived distress contagion" via the optimal Bayesian updating of the individual probabilities of disaster and recovery. The size of the Bayesian revision of p_t^h after a distress or recovery event depends on the parameters of the orchard, such as the degree of uncertainty in the economy and the 'quality' of the orchard connections, namely the degree of heterogeneity across good and bad economic states. Let $H_t = (H_t^1, H_t^2, \ldots, H_t^N)$ indicate the distress state across trees in the economy, i.e., $H_t^i := \mathbf{1}(x_t^i = \underline{x}^i)$, $i = 1, 2, \ldots, N$, so that $dH_t^i = 1$ ($dH_t^i = -1$) indicates a distress (recovery) shock for tree i. We denote by:

$$d\hat{H}_t^i := \frac{dH_t^i - \widehat{\lambda}_t^i dt}{\widehat{\lambda}_t^i}, \quad d\hat{K}_t^i := \frac{-dH_t^i - \widehat{\eta}_t^i dt}{\widehat{\eta}_t^i}, \tag{9}$$

the compensated process of distress and recovery events, such that $E[d\hat{H}_t^i|\mathcal{F}_t^{x,Y}] = 0$ and $E[d\hat{K}_t^i|\mathcal{F}_t^{x,Y}] = 0$, relative to investors' filtration. The filtered dynamics of posterior probability p_t^h for the common latent factor are given in the next technical lemma.

Lemma 1 *Let p_0^h denote investor's prior belief about the common latent factor being in the 'high' state. The posterior probability dynamics of p_t^h follows the stochastic differential equation:*

$$dp_t^h = \left[k_l - (k_l + k_h)p_t^h\right] dt + p_t^h(1-p_t^h) \sum_{i=1}^{N} \left[(\overline{\lambda}^i - \underline{\lambda}^i)(1 - H_{t-}^i)d\hat{H}_t^i + (\overline{\eta}^i - \underline{\eta}^i)H_{t-}^i d\hat{K}_t^i\right] \quad (10)$$

Expression (10) is intuitive. The stochastic components $d\hat{H}_t^i$ and $d\hat{K}_t^i$, $i = 1, 2, \ldots, N$, are the normalized unexpected innovations of distress and recovery realizations. If the distress (recovery) intensities were constant (i.e. $\overline{\lambda}^i = \underline{\lambda}^i$ and $\overline{\eta}^i = \underline{\eta}^i$), the events would be firm-specific and idiosyncratic: signals would be uninformative about the state of the common latent factor z_t. In this case, there would not be any cross-sectional learning effect due to the observation of a negative shock to a tree. When $\underline{\lambda} > \overline{\lambda}$ or $\overline{\eta} > \underline{\eta}$, however, the observation of a distress (recovery) to one tree leads to an downward (upward) revision of the posterior probability p_t^h of the common latent factor being in a "high" state. Note that distress and recovery signals are realized discretely over time. Therefore, posterior probabilities have discontinuous trajectories reflecting the discrete structure implied by distress events and recoveries for investors' information filtration. Moreover, the stochastic term in equation (10) are multiplied by the terms $1 - H_{t-}^i$ and H_{t-}^i, respectively: upon distress for tree i, H_t^i jumps from $H_{t-}^i = 0$ to $H_{t-}^i = 1$. Thus, the posterior probability of the common latent factor decreases due to the activation of the first term in square brackets. Symmetrically for an observation of a recovery event this probability increases.

Since distress and recovery innovations enter equation (10) weighted by the difference of disaster and recovery intensities in good and bad states, individual distress and recoveries have greater weight in investors' posterior distribution whenever the underlying individual intensity process is more volatile. In addition, all signals have a uniformly greater weight when overall Bayesian uncertainty is large, i.e., when the term $p_t^h(1 - p_t^h)$ is large. This occurs for $p_t^h \approx 0.5$, when investors face the largest degree of subjective uncertainty about the true common latent state of the economy. These features can generate interesting effects of perceived distress contagion via agents' optimal learning behavior: large revisions in the posterior intensity of a tree i, say, can arise because of the observation of a distress or recovery in another tree $j \neq i$, even in absence of any cash flow innovations for tree i. These effects arise because, following a distress of a tree, Bayesian optimal learning potentially affects the perceived posterior probabilities of distress of all trees. In order to measure more directly these learning contagion channels, we follow Frey, Schmidt, and Gabih (2007) and consider as a measure of distress contagion the variation of the instantaneous probability of distress of tree i after a distress of another tree j. Let τ_j be the timing of a distress event for tree j.

Using the posterior dynamics in Lemma 1, we obtain a simple direct measure of the degree of perceived distress contagion $\widehat{\lambda}^i_{\tau_j} - \widehat{\lambda}^i_{\tau_j-}$ in our economy:

$$\widehat{\lambda}^i_{\tau_j} - \widehat{\lambda}^i_{\tau_j-} = p^h_{\tau_j-}(1 - p^h_{\tau_j-})\frac{(\overline{\lambda}^i - \underline{\lambda}^i)(\overline{\lambda}^j - \underline{\lambda}^j)}{\widehat{\lambda}^j_{\tau_j-}} \tag{11}$$

Perceived contagion is high when there is high uncertainty about the current state of the economy and when the difference between distress intensities of tree i and j in the two states is large. The more a distress event of tree j is unexpected, that is, posterior intensity $\widehat{\lambda}^j_{\tau_j-}$ is low immediately before the distress of tree j, the larger the degree of perceived contagion. Note that if the intensities were constant across states, $\overline{\lambda} = \underline{\lambda}$, the last term would be equal to zero and there would be no perceived contagion so that any effects would be completely idiosyncratic. It is important to highlight, moreover, that a sufficient condition for the existence of a contagion channel is the incomplete information about the state of the common latent factor.

In the time spans elapsing between distress and recovery events, investors do not observe additional relevant information to update their beliefs. Therefore, in this time span their posterior probability dynamics is dominated by the drift component:

$$\frac{1}{dt}\mathbb{E}\left[dp^h_t|\mathcal{F}^x_t\right] = k_l - (k_l + k_h)p^h_t = (k_l + k_h)\left[\frac{k_l}{k_l + k_h} - p^h_t\right], \tag{12}$$

which implies a local linear reversion of p^h_t to the mean $\overline{p} = k_l/(k_l + k_h)$ at a speed $\theta = k_l + k_h$. When k_l and k_h are constant, $\overline{p}$ is simply the fraction of time that the economy spends in a good state in the long run. The speed of reversion to the mean is larger when the probability of transitions between good and bad states of the economy is large. In this context, it would be straightforward to include a set of unbiased continuous signals for the intensity of disasters and recoveries of any tree of the orchard. In this extended setting, posterior probabilities would include a stochastic component generated by the continuous signals also in the time spans between distress events and recoveries. However, for simplicity of notation and in order to focus on the pure interaction of distress risk and learning, we do not include any additional continuous signals in the model.

Figure 1 and 2 illustrate in more detail these dynamic features of posterior probabilities in our economy.

Insert Figure 1 about here

In Figure 1, we consider an economy in which for all trees the intensities of distress and recovery are only slightly different across the two relevant states of the economy. In this setting, the covariance between the true intensity λ_t and signals of distress or recovery is low. Therefore, all revisions of posterior probability p_t^h at times of a distress or a recovery tend to be small, and no large perceived distress contagion generated by agents' Bayesian belief updating emerges. Between distress and recovery events, the dynamics of p_t^h is driven by the deterministic mean reversion component in the drift of equation (10), which tends to pull p_t^h to the (local) mean value until a new signal from one of the trees in the orchard is observed.

Insert Figure 2 about here

Figure 2 presents an economy in which intensities are more different across the states of the economy. Therefore, sizable revisions of beliefs arise due to distress contagion through agents' Bayesian learning, which generates an additional relevant source of risk in the economy. As in the previous example, the economy is more likely to be in a good than in a bad state. Thus, the innovating content is larger for distress events than for recoveries. This feature explains why larger downward revisions of beliefs are observed in case of a distress, but only much smaller upward revisions are implied for recoveries. This asymmetric optimal revision pattern in beliefs generates an additional source of risk premium for (asymmetric) distress risk.

D. Self-Exciting Contagion and Clustering of Disasters

Even if distress and recoveries across trees are instantaneously independent, the potential dependence of their stochastic intensities on the normalcy or distress state of each tree can generate a wide degree of clustering of distress events and recoveries through self-exciting feed-back and contagion effects. These features have direct implications, e.g., for the pricing of contingent pay-offs that depend on the joint distress of a collection of trees over a given time-horizon. Figure 3 illustrates the effect of self-exciting contagion on distress clustering in our economy. We consider two settings. The first one features a constant transition matrix I with no feed-back effects. The second one models potential self-exciting contagion effects using a state dependent intensity matrix (6) that reflects the structure of the three- sectors economy depicted in (5).

Insert Figure 3 about here

For both economies, we compute the term structure of distress probabilities for (i) the event that a single distress is realized before maturity T (Panel 1) and (ii) the event that exactly two distress events are realized before maturity T (Panel 2), as shown in Lemma A.2 of the Appendix. As expected, the term structures of distress probabilities are monotonically increasing and they are higher in the economy with contagion, both for the event of two and one disasters before maturity T (straight lines in Panel 1 and 2, respectively). More interestingly, the increase in the probability of observing two distresses before maturity T is much more pronounced than the increase of the probability of a single distress. For instance, for a maturity of $T = 10$ years the probability of a single distress increases slightly from approximately 0.57 to 0.61 in the economy with contagion, but the probability for the event of two distress states increases dramatically from approximately 0.13 to 0.39. This is important from an asset pricing perspective since it affects the beta risk of the two economies.

III. Model results

The rest of the paper is organized in four parts. First, we investigate the properties of the aggregate consumption process and of the equilibrium state-price deflator. We analyze, in particular, the compensation demanded by the agent to bear the risk of distress/recovery events, that is, the market price of event risk. Second, we investigate the properties of a benchmark network: we derive analytical conditions that link the parameters of the economy to the behavior of interest rates, p/d ratios, the market equity risk premium, and the properties that lead to cross-sectional predictability. Third, we study heterogeneous networks in which trees differ at a more fundamental level, in terms of the endogenous or exogenous way in which they respond to shocks to other trees. Last, we discuss the term structure of dividend risk and the slope of the dividend swap curve.

A. Properties of Aggregate Consumption

Unless a distress, or a recovery from distress, occurs for some tree, the evolution in time of aggregate consumption coincides with the continuous evolution of the common component Y. When a distress (recovery) of tree i takes place, the persistent dividend component x_t^i falls (increases) to the state of distress (normalcy) $\underline{x}^i$ ($\overline{x}^i$). As a result, the following equation describes the evolution of aggregate consumption growth:[5]

$$\frac{dC_t}{C_{t-}} = \mu_Y dt + \sigma_Y dZ_t - \sum_{i=1}^{N} \left[(1 - H_t^i) \frac{(\overline{x}^i - \underline{x}^i) Y_t}{C_{t-}} - H_t^i \frac{(\overline{x}^i - \underline{x}^i) Y_t}{C_{t-}} \right] dH_t^i, \tag{13}$$

where C_{t-} denotes aggregate consumption immediately before time t. Expected consumption growth is the sum of the expected growth in the unitary supply Y, and the relative loss (gain) in consumption that a distress (recovery) of each tree would yield, weighted by the posterior probability of the event for each tree.

$$\mathbb{E}\left[\frac{dC_t}{C_{t-}} \middle| \mathcal{F}_t^{x,Y} \right] = \mu_Y - \sum_{i=1}^{N} \left[(1 - H_t^i) \frac{(\overline{x}^i - \underline{x}^i) Y_t}{C_{t-}} \widehat{\lambda}_t^i - H_t^i \frac{(\overline{x}^i - \underline{x}^i) Y_t}{C_{t-}} \widehat{\eta}_t^i \right] \tag{14}$$

[5]Remember that since $H_t^i = \mathbf{1}(x_t^i = 0)$ is the indicator of a disaster for tree i, $dH_t^i = -1$ if $H_t^i = 1$.

Similarly, consumption variance is the sum of the common diffusive variance σ_Y^2, and the squared relative consumption variation in case of distress or recovery of any tree, weighted by the perceived probabilities of these events.

$$\mathrm{Var}\left[\frac{dC_t}{C_{t-}}\middle|\mathcal{F}_t^x\right] = \sigma_Y^2 + \sum_{i=1}^{N}\left[H_t^i\left(\frac{(\overline{x}^i - \underline{x}^i)Y_t}{C_{t-}}\right)^2 \widehat{\eta}_t^i + (1-H_t^i)\left(\frac{(\overline{x}^i - \underline{x}^i)Y_t}{C_{t-}}\right)^2 \widehat{\lambda}_t^i\right] \tag{15}$$

It is useful to understand the determinants of these moments, as they determine the demand for securities and equilibrium security prices. Expected consumption growth is affected by the share sizes of aggregate endowment supplied by the trees and by incomplete information. The former aspect has been extensively discussed in Cochrane, Longstaff and Santa-Clara (2007) and Martin (2009), among others. Intuitively, if a high fraction of the current level of aggregate consumption is provided by a few trees, which are perceived as highly prone to distress, or unlikely to recover if currently in distress, then expected consumption growth is low. Similarly, when consumption is diversified between many supplying trees, which are scarcely likely to experience a distress or recover from it, consumption volatility is intuitively low. The effect of events on expected consumption growth is two-fold. Consider a distress event, hence a negative dividend shock of the persistent component x_t of some tree. On the one hand, the distress status induces direct predictability of dividend growth: expected consumption growth increases, because the potential recovery of the tree in distress is foreseen. On the other hand, the distress event decreases expected consumption growth, because the agent updates her posterior belief towards the 'low' state of the latent factor: the higher posterior distress probabilities estimate as likely negative dividend shocks of other trees. In our model, for perceived consumption growth to decrease following a distress event in one tree, the latter effect must be dominant over the former. It follows that even trees paying a small fraction of the aggregate dividend have the potential to catalyze sizable revisions of future consumption perspectives through the learning mechanism. This feature is important since it can reverse the effects of traditional models with no cross-sectional learning.

B. Interest rates dynamics with distress risk

A first important set of implications of our model can be casted in terms of the effect that the observation of a distress event on a tree has on the equilibrium short-term interest rate. According to the first order conditions for the optimal consumption problem of the representative agent, the equilibrium state-price density coincides with the intertemporal marginal rate of aggregate consumption substitution:

$$\xi_t = e^{-\delta t}\left(Y_t \sum_{i=1}^{N} x_t^i\right)^{-\gamma}. \tag{16}$$

The dynamics of the stochastic discount factor follows from Ito's lemma:

$$\frac{d\xi_t}{\xi_t} = -r_t dt - \kappa dZ_t + \sum_{i=1}^{N}(\theta_s^i - 1)\left[(1-H_t^i)(dH_t^i - \widehat{\lambda}_t^i) + H_t^i(-dH_t^i - \widehat{\eta}_t^i)\right]. \tag{17}$$

The stochastic term in equation (17) includes two components. The first is the market price κ of the diffusive risk. The second component arises because of the event risk dH_t^i. When $dH_t^i = 1$, tree i falls in distress, when $dH_t^i = -1$. $\theta_t^i - 1$ is defined as the *market price of event risk* for tree i. θ_t^i can be interpreted as the price per unit of volatility that the agent is willing to pay for an insurance contract that provides one unit of consumption in case an event happens next instant. Applying Ito's lemma to (16) and comparing the result to equation (17), we can identify both the equilibrium interest rate and the market price of risk for diffusive consumption variability: $\kappa = \gamma\sigma_Y^2$. The equilibrium market prices of distress and recovery events are, respectively:

$$\theta_t^i = \frac{1}{\widehat{\lambda}_t^i}\mathbb{E}\left[\frac{\xi_t}{\xi_{t-}}dH_t^i \middle| \mathcal{F}_t^{x,Y}\right] = \left(\frac{\underline{x}^i + \sum_{j\neq i} x_{t-}^j}{\overline{x}^i + \sum_{j\neq i} x_{t-}^j}\right)^{-\gamma}, \quad H_{t-}^i = 0 \tag{18}$$

$$\theta_t^i = \frac{1}{\widehat{\lambda}_t^i}\mathbb{E}\left[\frac{\xi_t}{\xi_{t-}}(-dH_t^i) \middle| \mathcal{F}_t^{x,Y}\right] = \left(\frac{\overline{x}^i + \sum_{j\neq i} x_{t-}^j}{\underline{x}^i + \sum_{j\neq i} x_{t-}^j}\right)^{-\gamma}, \quad H_{t-}^i = 1 \tag{19}$$

$Y \sum_{j\neq i} x_{t-}^j$ denotes the cumulative dividends paid by trees other than i, so that the terms

$$\frac{\underline{x}^i + \sum_{j\neq i} x_{t-}^j}{\overline{x}^i + \sum_{j\neq i} x_{t-}^j}, \quad \frac{\overline{x}^i + \sum_{\neq i} x_{t-}^j}{\underline{x}^i + \sum_{j\neq i} x_{t-}^j} \tag{20}$$

are the gross consumption growth due to a distress or a recovery of i, respectively. The former is always smaller than one, while the latter is always greater than one. The price per unit of volatility of this insurance is equal in equilibrium to the intertemporal marginal rate of substitution of the unit of consumption it pays-off in case of the occurrence of this event. The market price of event risk, $\theta_t^i - 1$, is positive for distress and negative for recovery events. Note that θ_t^i equals the risk adjustment to distress and recovery intensities that the agent would require if she were to act in a risk-neutral fashion:

$$\widehat{\lambda}_t^{*,i} = \widehat{\lambda}_t^i \theta_t^i. \tag{21}$$

Similarly, $\widehat{\eta}_t^i \theta_t^i$ is the risk adjusted instantaneous probability of recovery. Figure 4 shows this market price of risk for a given tree, as a function of the risk aversion coefficient and for two different fractions of aggregate consumption paid by the tree.

Insert Figure 4 about here

Panel 1 reports the market price of recovery risk, conditional on a distress status of the tree, while Panel 2 plots the market price of distress, conditional on normalcy state. A distress covaries positively with the state-price density: the extent of this covariation is increasing in the relative risk aversion coefficient and in the fraction of aggregate output that the tree provides. Hence the risk adjusted intensity of distress is higher than the objective intensity λ_t^i. Conversely, a recovery displays negative covariance with the stochastic discount factor, the risk aversion and the dividend share of the tree: the risk adjusted probability of recovery is smaller than the objective probability η_t^i. In a risk neutral world, the likelihood of each tree's distress or recovery depends also on the distress or normalcy condition of all trees.

The equilibrium interest rate reads:

$$
\begin{aligned}
r_t &= -\mathbb{E}\left[\left.\frac{d\xi_t}{\xi_t}\right| \mathcal{F}_t^{x,Y}\right] \\
&= \delta + \gamma\mu_Y - \frac{1}{2}\gamma(\gamma+1)\sigma_Y^2 + \sum_{i=1}^{N}\left\{ H_t^i \left[1 - \left(\frac{\overline{x}^i + \sum_{j\neq i} x_{t-}^j}{\underline{x}^i + \sum_{j\neq i} x_{t-}^j}\right)^{-\gamma}\right]\widehat{\eta}_t^i + \right. \\
&\qquad \left. (1 - H_t^i)\left[1 - \left(\frac{\underline{x}^i + \sum_{j\neq i} x_{t-}^j}{\overline{x}^i + \sum_{j\neq i} x_{t-}^j}\right)^{-\gamma}\right]\widehat{\lambda}_t^i \right\}.
\end{aligned}
\tag{22}
$$

The common diffusive dividend component Y gives rise to the standard intertemporal consumption substitution and precautionary savings effects of i.i.d. consumption growth. The term in curly brackets in (22) is the interest rate component due to event risk, which gives rise to an additional intertemporal substitution effect. In particular, the potential distress (recovery) of trees in normalcy (distress) state reduces (increases) the interest rate. The perceived likelihood of distress (recovery) of a tree decreases (increases) expected consumption growth, thereby increasing (decreasing) the willingness of the agent to substitute consumption over time investing in the risk-free asset. Since this asset is in zero-net supply and its current price is fixed, its rate of return decreases (increases) proportionally. It follows that the risk of distress of a tree that pays a sizable fraction of the aggregate dividend can generate a significant negative effect on the interest rate.

If information was complete, event risk would lead to the counterfactual prediction that interest rates are high(er) during periods of generalized distress for the economy. Without learning and incomplete information, any distress event would act as a persistent dividend shock implying a higher expected dividend growth. In the incomplete information economy this may not be the case, because a distress event unveils the possibility of similar events for other trees. This depends on the extent of posterior probability update for the 'low' state of the common factor – in the sense discussed in Section II.C. When this occurs, learning induces the agent to revise downwards expected consumption growth, thus interest rates to drop after observing a distress.

Insert Figure 5 about here

In Figure 5 we report a simulated trajectory of the equilibrium interest rate prevailing in a simple symmetric economy. In this example a few trees supply equal shares of aggregate consumption, hence the volatility of the interest rate is limited. Nonetheless, Figure 5 shows that during phases where one or more trees are in distress, learning lower interest rates. This is different and more realistic compared to the effect prevailing in a full-information economy.

In standard models interest rates are increasing in risk aversion, for realistic parameter choices. In our model, conditional on only a few or no sector being in distress, the distress perspective of additional trees can generate a reduction in interest rates that is increasing in risk aversion. This is potentially important, since this effect helps to address the well-known 'risk-free rate puzzle', weakening the equilibrium trade-off between the equity premium and the interest rate.

C. Security prices

The Euler equations arising from the representative agent's optimization imply that the equilibrium price of the claim to the $i-$th dividend process is: $P_t^i = \mathbb{E}\left[\int_t^\infty \xi_s D_s^i ds \middle| \mathcal{F}_t^{x,Y}\right]/\xi_t$, where ξ_t is the equilibrium stochastic discount factor given in (16). In the incomplete information economy, $\mathcal{F}_t^{x,Y}$ does not include information generated by the latent common factor driving intensities (λ_t, η_t). Hence the agent needs to form expectations about whether event intensities are in the 'high' or in the 'low' state.

The next Proposition describes the equilibrium link between p/d ratios in our incomplete-information economy and the full-information p/d ratios:

Proposition 1 *Let $P_t^i(Y_t, x_t)$ denote the price of the claim to the $i-$th endowment. Let also $\overline{P}^i(Y_t, x_t)$ and $\underline{P}^i(Y_t, x_t)$ denote the full-information price conditional on the 'high' and 'low' state of the latent factor, respectively. The incomplete information price-dividend ratio is equal to:*

$$\frac{P_t^i}{D_t^i} = p_t^h \frac{\overline{P}^i(Y_t, x_t)}{D_t^i} + (1 - p_t^h)\frac{\underline{P}^i(Y_t, x_t)}{D_t^i}, \tag{23}$$

where $D_t^i = Y_t x_t^i$, and $\overline{P}^i(Y_t, x_t)$, $\underline{P}^i(Y_t, x_t)$ are reported in the Appendix. If $\gamma > 1$, the full-information price of any security conditional on the 'low' state of the common factor is higher than the 'high' state price, i.e. $\underline{P}^i(Y_t, x_t) > \overline{P}^i(Y_t, x_t)$.

The p/d ratio of the $i-$th equity asset is a weighted average of full-information p/d ratios, with weights given by the posterior probabilities of the 'high' and the 'low' state of the latent factor. The price-dividend ratio depends on:

i) *the intensity of event risk* (λ_t^i, η_t^i) faced by firm i, relative to the rest of the cross-section. This characteristic is determined by (a) the size of distress and recovery intensities of the firm, and by (b) the extent to which realizations of λ_t^i and η_t^i are pro-cyclical. Higher price-dividend ratios obtain when the distress (recovery) intensity is low (high), relative to all other firms, and when the covariance of the firm's dividend with the common factor is low. The latter property arises when event intensities are similar across the different states of the latent factor.

ii) *The distress status of all sectors* (the vector x_t). In our model, distress and recovery events dividend shocks that cause a tree to enter enter a persistent distress or normalcy state, respectively.[6] Since persistent dividends' component x_t forecasts future dividend growth, it affects not only asset prices, but also price-dividend ratios. In particular, states of generalized distress are characterized by high expected consumption growth, because overall sectors' recoveries are foreseen. In this case, low future state prices (marginal utility) imply a scarce desire to invest in risky assets in order to substitute consumption intertemporally. This implies low price-dividend ratios.

iii) *The posterior belief* p_t^h about the state of the latent factor that drives distress-recovery intensities. According to Proposition 1, full-information price-dividend ratios are higher in the 'low' state than in the 'high' state of the common factor, if $\gamma > 1$. Expected dividend growth is lower for all the cross-section in the former state, and for sufficiently high risk aversion, the expected increase in state-prices following a drop in consumption more than compensates the lower expected cash-flows paid by the security. In the incomplete information economy, since the state of the common factor is unobservable, the equilibrium security price-dividend ratio depends also on the confidence of the agent about the current state.

[6] Remember that x_t takes 2^N possible values, ranging from a combination where no tree/firm is in distress – therefore all supply a high multiplier – to one where all are in distress and supply a low multiplier.

These considerations allow us to deduct the main properties of security returns in our model. In a full-information economy with i.i.d. dividend flows – as in Cochrane, Longstaff and Santa-Clara (2007) and Martin (2009) – the cross-sectional variability of price-dividend ratios in response to dividend shocks is a pure consequence of market-clearing effects. In this context, a distress of a tree likely leads to lower price-dividend ratios, hence prices, for the rest of the cross-section: a contagion effect. The mechanism relies on an increase in equilibrium risk premia: as the distressed asset pays a lower share of aggregate consumption, unaffected trees' shares rise, and their covariance with aggregate consumption rises, thus increasing their risk premium and reducing their valuation. In our model, we obtain a more heterogeneous cross-sectional response to dividend shocks because the combined effect of the persistence of dividend shocks (events) – see *ii*) above – and cross-sectional learning – see *iii*) above – induces dividend growth predictability. Full-information implies that any distress event leads to an increase in expected consumption growth. The agent would expect with complete confidence the economy to be already in the worst state, so that future consumption growth can only be higher. This implies that the agent is less willing to invest in *any* risky security for intertemporal consumption substitution. The higher the event risk and the market share of the distressed (recovered) tree,[7] the more pronounced the negative spill-over effect, because the increment of expected consumption growth is maximal. Under full information, this effect leads to a homogeneous return response of all firms.

Under incomplete information these effects are more nuanced. While a distress unambiguously leads to a negative contemporaneous return for all securities, it also leads to a upward revision of the posterior probability for the 'low' state of the economy, hence a higher perceived (i.e. posterior) correlation between endowments. In Section II.C we have described this feature as 'perceived distress contagion', which can lead to a lower expected consumption growth. The final result on asset prices is ambiguous and it depends on the difference between the update in the posterior beliefs about the 'low' state and the price difference, under full-information, of the security in the 'high' and 'low' states. This last effect depends on the the extent to which the distress intensity of the asset is procyclical, a feature that depends on the production technology of the tree and the way it is linked

[7]A synthetic indicator of a security's event risk must take into account distress and recovery intensities in the 'high' and 'low' states of the common factor, as well as the persistence of these states:

$$\tfrac{k_l}{k_h+k_l}\,\overline{\lambda}^i + \tfrac{k_h}{k_h+k_l}\,\underline{\lambda}^i \tag{24}$$
$$\tfrac{k_l}{k_h+k_l}\,\overline{\eta}^i + \tfrac{k_h}{k_h+k_l}\,\underline{\eta}^i \quad i = 1, 2, \ldots, N \tag{25}$$

For a given security, distress risk is high when the unconditional distress (recovery) intensity, reported in equation (24) ((25)), is high (low): in this case the tree has a high (low) chance of experiencing (recovering from) a distress in the most persistent economic state.

to the rest of the orchard. The lower the increase in its distress intensity, the higher the hedging demand for this asset. If the price variation in the two full-information economies is smaller than the update in the posterior, then for some assets the full-information result can be even reversed: a distress can lead to a positive contemporaneous return. We formalize this discussion in the next Proposition.

Proposition 2 *In the full-information economy, conditional on either the 'high' or the 'low' state of the latent factor, the price of any security in the cross-section decreases after a distress and increases after a recovery event of any tree. In the incomplete-information economy, the price of security i decreases after the distress of tree j if the following condition is satisfied:*[8]

CONTAGION:

$$p_t^h(1-p_t^h)\frac{\overline{\lambda}^j-\underline{\lambda}^j}{\widehat{\lambda}_t^j}(\overline{P}^i(Y_t,x_t-j)-\underline{P}^i(Y_t,x_t-j))<p_t^h\overline{\Delta P}+(1-p_t^h)\underline{\Delta P} \quad (27)$$

where $\overline{\Delta P}=\overline{P}^i(Y_t,x_t)-\overline{P}^i(Y_t,x_t+j)$, *and* $\underline{\Delta P}=\underline{P}^i(Y_t,x_t)-\underline{P}^i(Y_t,x_t+j)$. *Conversely, the price of security i increases if the following holds:*

FLIGHT-TO-QUALITY:

$$p_t^h(1-p_t^h)\frac{\overline{\lambda}^j-\underline{\lambda}^j}{\widehat{\lambda}_t^j}(\overline{P}^i(Y_t,x_t-j)-\underline{P}^i(Y_t,x_t-j))>p_t^h\overline{\Delta P}+(1-p_t^h)\underline{\Delta P} \quad (28)$$

If (28) holds, a distress of firm j has a flight-to-quality rather than a contagion effect, on firm i: the combined effect of learning (the distress of firm j signals lower consumption growth) and of the intrinsic hedging potential of firm i's equity against bad economic states leads to an increase in the demand and price of equity i. Likely candidates for condition (27) to be violated are securities with high price-dividend ratios. Suppose that the firm that experiences a negative dividend shock, firm j, has lower price-dividend ratio than firm i. According to the discussion in point i) above, event intensities of low p/d ratio firms are

[8]Similarly, it increases (decreases) after the recovery of tree j if the following condition is (not) satisfied:

$$p_t^h(1-p_t^h)\frac{\overline{\eta}^j-\underline{\eta}^j}{\widehat{\eta}_t^j}[\overline{P}^i(Y_t,x_t+j)-\underline{P}^i(Y_t,x_t+j)]>p_t^h\overline{\Delta P}+(1-p_t^h)\underline{\Delta P}, \quad (26)$$

where $\overline{\Delta P}=\overline{P}^i(Y_t,x_t)-\overline{P}^i(Y_t,x_t+j)$, and $\underline{\Delta P}=\underline{P}^i(Y_t,x_t)-\underline{P}^i(Y_t,x_t+j)$ Condition (27) can be alternatively stated as follows: *posterior belief update*$\times$ *difference of post-distress full-information prices across states of the economy* $<$ *difference between pre- and post-distress partial information prices ignoring the posterior probability update* more disperse in the high and low states of the latent factor. Thus, not only firm i has lower propensity to distress events, but it has similar intensities across the states of this factor, thus less procyclical. This implies that a negative shock to firm j unfolds an adverse consumption state that the equity of firm i is ideal to hedge, because of its lower correlation with aggregate consumption. These properties imply that firm i is a likely candidate to violate condition (27), because of two reasons: (a) the full-information price of firm i in the 'low' state is higher than its full-information 'high' state price, and its post-distress 'low' state price may be even higher than its pre-distress 'high' state price. (b) A major posterior probability update occurs after the negative dividend shock to firm j – which is more procyclical – as discussed in Section II.C. Intuitively, if a negative shock is experienced by its own dividend, firm i cannot violate condition (27), so that its equity price and p/d ratio always decline.

Example. Consider a simple three-sector economy in which differences across trees are exclusively due to event risk intensities, proxied by the parameter λ^i. The orchard is symmetric across all other dimensions: cash flow shocks, x^i, and recovery intensities, η^i, are the same across trees. Let's consider the simple case in which the intensity of event risk is ranked highest to lowest from sector 1 to 3, i.e. $\lambda^1 > \lambda^2 > \lambda^3$. Figure 6 illustrates the effect of learning on asset returns in this stylized economy. Given the symmetric structure of the network, differences in p/d ratios are implied by the intensity of event risk.

Insert Figure 6 about here

In this example, sector 3 displays the highest p/d ratio because of its low unconditional intensity of distress – thus high dividend growth – and low covariance with the common factor – thus small covariance with aggregate consumption. As a result, Figure 6 shows two results. First, the equity return response of sector 3 to a distress event of sector 1 is less negative than the response of sector 2, whose cash-flow is more pro-cyclical and prone to distress risk. Second, and more importantly, cross-sectional learning is the main channel responsible for the heterogeneity in responses: the effect on sector 2 to a dividend shock of sector 1 is almost insensitive to the uncertainty about the state of the latent factor. Indeed, incomplete information has a minor effect on the return to this sector: the scarce propensity to hedge adverse consumption states of this sector implies that it is no more attractive if the economy is currently in the 'bad' rather than in the 'good' state. In the full-information economy conditional prices of sector 2 are similar, therefore in the incomplete information economy uncertainty about the state of the latent factor does not matter. Sector 3, instead, is significantly more valuable in the 'low' state of the common factor, where dividend growth is deemed low for all the cross-section. Cross-sectional learning then implies a higher return for sector 3, the more the distress of sector 1 signals that the 'good' state was erroneously assumed likely – high p_t^h.

It is instructive to compare this result to a traditional *one-tree* economy. In this case, an aggregate consumption disaster would lead both the price and the price-dividend ratio of the market portfolio to fall abruptly, because this event unarguably predicts higher dividend growth. Absent cross-sectional learning and/or cross-sectional network heterogeneity, the agent simply updates the future outlook of the sector and the range of possible implications are naturally more limited. Extending the standard one tree economy indeed enrich the possible range of outcomes.

It is also interesting to understand the role played by the persistence induced by the Markov chain in the event intensities with respect to effect generated by pure jump innovations. Imagine to extend the one-tree Veronesi (2000) economy to include rare events in the form of Poisson jumps in aggregate consumption, occurring at an unknown intensity driven by an unobservable factor. In such (hypothetical) setting, the occurrence of a negative cash flow shock implies a drop in the asset value and a drop in conditional expected consumption growth. Since the jump event is transitory, agents *increase* their investment in risky assets to finance future consumption, thus inducing an increase in the equilibrium p/d ratio and depressing expected returns. Even though Poisson jumps are transitory, learning has the effect of a permanent increase in the expected value of event intensities, as the agent uses observations about dividend shock to revise her posteriors about the jump intensities. In our context, however, the dynamics allows for a level of persistence. As a result, some implications can reverse sign. Because of the Markov nature of the state dynamics of the event intensities, a negative realization induces an *increase* in expected consumption growth given the perceived temporary nature of the event. This leads to a *decrease* in the demand for risky assets, a reduction in the p/d ratio and an increase in risk premia. This effect partially addresses the puzzle that learning counterfactually generates a reduction in the equity risk premium, at least in the context of a simple one-tree economy. We investigate in more details the implications for the equity risk premium in the next section. For convenience we summarize in Table 1 a comparison of the predictions of previous models and our specifications.

Insert Table 1 about here

D. Risk premia

The equilibrium risk premium of the $i-$th security is:

$$\mu_t^i = \mathbb{E}\left[\left.\frac{dV_t^i + \epsilon_t^i}{V_t^i}\right| \mathcal{F}_t^{x,Y}\right] - r_t = -\mathrm{Cov}\left[\left.\frac{d\xi_t}{\xi_t}, \frac{dV_t^i}{V_t^i}\right| \mathcal{F}_t^{x,Y}\right]. \tag{29}$$

Our framework allows for a straightforward decomposition of equilibrium risk premia in terms of the sources of risk priced in the economy, as discussed in the following Proposition.

Proposition 3 *The equilibrium risk premium of the $i-th$ security can be decomposed into a premium for diffusive risk, a premium for distress risk (μ^i_λ) and a premium for recovery risk (μ^i_η). These components read explicitly as follows:*

$$\mu^i_t = \gamma\sigma^2_Y + \mu^i_\lambda + \mu^i_\eta \tag{30}$$

$$\mu^i_\lambda = -\sum_{j=1}^N (1-H^j_t)\left[q^h_t\left(\frac{\overline{P}^i(Y_t,x_t-j)}{\overline{P}^i(Y_t,x_t)}\overline{\lambda}^j\tilde{\theta}^j_t\right)+q^l_t\left(\frac{\underline{P}^i(Y_t,x_t-j)}{\underline{P}^i(Y_t,x_t)}\underline{\lambda}^j\tilde{\theta}^j_t\right)-\widehat{\lambda}^j_t\tilde{\theta}^j_t\right] \tag{31}$$

$$\mu^i_\eta = -\sum_{j=1}^N H^j_t\left[q^h_t\left(\frac{\overline{P}^i(Y_t,x_t+j)}{\overline{P}^i(Y_t,x_t)}\overline{\eta}^j\tilde{\theta}^j_t\right)+q^l_t\left(\frac{\underline{P}^i(Y_t,x_t+j)}{\underline{P}^i(Y_t,x_t)}\underline{\eta}^j\tilde{\theta}^j_t\right)-\widehat{\eta}^j_t\tilde{\theta}^j_t\right] \tag{32}$$

where $\tilde{\theta}^j_t = \theta^j_t - 1$ is the market price of risk for the distress (if $H^j_t = 0$) or recovery (if $H^j_t = 1$) event of the $j-th$ tree at time t, reported in (18). $\overline{P}^i(Y_t,x_t-j)$ ($\overline{P}^i(Y_t,x_t+j)$) denotes the full-information $i-th$ security price at time t, conditional on the 'high' state of the common factor, if tree j has an immediate distress (recovery), as reported in Proposition 1. Similarly, $\underline{P}$s denote full-information prices conditional on the 'low' state of the common factor. Finally, q^h_t denotes the following 'value adjusted' posterior probability of the 'high' state of the common factor:

$$q^h_t = \frac{p^h_t\overline{P}^i(Y_t,x_t)}{p^h_t\overline{P}^i(Y_t,x_t)+(1-p^h_t)\underline{P}^i(Y_t,x_t)} \tag{33}$$

and $q^l_t = 1 - q^h_t$.

The term $\gamma\sigma^2_Y$ is the compensation for the diffusive risk of the common dividend component Y. The distress risk premium, μ_λ, comprises two layers of reward. [9] First, a direct compensation for distress risk, due to the impact of the persistent dividend component x_t on the price-dividend ratio. This layer holds also in the full-information economy. Second, a component due to learning. To describe the first channel, assume full-information, hence no learning premium. Since, in this case, either $p^h_t = 1$ or $p^h_t = 0$, depending on the observed state of the factor, the risk premium (30) becomes:

$$\mu^i_\lambda = \sum_{j=1}^N (1-H^j_t)\lambda^j_t\tilde{\theta}^j_t\left(1-\frac{\widetilde{P}^i_t(Y_t,x_t-j)}{\widetilde{P}^i_t(Y_t,x_t)}\right) \quad \widetilde{P} = \overline{P}\, or\, \underline{P}. \tag{34}$$

[9] In the following discussion we consider the distress risk premium μ^i_λ. The intuition for the recovery premium μ^i_η is similar.

As discussed in Section III.D, the transition to distress of any tree j gives rise to a contemporaneous negative return for any security i. Therefore $\widetilde{P}_t^i(Y_t, x_t - j)/\widetilde{P}_t^i(Y_t, x_t) < 1$, so that each summand in (34) is positive and increasing in the distress intensity. This direct reward for event risk is due to both the cash-flow and valuation betas of the security. The former is positive and it follows from the covariance between the dividend shock associated to the event and aggregate consumption growth. The latter is also positive and follows from the impact of a distress event on the price-dividend ratio, which induces positive covariance between the latter and aggregate consumption. Ultimately, this effect generates high equity premia even for small levels of risk aversion.

The term μ_λ also include a learning component. To gain insight on this part of the term, assume that the first layer is switched off, so that learning vehicles all the compensation for trees' distress risk. This entails assuming that $\overline{P}^i(Y_t, x_t - j)/\overline{P}^i(Y_t, x_t) = 1$ and $\underline{P}^i(Y_t, x_t - j)/\underline{P}^i(Y_t, x_t) = 1$, that is, that the full-information price-dividend ratio is independent of the persistent component x.[10] Expression (30) reduces to:

$$\mu_\lambda^i = -\sum_{j=1}^{N}(1 - H_t^j)[(q_t^h - p_t^h)\overline{\lambda}^j\tilde{\theta}_t^j + (q_t^l - p_t^l)\underline{\lambda}^j\tilde{\theta}_t^j] \tag{35}$$

Notice that q is the 'value adjusted' posterior belief of Veronesi (2000). This probability distribution, reported in expression (33), puts more weight than the original posterior belief p on the state of the common factor where the asset is valued the most. As seen in Section III.D, this is the 'low' state if $\gamma > 1$. The learning premium (35) is negative, because the value-adjusted probability of the 'low' state is higher than the objective one, and in this state the distress intensity λ is higher. Expression (35) coincides with the risk premium of Veronesi (2000). The learning premium is part of the valuation beta of the security, because it derives from the covariance between the (posterior) price-dividend ratio – i.e. the valuation ratio – and aggregate consumption. The intuition is that, as remarked in Section III.D, because of learning the dividend shock associated to a distress event has a positive effect on (posterior) price-dividend ratios; this effect is larger for securities that have hedging potential against those adverse consumption states that the distress predicts as likely. The dividend shock coupled to a variation of the opposite sign in the price-dividend ratio implies a negative covariance between the latter and aggregate consumption, hence a negative learning premium.

In summary, the risk premium (30) combines these two opposing forces. The next Proposition clarifies why this interaction can generate high premia for moderate levels of risk aver-

[10]This would be the case if events of the cross-section were governed by an IID jump process x, rather than a persistent Markov chain.

sion: the risk premium is high as long as each distress gives rise to sufficient contagion in the cross-section, and when the risk aversion increases contagion is more likely.

Proposition 4 *i) In the premium component due to distress (recovery) risk, μ_λ^i (μ_η^i), the contribution of a given tree j is positive if condition (27) ((26)) is satisfied.*

ii) The risk premium increases unboundedly when the risk aversion increases.

In the full-information economy the better (worse) future consumption perspective forecasted by a distress (recovery) event dampens (enhances) the propensity of the agent to save and precautionally invest in the securities to hedge adverse consumption states. The reduced (increased) demand implies that negative (positive) consumption shocks are fully translated into negative (positive) contemporaneous returns. This high covariance between consumption and returns is the reason for the distress (recovery) premium charged. When information is incomplete, a dividend fall forecasts additional likely distress events for other firms, because of an increase in the posterior distress correlation between the firms' dividends. If the resultant of this learning process is lower expected consumption growth, the hedging demand for the assets increases. Depending on the hedging abilities of a specific security, in the sense clarified in Section III.D, this effect may counteract the reduced demand for the asset arising from a lower expected cash-flow. Depending on whether (27) is satisfied (or not), the asset return is negative (positive) and the distress premium positive (negative). The same conclusion and a similar intuition hold for the recovery risk premium.

Proposition 4 *ii*) reports an important contribution of our paper. It explains why the C-CAPM that we advocate is able to cope with the equity premium puzzle. When the coefficient of relative risk aversion increases, the direct reward for event risk prevails on its learning component and risk premia increase unboundedly. If a distress occurs and the risk aversion is high, the abrupt output fall leads to a jump in current marginal utility, hence to an immediate lower desire to postpone consumption intertemporally. In this situation the agent's behavior is determined mainly by the higher expected consumption growth induced by the foreseen recovery - the direct effect of persistent distress events – and less by the more likely perspective of additional future distress events – the learning effect. Indeed price-dividend ratios fall abruptly after a distress, and returns are strongly correlated with aggregate consumption risk, for very high risk aversion. This leads to increasing premia for distress risk. The intuition for recovery risk premia is similar.

The previous result is interesting and not obvious at first sight, given that the current literature mainly suggests otherwise. Panel 2 of Table 1 compares our event risk premium with those arising in related modeling frameworks, which are described in the last paragraph of Section III.D. In the single-tree version of our model, the absence of cross-sectional learning implies that the learning component of the equity premium is minor. As a result, the event risk premium is high both with full and incomplete information. In the event-risk analog of Veronesi (2000), instead, the valuation beta comprises a pure learning component that is negative and decreasing in the relative risk aversion coefficient. This feature makes the risk premium bounded in the parameter of risk aversion, thus making – as the author highlights and discusses – the single-tree economy difficult to be reconciled with empirical evidence on the equity risk premium. In the context of our multiple-tree model, on the other hand, the learning component of the valuation beta is also negative, but Proposition 4 *ii*) states that the direct compensation for event risk – due to the impact of the persistent component – is dominant over the learning component for increasing risk aversion, thus making the risk premium unbounded in this parameter.

IV. Analysis of the cross-section of expected returns

In this Section, we show under which conditions cross-sectional learning can help to reconcile the theoretical implication of economies with event risk with the empirical properties of the cross-section of expected returns .

In our model stocks with high price-dividend ratios have lower expected excess returns than stocks with low price-dividend ratios. The reason is that the negative impact of learning on risk premia is most effective for assets with high price-dividend ratios. The intuition is simple. Briefly recalling the discussion of Section III.D, these assets are characterized by low and idiosyncratic event risk – that is, similar distress and recovery intensities across states of the common factor – and limited loss of dividend growth upon distress – that is, small $(\overline{x} - \underline{x})/\overline{x}$. They are less prone to being in distress and they suffer a lower cash-flow reduction in bad aggregate states of the world. When incomplete information is coupled with a heterogeneous network of trees, negative shocks can generate asymmetric effects: not only because trees with different hedging properties will be subject to different hedging demands, but also because of the way they affect other trees. When a distress of some tree downgrades perceived dividend growth, the additional hedging portfolio demand is directed mainly towards assets with high price-dividend ratios - that is, high hedging potential. Hence cross-sectional learning mostly decreases the premia of these assets. The next Proposition formalizes this discussion in two steps. First it shows that in our model, even with full-information, a sufficient condition for stocks with higher price-dividend ratio to command a lower conditional risk premium is that the risk aversion is 'low', that is, smaller than a given threshold. Then it shows that the higher the price-dividend ratio of an asset, the more cross-sectional learning decreases its full-information premium.

Proposition 5 *Assume that the full-information equilibrium price-dividend ratio of the claim to dividend stream j is higher than the full-information price-dividend ratio of the $i-th$ security, i.e.*

$$\frac{\overline{P}^j}{D_t^j} \geq \frac{\overline{P}^j}{D_t^i}, \qquad \textit{and} \qquad \frac{\underline{P}^j}{D_t^j} \geq \frac{\underline{P}^j}{D_t^i}. \tag{36}$$

Let $\overline{\mu}_v^i$ and μ_t^i denote, respectively, the full-information risk premium of the $i-th$ security in state v of the common factor, $v = h, l$, and the incomplete information risk premium of the same security. Then:

i) If the following condition is satisfied:

$$\gamma \leq \min_{i,j} \left[\frac{\overline{x}^i + \sum_{u \neq i} x_{t-}^u}{\overline{x}^i}, \frac{\overline{x}^j + \sum_{u \neq j} x_{t-}^u}{\overline{x}^j} \right] \tag{37}$$

we also have:

$$\overline{\mu}_v^i \geq \overline{\mu}_v^j, \qquad v = h, l. \tag{38}$$

ii) With incomplete information, the premium of the $i-th$ security decreases more than the premium of the $j-th$ security with respect to the full-information premium, i.e.

$$\overline{\mu}_v^j - \mu_t^j \geq \overline{\mu}_v^i - \mu_t^i \qquad v = h, l \tag{39}$$

It is interesting to interpret our results from the perspective of Santos and Veronesi's (2009) model. Their equilibrium model features a stochastic discount factor implied by nonlinear external habit formation preferences and each tree is modeled by means of a share (of aggregate consumption) process which is specified as an exogenous diffusion. They show that habit persistence counterfactually generates higher expected returns for stocks with high price-dividend ratios, if firms/trees are allowed to differ only in terms of their expected dividend growth. In the context of their model, the reason is that firms with high price-dividend ratios have high expected dividend growth, hence pay the majority of their cash-flows far in the future, and their returns have higher (in absolute value) covariance with the stochastic discount factor because they are more sensitive to shocks of the latter. Therefore, the higher valuation beta induces a higher premium. This implication is counterfactual with respect to the findings of the empirical literature. However, they show that if firms are allowed to differ in the covariance of their dividends with aggregate consumption, then a lower premium for firms with high price-dividend ratio can be restored. Higher covariance with aggregate consumption implies lower price-dividend ratio and a higher premium in the form of a cash-flow beta. In our framework, Proposition 5 delivers a result that is consistent with the data, even though preferences are not time-varying and dividend shocks are solely responsible for the variability. In our model, assets with high price-dividend ratio have both high expected dividend growth - due to low (high) distress (recovery) intensity - and modest covariance with the common factor - that is, similar event intensities across its states. Consequently, they feature both high valuation betas and low cash-flow betas. With full-information the former property dominates, giving rise to higher risk premia, unless the risk aversion is 'low', in the sense of Proposition 5 *i*), so the variation of state prices following dividend shocks is not too severe. With cross-sectional learning, a negative dividend shock signals that the 'low' state of the common factor is more likely. Since the cash-flow risk of assets with high price-dividend ratios is scarcely correlated with the common factor, their posterior covariance with aggregate consumption is lower than that of assets with low price-dividend ratios. Ultimately, the higher the price-dividend ratio, the more learning reduces the cash-flow beta, hence the overall risk premium of the asset.

V. Heterogeneously connected networks

The second channel that we want to explore is the cash-flow connectivity structure among different trees in the orchard. In Santos and Veronesi (2009), a key ingredient for the cross-sectional predictability is the role played by the dynamics of the exogenous consumption share process. In our setting, we rely on the structural characteristics of connectivity of a tree with the rest of the network. This determines the extent to which cross-sectional learning generates predictability. Our analysis has so far exploited the properties of the simplest form of network, where the degree of interdependence is given by the influence on different sectors of a common exogenous factor. Each tree shares the same characteristics of connectivity with the rest of the orchard: we call this a *symmetric network*. In Panel 1.a of Figure 9 we report a stylized diagram of this structure:

Insert Figure 9 about here

A useful feature of our modeling approach is the flexibility in modeling a variety of different network connectivity characteristics. Examples include (a) economies in which the latent factor is not exogenous with respect to distress and recovery events of sectors, so that a distress event of a sector can have a systematic impact on the economy (endogenous feed-back effects); (b) economies in which shocks to individual sectors can directly influence the cash-flows of remaining sectors, that is, sectors are pair-wise connected but the connections can be asymmetric. We call such general forms of dependence structure an *asymmetric network*. The first effect (a) is achieved by letting the transition probabilities of the common factor depend on the state variable x_t: $k_u = k(x_t)$, $u = h, l$. In this manner, the 'good' and the 'bad' states of the factor become more or less persistent depending on the health status of specific sectors. We account for the second effect (b) by allowing sectors' distress and recovery intensities to depend on x_t: $\lambda_t^i = \lambda_t^i(x_t)$, $\eta_t^i = \eta_t^i(x_t)$, so that the distress likelihood of sector i depends on the distress or recovery status of the rest of the network. Panel 1.b reports a diagram of a general asymmetric network structure. The example in Panel 1.c depicts a vertically integrated industry, where sectors are specialized in successive stages of the manufacture of a product. As a laptop manufacturer does not directly source parts from a silicon wafer maker, but rather indirectly through its supply agreement with a microprocessor manufacturer, similarly output shocks to base sectors (sector 3) propagate to final sectors (sector 1) by means of the supply chain (sector 2). Since the industrial relation between sectors is asymmetric, shocks (events) originating from sector i do not impact j in the same manner as shocks originating from j impact i. In Panel 1.c we depict this asymmetry distinguishing between dotted and solid arrows, while in analytical terms we model it by assuming that $\frac{d\lambda_t^i(x_t)}{dx_t^j} \neq \frac{d\lambda_t^j(x_t)}{dx_t^i}$.

We study the cross-sectional characteristics of a simple asymmetric network, the three-sectors economy depicted in (5). Its diagram is reported in Panel 2 of Figure 9. Table 7 reports the numerical values of the parameters that give rise to this structure.

Insert Table 7 about here

In this economy, a distress of the Banking sector has a systematic impact, because the chance k_h (k_l) of a regime switch to the 'low' ('high') latent state becomes 5 times higher (smaller). In addition, upon a distress of Banking there is a chance of direct contagion, because its role of credit provider makes this sector directly connected to the remaining two: the intensities of distress for Housing and Manufacturing become a_h (a_l) times higher if the common factor is in the 'high' ('low') state. It is reasonable to assume $a_l > a_h$, so that contagion effects are more pronounced in the bad state of the economy. The connectivity of the network is asymmetric: the impact of a distress for Housing or Manufacturing is not systematic, in that it does not affect the common factor dynamics. Housing is actively connected to Banking alone, due to financial securitization reasons: a Housing distress amplifies b_h (b_l) times the intensity of distress for Banking in the 'high' ('low') economic state. The Manufacturing sector is 'remote', in the sense that it does not directly influence any sector. The network asymmetry emerges from the heterogeneity of network connections, and from the assumption that $b_u > a_u$, $u = h, l$, so that shocks to the Banking sector have the highest degree of propagation to the rest of the network.

While in the previous literature multiple trees economies have been studied through the specification of share processes, here we study the implications of the mutual influence of sectors and the differential impact of the latent factor on sectors. To highlight the importance of this different channel, in the analysis and numerical examples to follow, we assume that the initial dividend process is identical across sectors, so that the relative size effect is muted. This allows us to investigate the marginal role of the degree of 'active' connection of each sector to the rest of the economy as a fundamental determinant of expected returns. This characteristics is the extent the dividend shocks of a sector forecast dividend shocks of other sectors, but not the opposite. We call 'exogenous' sectors those that are 'actively' connected.[11] In what follows, we introduce a synthetic measure of this property.

Definition 3 *For any tree i of the population, consider the following measure:*

$$ex^i_{s,T} = \frac{1}{T-s}\left(\mathbb{E}\left[\int_s^T \mathbf{1}\left(\bigcup_{z\neq i}(x^z_u = \underline{x}^z)\right) du \middle| (x^i_s = \underline{x}^i, x^j_s = \overline{x}^j, \forall j \neq i), \mathcal{F}^{x,Y}_s\right] - \right.$$
$$\left. \mathbb{E}\left[\int_s^T \mathbf{1}(x^i_u = \underline{x}^i) du \middle| \left(x^i_s = \overline{x}, x^j_s = \underline{x}^j, \forall j \neq i\right), \mathcal{F}^{x,Y}_s\right]\right) \quad (40)$$

Tree i is deemed more exogenous than tree j on the horizon $T-s$ if $ex^i_{s,T} > ex^j_{s,T}$.

$ex^i_{s,T}$ is the expected fraction of the time horizon (s,T) where some sector is in distress, but not sector i, conditional on the initial distress status of sector i alone, minus the expected fraction of time where sector i is in distress conditional on an initial distress of all sectors j excluding i.[12] Intuitively, the more a sector's distress status forecasts future distress events for the rest of the economy, the higher the probability of future distress of some tree conditional on a past distress of the sector. The measure $ex^i_{s,T}$ is high when a sector is 'exogenous', i.e. when its future distress events are scarcely predicted by past events of the remaining trees. The probabilities of future distress reported in the expression for $ex^i_{s,T}$ are the entries of the probability transition matrix of the persistent sectors' dividend component x_t: $\exp(-\mathbf{A}^H(T-t))$. Its rows represent conditional (initial) sectors' distress

[11] This concept of economic 'exogeneity' is related to the statistical concept of causality.

[12] Expression (40) can be rewritten in terms of conditional probabilities of distress:

$$ex^i_{s,T} = \frac{1}{T-s}\left(\int_s^T \mathbb{P}\left[\bigcup_{z\neq i}(x^z_u = \underline{x}^z) \middle| (x^i_s = \underline{x}^i, x^j_s = \overline{x}^j, \forall j \neq i), \mathcal{F}^{x,Y}_s\right] du - \right.$$
$$\left. \int_s^T \mathbb{P}\left[(x^i_u = \underline{x}^i) \middle| \left(x^i_s = \overline{x}, x^j_s = \underline{x}^j, \forall j \neq i\right), \mathcal{F}^{x,Y}_s\right] du\right) \quad (41)$$

or normalcy states, while its columns represent terminal distress or normalcy states. The matrix $\mathbf{A}^H$ is the sectors' infinitesimal transition probability matrix: it takes into account the connectivity property of the network through the functional form of the distress (recovery) intensities λ (η), and of the regime switch probabilities (k_h, k_l). Sectors with low exogeneity are highly 'passively' connected to the rest of the economy, in that their distress and recovery intensities are strongly cyclical - thus being exogenously influenced by the common factor - but their cash-flow record does not affect other sectors either by direct distress contagion or indirectly, acting on the common factor. It should be noticed that since shocks in our economy propagate dynamically, depending on the extent of amplification of absorption due to the network structure, the previous measure of 'active' connection has a time dimension and it should be thought as a term structure characteristics.

One of the most debated empirical regularities of the cross-section of returns is that stocks with high price-to-fundamental ratios, growth stocks are characterized by lower average expected returns than stocks with low price-to-fundamental ratios, value stocks. In our model the 'growth' or 'value' property of a sector depends on its degree of exogeneity, as defined in Definition 3. The next Proposition formalizes this link.

Proposition 6 *If, given the information set at time s, tree i is more exogenous than j at any horizon $T > s$, and $\overline{x}^i = \overline{x}^j$, $\underline{x}^i = \underline{x}^j$, the p/d ratio of i at time s is greater than the p/d ratio of j.*

Exogenous sectors lead the business cycle as their dividend shocks are more likely to become systematic, while they are more immune to external shocks. They are less procyclical and their cash-flows have limited covariance with aggregate consumption, thus lower cash-flow risk. Expression (40) also mandates that exogenous sectors have low unconditional risk of distress due to their lower distress persistence. This implies higher dividend growth, hence cash-flow duration. Higher exogeneity at all time-horizons unambiguously predicts higher p/d ratios and, according to Proposition 5, lower expected excess returns. While this effect is known in the literature as the value/growth premium, in the context of our economy this property emerges as an implication of the network connectivity.

To investigate further the link between the characteristics of exogeneity of a network and the cross-section of p/d ratio and the expected returns, in what follows we consider two different network structures. In the first one, we consider an economy in which a sector (Banking) is more actively connected (exogenous) to the rest of the network. In the second one, we consider the case of state-dependent heterogeneous network.

CASE 1 (Heterogeneous Intensity Propagation). In the first case economy (see Panel 1 of Figure 10), we choose a calibration in which a distress of the Banking sector amplifies 10 times the intensity of distress for both manufacturing and housing, regardless of the state of the common factor, i.e. $a_h = a_l = 10$. On the other hand, the banking sector has (only) twice the chance of falling in distress after a housing distress, i.e. $b_h = l_l = 2$. Panel 2 of Figure 10 plots the three term structure of sectors' exogeneity, $ex^i_{s,T}$, for the three sectors in this economy. The connectivity structure implies that the Banking sector is the most exogenous, because its distress events strongly affect shocks of remaining sectors, at all times horizons. At the same time, its future distress events are scarcely determined by past shocks of the other sectors. The Manufacturing sector is the most 'passively' connected (or least exogenous).

Insert Figure 10 about here

Two important results emerge. First, we find that everything else equal, the asymmetry generates significant dispersion in both p/d ratios and equity premia. When no sector is in distress (solid line) the p/d ratios range from 5.2 of Manufacturing to 8 of Banking, while the risk premia range from 6% of Manufacturing to 4% of Banking. Conditional on a distress of the banking sector (dashed line), the cross-sectional variability is even higher: p/d ratios range from 3 of Manufacturing to 5 of Banking and premia range from 7.5% of Banking to 13% of Manufacturing. This is important since it shows that the network structure plays a key role in the implied dispersion of equity risk premia, which is usually not granted in full-information symmetric economies. Second, the degree of exogeneity of a sector is linked to its p/d ratio. Indeed, the higher a sector exogeneity, the higher its price-to-fundamental ratio and the lower its expected returns. The intuition is simple. For a sector to be highly valued relative to its fundamentals - and demand a low premium, according to Proposition 5 - it needs to provide high dividend flows when the rest of the sectors are in distress, and aggregate consumption is low. This is hardly the case for the Manufacturing sector, the least exogenous, because a distress of remaining sectors has likely thrown the economy in the 'low' state or spread to connected sectors by direct contagion on intensities, so that a distress for this sector is also to be expected. The Manufacturing sectors lags the business cycle. This mechanism is mostly effective when the banking sector is in distress (dashed line). In this state, the risk of events for the exposed sectors is imminent, hence equity premia are high and expected consumption growth is modest. The desire to substitute consumption intertemporally, coupled with the poor growth perspectives of the exposed Housing and Manufacturing sectors, imply that the investor looks for shelter in the recovery perspectives of the Banking sector, thus widening the gap between equity premia.

To investigate further the role played by the asymmetry in the network structure and the link between exogeneity and p/d ratios, we also consider an economy in which the Banking sector is not only more exogenous on average than the other two sectors, but the previous amplification mechanism is asymmetric and larger in 'low' states.

CASE 2 (State-dependent Heterogeneity). In Panel 1 of Figure 11 sectors' characteristics are as in Figure 10, but the direct contagion effect of a Banking or Housing distress is heterogeneous across states of the common factor. Shocks to the Banking sector are amplifies more in 'low' states. Compared to the economy in Figure 10, the amplification is more severe in the 'low' state - because $a_l = 15$ and $b_l = 3.5$ - and milder in the 'high' state - because $a_h = 5$ and $b_h = 1.5$.

Insert Figure 11 about here

This additional layer of network asymmetry generates additional cross-sectional dispersion of equity premia. For instance, conditional on a distress for the Banking sector, premia range from 0.082 for Banking to 0.15 of Manufacturing. The reason for this additional variability is cross-sectional learning. Consider the Manufacturing sector, which has the lowest p/d ratio. If distress contagion is more severe in the 'low' state of the factor, the output of Manufacturing is more pro-cyclical and correlated with aggregate consumption, thus less useful to hedge adverse aggregate consumption states. This implies that learning about a more likely 'low' state of the economy brings almost no additional demand for this sector, hence does not lower its premium significantly. It does, however, decrease significantly the premium of sectors with high p/d ratios, as the Banking sector, because, while they lead the contagion, they do not suffer from the effect mentioned above.

It is important to highlight the link between sector exogeneity and risk premia. As Panel 2 of Figure 11 shows, the Banking sector is more exogenous than in CASE 1 at all time horizons; the Housing and, to a lesser extent, the Manufacturing sector are less exogenous. This explains why the impact of cross-sectional learning is more pronounced when the network is asymmetric, especially for sectors with high p/d ratios. The Banking sector has high hedging potential, its price decline following the Manufacturing distress is partially dampened, since the persistence of the 'bad' state creates a higher hedging demand against lower expected future consumption. The opposite occurs for the Manufacturing sector. Notice that this effect and its implications in terms of equilibrium cross-sectional p/d ratios would not emerge in a full-information economy with a symmetric network structure (see Julliard and Gosh, 2008)

Insert Figure 12 about here

From this analysis we learn that 'value' sectors are the least exogenous sectors and our model predicts higher expected excess returns for 'value' sectors. It is important to notice that this occurs even in absence of a consumption share effect, which is the traditional channel investigated in the existing literature. Incomplete information and learning play the key role in generating this effect.

VI. An empirical analysis

While a Consumption CAPM featuring disaster risk and full-information can be calibrated to produce large equity market risk premium (Barro and Ursua, 2008), it typically fails to explain the cross-section of expected returns (Julliard and Ghosh, 2008). Analyzing the 25 Fama-French portfolios, Julliard and Ghosh (2008) find that a consumption-based pricing kernel can explain only a small percentage of the cross-sectional variation of expected returns, after the rare disaster hypothesis is imposed on aggregate consumption data. As noted by the authors, and indeed confirmed by our theoretical analysis, this framework yields a narrow cross-sectional range of covariances with consumption risk, thus of expected returns. This leads to the documented disappointing cross-sectional empirical pricing performance. In our model two new channels induce additional cross-sectional dispersion of risk premia: learning and the asymmetric connectivity structure. These create the potential to improve with respect to a Consumption CAPM featuring disaster risk alone. In what follows, we quantify the extent to which each of these two channels can offer some help.

A. Data and Calibration Procedure

We collect a time-series of quarterly dividend distributions (from CRSP database) and share repurchases (from Compustat database) on 12 US industries portfolios from 1947 to 2007. Share repurchases are defined as firms' expenditure for the purchase of common and preferred stock minus any reduction in value on the number of preferred stock outstanding. At the beginning of each quarter we assign a firm to a given portfolio based on its CRSP four-digit SIC code. The industry composition follows the definition of Kenneth French.[13] Value-weighted quarterly portfolio cash-flows and prices are obtained according to the procedure of Mentzly, Santos and Veronesi (2004).[14] This procedure leads to portfolio cash-flows that are consistent with an initial investment in each industry. Distress and recovery intensities are difficult to estimate from a frequentist perspective, because of the rarity of events in a relatively short time-series.[15] We identify a sector's likelihood of being in distress over τ

[13]'17 Industry Portfolios' as appearing on K. French's website, after grouping 'drugs, soap, pfs and tobacco', 'food', 'mining and minerals' , 'steel works', 'textile, apparel and footwear' and 'other' into 'other'.

[14]See their Appendix for details.

[15]Barro and Ursua (2008) use a dataset of aggregate consumption expenditure for several countries and, assuming a common disaster intensity, they estimate this parameter using the cumulative number of years

years by calibrating its historical $\tau-$years default rate on the issued debt using data from Moody and Standard and Poor's. Since dividend distributions are subordinated to debt payments, dividend distress events are at least as frequent as corresponding corporate debt defaults. This procedure, therefore, produces a lower bound for the frequency of distress events.

In a first stage, we focus on the symmetric-network version of our economy. This allows us to abstract from the connectivity structure and gouge the pricing performance enhancements due to the learning channel. Then, we investigate the marginal effect of the connectivity structure.

We obtain parameter values by means of a sequential method of moments calibration procedure. Each step of the procedure takes as given the parameters calibrated at the previous stage. First, we obtain the parameters $\theta_1 = (\mu_Y, \sigma_Y)$ of the diffusive dividend component Y from aggregate consumption data. Second, we extract the transition intensities $\theta_2 = (k_h, k_l)$ of the common factor from the related estimation of Ribeiro and Veronesi (2002). Third, as briefly mentioned above, we calibrate distress and recovery intensities $\theta_3 = (\overline{\lambda}, \underline{\lambda}, \overline{\eta}, \underline{\eta})$ based on Moody's and Standard & Poor's historical default rates on corporate debt by Industry group. Fourth, we obtain persistent dividend components $\theta_4 = (\overline{x}, \underline{x})$ from sector-specific empirical dividend growth rates. Finally, we set the coefficient of relative risk aversion γ in order to allow the model to match average p/d ratio of the equally weighted market portfolio. The details of each step are reported in the Appendix. This sequential procedure has two important advantages. First, it allows us to obtain a sequence of exactly identified sets of moments conditions, which greatly simplifies and robustifies the parameter estimation. Second, it avoids finding structural parameters by matching price-based information: in our exercise, the properties of financial prices are an outcome rather than an input.

The next Table reports parameter values and some implied event risk indicator for each industry.

Insert Table 2 about here

The main properties of the calibrated economy are as follows:

i) We find considerable heterogeneity among sectors in both the average duration of distress events ($\overline{\tau}$) and in the expected fraction of time spent in distress (FTD, Table

spent in distress, where a period of distress is identified according to a given consecutive GDP fall. We don't follow this methodology because of the impossibility to impose a common event likelihood among sectors.

2).[16] For the Financial sector distress events are rare and persistent, with an expected duration of 6.75 years and an expected permanence in distress of only 2.5 years out of 100. The Housing and especially the Manufacturing sectors, instead, are prone to more frequent and less persistent events: for Construction and Construction Materials we observe $\overline{\tau} = 6.31$ and $FTD = 11.54$, for Automotive we have $\overline{\tau} = 4.72$ and $FTD = 11.68$, for Durables $\overline{\tau} = 5.06$ and $FTD = 9.26$, and for Fabricated Products we observe $\overline{\tau} = 6.22$ and $FTD = 15.11$.

ii) The dividend loss upon distress, $(\overline{x} - \underline{x})/\overline{x}$, is also quite heterogeneous. On average, we find that sectors whose events are more rare experience more severe losses. The Financial sector is expected to lose approximately 60% of dividend pay-outs at each event – which is consistent with the default of firms within the sector during times of distress – while the Housing and the Manufacturing sectors experience a loss of 12% and 21%, respectively.

iii) The difference $(\overline{\lambda} - \underline{\lambda})$ drives the correlation between sectors' dividend growth and the business cycle, hence it is indicative of the correlation between sectors' dividend growth and aggregate consumption growth – cash-flow risk. We find that the Financial sector has a more idiosyncratic dividend loss due to distress events than the Housing and, to a greater extent, the Manufacturing sectors.

B. *Simulation study: Fama-McBeth Regressions*

To investigate the implications of our framework we use combined time-series and cross-sectional information (Fama and McBeth (1973)). To this end, we first mimic the exercise

[16]These quantities are computed using Lemma A.2 in the Appendix. The unconditional fraction of time that a sector spends in distress (FTD) is found adapting Lemma A.2 (*iii*) to the required event and letting $T - s \to \infty$. The average persistence of each distress ($\overline{\tau}$) is found computing the conditional expected time left until recovery of the sector, from Lemma A.2 (*ii*), and then applying a simple numerical procedure to compute the unconditional expected time. This procedure exploits the ergodic property of the hidden Markov chain that governs events: it consists in simulating a long path of T years of distress, recovery events, and posterior probabilities, evaluate the conditional expected recovery times at each time point and average them across time. The unconditional expected number of distress events over a given time horizon τ is

$$E\left[\int_t^{t+\tau} \widehat{\lambda}_s^i \mathbf{1}(H_s^i = 1) ds\right].$$

The unconditional expectation is computed according to the same numerical procedure outlined above.

of Mentzly, Santos and, Veronesi (2004)[17] and consider a regression of the form:

$$\frac{P_s^i}{D_s^i} = \alpha^i + \beta_i \left(\frac{P_s^i}{D_s^i}\right)^* + \varepsilon_s^i \quad i = 1, 2, \ldots, 12, \tag{42}$$

where P_s^i/D_s^i denotes the price-dividend ratio of the $i-$th industry portfolio observed at date s, whereas $(P_s^i/D_s^i)^*$ denotes the corresponding model-implied price-dividend ratio. We then report the extent to which the model-implied consumption-betas can explain the cross-section of excess returns by considering a regression of the form:

$$R_s^{i,e} = \alpha^i + \omega^i \frac{-\text{Cov}\left(\frac{dU'(C_s)}{U'(C_s)}, \frac{dP_s^i}{P_s^i}\right)}{\mathbb{E}\left[\frac{dU'(C_s)}{U'(C_s)}\right]} + \varepsilon_s^i \tag{43}$$

$R_s^{i,e}$ denotes the return on the $i-$th industry portfolio in excess of the 3-month T-Bill rate observed at time s, while the ratio appearing on the right-hand-side is the consumption beta implied by our model. This last quantity is computed in closed-form and corresponds to the equity premium of the portfolio should the CCAPM advocated by our paper hold exactly. We simulate a long time-series of posterior beliefs (p_s^h) and multipliers x_s, and we split it into 2500 quarterly histories, each one having the length of the available sample. For each history, we compute theoretical price-dividend ratios and premia using the observed dividends and simulated values of unobservables – p_s^h and x_s – and we perform regressions (42) and (43) on the cross-section of industry portfolios at each point in time. We then compute mean coefficients across time and finally average them over the simulated histories to integrate out the dependence on unobservables. To compare the results to the complete information case, we apply the same methodology to obtain model-implied price-dividend ratios and premia under complete information. In this case the latent factor is assumed observable by the agent but unobservable by the econometrician.

1. The Cross-Section of Price/Dividend Ratios

Table 5 reports the results of the regression exercise (42).

Insert Table 5 about here

In Panel 1 model-implied prices are obtained under incomplete information and learning. While the model does not provide a perfect explanation of the data - which would demand a zero intercept and a unitary slope coefficient - it captures a significant portion of the

[17]See also Bansal, Dittmar and Lundblad (2005), Hansen, Heaton and Li (2008), Campbell and Cochrane (1999), and, for the rare disaster literature, Julliard and Gosh (2008).

cross-sectional variability of price-dividend ratios. Note that in the calibration procedure, the relative risk aversion coefficient has been calibrated to match the unconditional p/d ratio of the global equally weighted portfolio. No sector-specific price information has been used. The estimate for the slope coefficient, 0.49, is statistically different from both 0 and 1 at the 5% level. The intercept is not different from zero at the 5% level. The results can be compared to what would be obtained in an economy with complete information (see Panel 2). While the estimate of the slope coefficient is also in this case statistically different from zero at the 5% level, we find that the incomplete information economy produces an R^2 of 24%, compared to 15% for the full information economy. This documents the marginal contribution of cross-sectional learning to reproduce the cross-section of price-dividend ratios.

2. The Cross-Section of Expected Returns

Table 6 reports the results of the regression (43) for expected returns. In Panel 1, we analyze the general ability of the model-implied consumption betas to explain the cross section of equity expected returns. It should be noted that no information on the cross-section of expected returns has been used in the calibration. We find that estimate of the slope coefficient, 0.34, is statistically different from both 0 and 1, while the intercept is not statistically different from zero.

Insert Table 6 about here

When we compare the results to the case of full-information (see Panel 2),[18] two facts emerge. First, learning implies a higher cross-sectional dispersion of consumption betas. The ratio of the variance of consumption beta and the variance of excess returns is 21% in the incomplete information economy, and it reduces to 8.4% with full information. Second, the goodness of fit improves and the R^2 increases from 6% to 19%. It can be noticed that the R^2 obtained under incomplete information is similar in magnitude to the empirical R^2 usually found in Fama-French portfolios using consumption series. This result is due to the full-information consumption betas being less responsive to dividend shocks than their partial-information counterparts. Under partial information, dividend shocks affecting a tree predict dividend growth of other trees. This effect is muted under full information: regime switches of the state of the economy, which are now observable, act homogeneously across the cross-section

[18] Full-information consumption betas are those arising in our model when the agent can observe the latent state of the economy. These are evaluated at the parameter values reported in Table 2, which are inferred assuming partial information. Towards a more rigorous assessment, we should recalibrate parameters using a full-information method of moments. We avoid this step for simplicity, convinced that parameters calibrated under full-information would lead to an even worse explanatory power of full-information consumption betas.

of consumption betas. To further investigate this issue, Figure 7 looks at the cross-section of partial and full-information consumption betas arising in our calibration, as a function of the sector specific price-dividend ratio.

Insert Figure 7 about here

When we assume a coefficient of relative risk aversion higher than the critical level defined in (37), the full-information model fails to generate a value premium, i.e. high conditional full-information betas for assets with low price-dividend ratios. The partial information economy with cross-sectional learning produces more realistic results. In Figure (7) no sector is in distress: in such a state, sectors' equity premia are determined by the chance that a distress for some sector may occur. Expected dividend growth is at its lowest level and the demand for risky assets, driven by intertemporal consumption substitution motives, is at its highest. In this state and with full information the cross-sectional variability of premia is poor, to the extent that sectors' equity premia lie within a range 0.5% wide: higher consumption growth perspectives imply that sectors with high price-dividend ratios are not more attractive than sectors with low price-dividend sectors. With partial information, however, the range of variation of sectors' premia becomes 2.5%. The intuition behind this result is simple. The potential negative dividend shock brought about by a distress on one sector increases the signals of the possibility of a bad economic environment (i.e. more modest consumption growth) as the agent expects higher distress intensity for additional sectors. The increased incentive to substitute consumption over time benefits the demand for those sectors whose equity is more apt at hedging adverse consumption states, that is, sector with high price dividend-ratios. This additional demand induced by learning implies that these high p/d ratio sectors experience a less negative (or positive) return following the consumption loss caused by the distress event. Hence, these sectors demand a lower risk premium. The negative slope of the partial information curve appearing in Figure 7 confirms the consistency of learning with the 'value vs growth' anomaly. We extensively analyze this topic in the sequel.

C. Empirical Implication with Asymmetric Network

The second channel that contributes to the cross-section of expected returns is due to the characteristics of the connectivity structure. We now slightly change the calibrated economy to generate a simple form of network asymmetry, which comprises feed-backs between sectors and the common factor. Our aim is to study the link between exogeneity and the cross section of expected returns. Following the analysis in Section V, we accomplish this task by letting transition intensities of the common factor to depend on sectors' distress status:

$$k_h = 0.2418 \prod_{i=1}^{N} [1 + a_i \mathbf{1}(x_t^i = \underline{x}^i)] \qquad k_l = \frac{1.3109}{\prod_{i=1}^{N} [1 + a_i \mathbf{1}(x_t^i = \underline{x}^i)]}, \tag{44}$$

where 0.2418 and 1.3109 are the transition intensities in the symmetric network. We set $a_i > a_j$ if the $i-$th sector turns out to be more exogenous than the $j-$th in the symmetric calibration: this means that a distress of sector i determines a more likely transition to the bad state of the common factor than a distress of sector j, which increases the exogeneity gap between sectors i and j with respect to the symmetric case. The cross-sectional dispersion of sectors' exogeneity has thus been enhanced. The rest of the parameters is unchanged. Choosing coefficients a as in Table 3, we keep the unconditional event frequencies close to those of the base symmetric case.

Insert Table 3 about here

Figure 8 reports the cross-section of partial information consumption betas arising in the both the asymmetric and the symmetric network, as a function of the sector specific measure of exogeneity.

Insert Figure 8 about here

Expected excess returns decrease (increase) for those sectors for which network asymmetry implies higher (lower) exogeneity. Risk premia are very sensitive to the degree of exogeneity of a sector: a limited difference in the cross-sectional dispersion of exogeneity in the two networks generates a substantial increase in the dispersion of premia. In the symmetric network, the highest risk premium is 37% larger than the lowest premium, whereas in the asymmetric network economy it is 87% larger. In the data, this figure amounts to 106%. Indeed network asymmetry, brings our model one step closer to explaining the empirical cross-section variability of returns on sectors' equity. We conclude that the value premium - in a reduced form regression - can be interpreted as an exogeneity premium in a structural setting.

VII. The Term Structure of Equity Premia and the Dividend Strip Curve

A recent literature shows the importance to investigate the link between the ability to explain the cross-section of expected returns and the term structure of expected equity returns. This has been made possible by a market on dividend strips, securities which are claims to dividends paid until a future date, which allows to distinguish the properties of short-term equity claims with respect to market returns. Van Binsbergen, Brandt, and Koijen (2010) compare the empirical properties of expected returns in the dividend strips market with those implied by several leading asset pricing models: the Campbell and Cochrane (1999) external habit formation model; the Bansal and Yaron (2004) long-run risk model; the Barro-Rietz rare disasters framework (Barro (2009)) as explored by Gabaix (2009) and Wachter (2010). They show that most of these models find it difficult to reproduce the observed empirical properties of the dividend strip curve. The habit model and the published specification of the long-run risk model imply that the term structure of the dividend strip curve is upward sloping (i.e. the risk premium on long-term dividend claims are higher than for short-term claim). In the rare disasters model, expected returns are flat across maturities. These implications contrast with the empirical evidence of a downward sloping terms structure of expected returns. Lettau and Wachter (2007) propose a reduced-form specification that is designed to be consistent with the empirical evidence. Their model assume shocks to expected and unexpected dividend growth that are negatively correlated, thus implying that long-maturity dividend claims are on a per-period basis less risky than short-horizon claims. However, as also pointed out by Lettau and Wachter (2010), the model is not a full-fledged equilibrium model. In our structural model, we can take the next step of thinking the micro foundations that can give rise to these properties of the stochastic discount factor.

We investigate the link between the cross-section (value premium) and term structure of equity premia and analyze the role of the connectivity of the network and the implied cross-sectional learning effects. We find that even in the context of very simple time-additive preferences, which imply by construction a negative correlation between market price of risk and dividend shocks, the 'value premium' can be reconciled with a downward-sloping (at some or all time horizons) term structure of equity premia. Interestingly, the model implies a downward-sloping term structure for growth stocks and upward sloping for value stocks. This is due to the fact that the shape of the term structure is linked to the same exogeneity property that is responsible for the value premium.

To study this link, let us consider the term structure of unconditional risk premia of dividend strips in the context of the stylized 3-sectors economy of Section V, featuring asymmetric feed-back effects. The $T-$maturity dividend strip of the $i-$th sector/tree, as evaluated at time s, is the claim to the $i-$th sector's dividend stream paid from s to T. Its conditional price can be computed as:

$$P_{s,T}^{D^i} = \frac{1}{\xi_s}\mathbb{E}\left[\int_s^T \xi_s Y_s x_s^i ds \middle| \mathcal{F}_t^{x,Y}\right] \tag{45}$$

$$= \frac{Y_s}{(\sum_{i=1}^N x_s^i)^{-\gamma}}(p_s^h, 1-p_s^h, \overline{0}_{\mathcal{N}-2})(\mathbf{a}+\mathbf{A}^H)^{-1}\left(I_d - e^{-(\mathbf{a}+\mathbf{A}^H)(T-s)}\right)\mathbf{C}^i \tag{46}$$

where $\mathbf{A}^H$ is the instantaneous transition probability matrix for sectors' joint normalcy and distress states.[19] Conditional risk premia are reported in the Appendix, together with a simple procedure to compute unconditional premia. Proposition 6 mandates that, for a given maturity, p/d ratios of dividend strips (risk premia) are increasing (decreasing) in the measure of sector's exogeneity. The results are shown in Figure 13 (Panel 1: the term structure for the Banking sector; Panel 2: the Manufacturing sector) where we adopt the parameterization in Figure 10. Full-information premia are reported as dashed lines, while premia under incomplete-information are solid lines.

Insert Figure 13 about here

As a matter of example, we consider a setting in which, the Manufacturing sector is not 'actively' connected to the rest of the orchard. By construction, this makes Manufacturing an endogenous sector as, according to Proposition 6, it is mainly affected by shocks of other sectors. As seen in Section V, the equilibrium implications of these properties imply that Manufacturing would be classified as a value sector, i.e. with low p/d ratio. Banking, instead, is highly exogenous sector and would be classified as a growth sector. We use this structure to investigate two questions. First, what is the role played by partial information and cross-sectional learning in determining the slope/shape of the dividend swap curve? Second, what is the link between differences in sector connectivity (exogeneity) and the shape of the term structure of dividend swaps ?

In full-information, Banking's high dividend growth simply implies high cash-flow duration: this feature would lead to risk premia that are increasing in time to maturity, as the dashed lines in Panel 1 and Panel 2 show. With incomplete-information, however, the shape of the term structure is different and can be negative or hump-shaped: when a distress of some other sector takes place, and the 'bad' state is regarded more likely, the posterior update significantly decreases (increases) the perceived covariance between Banking dividend growth and aggregate consumption growth (market price of risk), because Banking's dividend flows are weakly correlated with the common factor. At some time horizon this reduction of cash-flow risk due to the learning mechanism can turn the slope of the term structure negative, consistent with the data (see Panel 1, solid line). In the short run, the unconditional probability of contemporaneous distress for sectors is low and, as a result, the Banking equity premium is initially increasing. To gain intuition, in Panel 2 of Figure 10 we report the exogeneity measure $ex^i_{s,T}$ for the three sectors as a function of the time-horizon $T - s$. Banking's exogeneity is increasing in the short-medium run and afterwards converges to a steady-state. When the sector exogeneity reaches a sufficiently high level, the long-

[19]This expression assumes, without loss of generality, that no sector is in distress at time s. The rest of the notation is reported in the Appendix.

run cash-flow risk of this sector is perceived smaller. This creates a link between the term structure of exogeneity and the term structure of equity premia.

To gain additional intuition on this link, in Panel 3 of Figure 13 we compare the term structure of dividend swap premia for the Banking sector for two different networks in which the Banking sector has different levels of exogeneity. For simplicity, we use the two network structures discussed in Section V: in CASE 2 (solid line) the Banking sector is more exogenous than in CASE 1 (dotted line). Panel 2 in Figure 10 and Panel 2 in Figure 11 clearly show that the exogeneity measure of the sector is higher in CASE 2 at all time-horizons. This is achieved via an asymmetric connectivity that makes Banking more 'active'. We find that as exogeneity increases (from CASE 1 to CASE 2) the hump occurs earlier and the slope of the dividend swap curve becomes progressively more negative. The lower the risk of distress of a sector relative to the rest of the cross-section, the shorter the time horizon at which the inversion takes place. Panel 4 reports the slopes of the term structures of Banking equity premia that we obtain when we let the long-term exogeneity of the sector vary, keeping the connectivity structure unaltered.[20] We define the slope of the term structure as the difference between the 30-year and the 6-month equity premium. Panel 4 shows that a lower slope corresponds to a higher exogeneity. This suggests that the connectivity structure of the orchard, coupled with cross-sectional learning, plays an important role in influencing the term structure of risk premia, even in the absence of more flexible non time-additive preferences.

Remark 4 *It has been noticed that a way to make the long-run risk model of Bansal and Yaron (2004) consistent with the observed term structure of equity returns is to assume that dividend shocks are negatively correlated with expected dividend and consumption growth, i.e.* $Corr(E[dD], dD) < 0$ *and* $Corr(E[dC], dD) < 0$*, as opposed to a zero correlation as in the published paper. It is interesting to notice that, in our context with cross-sectional learning, this assumption is satisfied for exogenous firms. i.e. growth stocks.*

The argument of the previous result is reinforced when we look at a different sector, i.e. Manufacturing. The connectivity properties of the previous economy makes the term structure of the risk-premium for the Manufacturing sector monotonically increasing and learning does not modify this shape. The reason is simple: the Manufacturing sector is 'passively' connected (endogenous) to other sectors dividend shocks so that the agent knows

[20]As in CASE 2 of Section V, compared to CASE 1, we obtain increasing exogeneity for the Banking sector by widening the difference between a_l and a_h, and between b_l and b_h. This additional cyclicality in the propagation of Banking shocks makes Banking increasingly 'active'.

that this sector will be subject to negative dividend shocks exactly at times of low aggregate consumption (as Panel 2 of Figure 10 reports, its term structure of exogeneity is decreasing). This makes Manufacturing a high cash-flow risk sector at all time horizons. Learning does not modify the monotonically increasing term structure since – as Panel 2 shows – the high correlation with the common factor implies that perceived (posterior) cash-flow risk is not significantly smaller than cash-flow risk under full-information.

To summarize, we find three results. First, the 'value premium' can be reconciled with the existence of a negatively sloped term structure for growth stocks and increasing for value stocks. An empirical test of this prediction is an interesting venue for future research.[21] Second, partial information and cross-sectional learning play an important role in determining the shape of the term structure of dividend swaps. Third, the network structure and the extent to which a sector connectivity is 'active' determines the shape of the term structure of dividend swaps.

VIII. Conclusions

We study an equilibrium asset pricing model with several Lucas (1978) trees subject to distress events. Our goal is to reconcile the well-established ability of rare disasters to explain the equity premium with the properties of the cross-section of expected returns. The success of the model depends on a notion of distress that emphasizes its role in the formation of expectations about fundamentals. Distress events spread over the rest of the cross-section *i*) because they are persistent, *ii*) because agents learn from them the future probabilities of distress of other sectors, and *iii*) because sectors are connected by technological relations which imply a direct channel of propagation for cash-flow shocks. The first feature allows the model to help explain empirical equity premia and interest rate dynamics even in a context with time-separable preferences and small levels of risk aversion. The second and third features give rise to a wider cross-sectional dispersion of excess returns, in a way that is consistent with the 'value premium'. We find that sectors whose dividends are more 'exogenous' as a result of the orchard connectivity structure – that is, their dividends are scarcely influenced by other sectors' dividends – have a high price-to-fundamental ratio (they are 'growth' sectors) and gain lower excess returns on average. A key driving force of this result is cross-sectional learning, which implies lower perceived correlation between dividends and aggregate consumption (i.e. cash-flow risk) for growth sectors. We can thus

[21]We are not aware of any empirical evidence, at the moment, on this testable implication of the model. To this purpose, one may apply the methodology of Van Binsbergen, Brandt, and Koijen (2010) to individual stock options.

link reduced-form assumptions of cash-flow risk heterogeneity to the structural properties of the orchard. Finally, we investigate the link between the 'value premium' and the the term structure of equity premia of the stocks in the cross-section. We find that the 'value premium' can be reconciled with a downward-sloping (at some or all time horizons) term structure of equity premia for more exogenous stocks (which also have high p/d ratios) and upward sloping for less exogenous stocks (which also have low p/d ratios).

There are several directions along which our model can be generalized, both at the expense of tractability and at the benefit of empirical performance. We refer, in particular, to a more flexible modelization of distress events and recoveries, such as considering distress events of different amplitudes and more states for the latent common factor - in order to allow for a more pervasive time variation of dividend correlations - and to a more general choice of preferences for the representative agent. We believe that the most parsimonious framework has allowed us to better convey our message. At an empirical level, it would be interesting to estimate a structural version of the model and test what type of structural cash-flow connectivity allows to explain both the dynamics of expected returns and the properties of the term structure of dividend swap curves conditional on the characteristics of different sectors. We reserve these questions to future research.

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Appendix A: Proofs

This Appendix contains the proofs of Lemma 1 and 2 and Proposition 1 to 7. Auxiliary lemmas that are used throughout the proofs, and two lemmas that compute some expression quoted in the main text, are reported at the end of this Appendix as 'Auxiliary Results' and labelled as 'Lemma A.#', # being the number of the Lemma.

Proof of Lemma 1

Let $H_t^i = \mathbf{1}(x_t^i = 0)$. It is convenient to express the dynamics of the continuous-time Markov chain x_t^i in the following manner:

$$x_t^i = \overline{x}^i - \int_0^t (\overline{x}^i - \underline{x}^i) dH_s^i$$

where the $\mathcal{F}_t-$intensity of the compound Poisson process dH_t^i is $-H_t^i\eta_t^i + (1-H_t^i)\lambda_t^i$. Observation of distress or normalcy indicators H_t is then equivalent to observing dividends D_t, because the continuous unitary dividend Y_t conveys no information about hidden states. Let $\psi_t = [H_t^1, H_t^2, \ldots, H_t^N]'$ denote the vector of observation processes. Let also $\mathcal{F}_t^{x,Y}$ denote the sigma field that the observation process generates. It follows from Theorem 18.3 in Lipster and Shyriaev (2001) that H_t^i is an $\mathcal{F}_t^{x,Y}-$point process with compensator

$$\widehat{\lambda}_t^{H^i} = -H_t^i[\overline{\eta}^i p_t^h + \underline{\eta}^i(1-p_t^h)] + (1-H_t^i)[\overline{\lambda}^i p_t^h + \underline{\lambda}^i(1-p_t^h)] \tag{A.1}$$

By Lemma 9.2 in Lipster and Shyriaev(2001) we have that, for $j = h, l$, the random process

$$y_t^j = \mathbf{1}(\lambda_t = \overline{\lambda}^j) - \mathbf{1}(\lambda_0 = \overline{\lambda}^j) - \int_0^t [-\mathbf{1}(\lambda_s = \overline{\lambda}^j)k_j(\mathbf{x}_s) + \mathbf{1}(\lambda_s = \overline{\lambda}^{j^c})k_{j^c}(\mathbf{x}_s)]ds$$

is an $\mathcal{F}_t-$martingale, where j^c denotes the complement of j. Taking conditional expectations with respect to $\mathcal{F}_t^{x,Y}$ in the definition of y_t^j, we obtain:

$$p_t^h = p_0^h + \int_0^t [-p_s^h k_j(\mathbf{x}_s) + p_s^h k_{j^c}(\mathbf{x}_s)]ds + \mathbb{E}[y_t^j|\mathcal{F}_t^{x,Y}] \tag{A.2}$$

We can now apply the martingale representation theorem in Lemma A.1 above to the martingale $\mathbb{E}[y_t^j|\mathcal{F}_t^{x,Y}]$ and identify stochastic integrands as in Lipster and Shyriaev (2001), Theorem 19.5. We end up with the representation given in the Proposition. This ends the proof.

Proof of Proposition 1

We assume without loss of generality that none of the N trees has yet (at time t) undergone a distress. The price of the claim to the $i-$th endowment process is:

$$\begin{aligned} P_t^i &= \frac{1}{\xi_t}\mathbb{E}\left[\int_t^\infty \xi_s Y_s x_s^i ds \middle| \mathcal{F}_t^{x,Y}\right] \\ &= \frac{Y_t}{(\sum_{i=1}^N x_t^i)^{-\gamma}}\mathbb{E}\left[\mathbb{E}\left[\int_t^\infty e^{-a(s-t)} x_s^i \left(\sum_{j=1}^N x_s^i\right)^{-\gamma} ds \middle| \mathcal{F}_t\right] \middle| \mathcal{F}_t^{x,Y}\right] \end{aligned} \tag{A.3}$$

where

$$a = \delta - \mu_Y(1-\gamma) + \frac{\sigma_Y^2}{2}(1-\gamma)\gamma.$$

Equation (A.3) follows from the independence of Y_t and x_t. Assume the economy is in a high state, so that $\lambda_t = \overline{\lambda}$ and $\eta_t = \overline{\eta}$. Let $V_h^i(H_t)$ denote the inner (full information) conditional expectation in (A.3). The inner expectation in (A.3) is computed similarly after the obvious modifications. By the law of iterated expectations

$$\int_0^t e^{-as} x_s^i \left(\sum_{j=1}^N x_s^i\right)^{-\gamma} ds + e^{-at} V_h^i(H_t) \tag{A.4}$$

is an $\mathcal{F}_t-$martingale, therefore the 'drift' component of its Ito representation must vanish. We use the same notation of the proof of Proposition 2 to identify the collection of trees that are in distress state and those that are not. We apply Ito's lemma to (A.4), take conditional expectations and impose the martingale property. Applying this argument also to the full information price of the market portfolio conditional on the 'low' state of the economy, we obtain the following system of equations:

$$
\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left(\begin{bmatrix} -a - \sum_{j=1}^{N} \overline{\lambda}^j & 0 \\ 0 & -a - \sum_{j=1}^{N} \underline{\lambda}^j \end{bmatrix} + I \right) \begin{bmatrix} V_h^i(H_t) \\ V_l^i(H_t) \end{bmatrix} +
$$

$$
\begin{bmatrix} \sum_{j=1}^{\#(\mathcal{S}(N,N-1))} \overline{\lambda}^j V_h^i(\mathcal{S}(N,N-1)_j) \\ \sum_{j=1}^{\#(\mathcal{S}(N,N-1))} \underline{\lambda}^j V_l^i(\mathcal{S}(N,N-1)_j) \end{bmatrix} + \begin{bmatrix} \overline{x}^i \left(\sum_{j=1}^{N} \overline{x}^j \right)^{-\gamma} \\ \overline{x}^i \left(\sum_{j=1}^{N} \overline{x}^j \right)^{-\gamma} \end{bmatrix} \tag{A.5}
$$

Using the notation of the proof of Proposition 2, this system of equations can be written compactly as follows:

$$
\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = -(\mathbf{a} + \mathbf{A}^H) \begin{bmatrix} \mathbf{V}^i(H_t) \\ \mathbf{V}^i(\mathcal{S}(N,N-1)_1) \\ \vdots \\ \mathbf{V}^i(\mathcal{S}(N,N-1)_N) \\ \mathbf{V}^i(\mathcal{S}(N,N-2)_1) \\ \vdots \\ \mathbf{V}^i(\mathcal{S}(N,1)) \\ \mathbf{V}^i(\mathcal{S}) \end{bmatrix} + \mathbf{C}^i \tag{A.6}
$$

where $\mathbf{V}^i(\,\cdot\,) = [V_h^i(\,\cdot\,), V_l^i(\,\cdot\,)]'$, $\mathbf{V}^i(\mathcal{S})$ denotes the function $\mathbf{V}^i$ conditional on all trees being in distress state, and

$$
\mathbf{C}^i = \begin{bmatrix} \overline{x}^i \left(\sum_{j=1}^{N} \overline{x}^j \right)^{-\gamma} \overline{1}_2 \\ \overline{x}^i \left(\underline{x}^1 + \sum_{j=2}^{N} \overline{x}^j \right)^{-\gamma} \overline{1}_2 \\ \vdots \\ \underline{x}^i \left(\underline{x}^i + \sum_{j \neq i} \overline{x}^j \right)^{-\gamma} \overline{1}_2 \\ \vdots \\ \underline{x}^i \left(\sum_{j=1}^{N} \overline{x}^j \right)^{-\gamma} \overline{1}_2 \end{bmatrix}. \tag{A.7}
$$

where $\overline{1}_2$ is a 2-dimensional column vector of ones. Finally:

$$
P_t^i \quad = \quad Y_t \left(\sum_{i=1}^{N} x_t^i \right)^{\gamma} (p_t^h, 1 - p_t^h, 0_{\mathcal{N}-2}) \cdot (\mathbf{a} + \mathbf{A}^H)^{-1} \mathbf{C}^i \tag{A.8}
$$

where $\mathcal{N} = 2^N$ and $\mathbf{a}$ is a $\mathcal{N}$-dimensional diagonal matrix with a on the main diagonal. Now redefine the vector $\mathbf{C}^i$ as $\widetilde{\mathbf{C}}^i = \mathbf{C}^i / (x_t^i (\sum_{i=1}^{N} x_t^i)^{-\gamma})$ and let x_t correspond to the $j-$th of the $\mathcal{N}$ possible combinations. Then the formula reported in (23) of the Proposition holds with the full-information conditional price-dividend ratios, $F_h^i(x_t)$ and $F_l^i(x_t)$ given by:

$$
F_h^i(x_t) \quad = \quad [\overline{0}_{2j-1}, 1, \overline{0}_{2^{N+1}-2j}] (\mathbf{a} + \mathbf{A}^H)^{-1} \widetilde{\mathbf{C}}^i \tag{A.9}
$$

$$
F_l^i(x_t) \quad = \quad [\overline{0}_{2j-2}, 1, \overline{0}_{2^{N+1}-(2j-1)}] (\mathbf{a} + \mathbf{A}^H)^{-1} \widetilde{\mathbf{C}}^i \tag{A.10}
$$

where $\overline{0}_i$ denotes a $i-$dimensional row vector of zeros.

Let $\overline{C}^i(x_t) = (\overline{C}_h^i(x_t), \overline{C}_l^i(x_t))$, where $\overline{C}_h^i(x_t)$ and $\overline{C}_l^i(x_t)$ are the expected discounted cash-flows conditional on the current multiplier x_t and a 'high' and, respectively, 'low' state of the world. These are the two contiguous entries of $(\mathbf{a} + \mathbf{A}^H)^{-1}\mathbf{C}^i$ corresponding to the specific combination of trees in distress and normalcy state contained in x_t.

We rewrite the price of the claim to the $i-$th endowment as follows.

$$
P_t^i \quad = \quad \frac{1}{\xi_t} \mathbb{E} \left[\int_t^{\infty} \xi_s Y_s x_s^i ds \,\middle|\, \mathcal{F}_t^{x,Y} \right] \tag{A.11}
$$

$$
\quad = \quad \frac{Y_t}{(\sum_{i=1}^{N} x_t^i)^{-\gamma}} \int_t^{\infty} e^{-a(s-t)} \mathbb{E} \left[x_s^i \left(\sum_{j=1}^{N} x_s^i \right)^{-\gamma} \,\middle|\, \mathcal{F}_t^{x,Y} \right] ds \tag{A.12}
$$

$$
\quad = \quad \frac{Y_t}{(\sum_{i=1}^{N} x_t^i)^{-\gamma}} \int_t^{\infty} e^{-a(s-t)} (p_t^h, 1 - p_t^h, \overline{0}_{\mathcal{N}-2}) \mathbf{B}_{s-t} \mathbf{C}^i ds \tag{A.13}
$$

$\mathbf{B}_{s-t}$ is the $(\mathcal{N}+1)\times(\mathcal{N}+1)$ full-information conditional joint transition probability matrix of the vector of supplying trees multipliers x and of the state of the economy from time t to time s, in other words

$$\mathbf{B}_{s-t}=\begin{pmatrix} \mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^1,\overline{\lambda})\Big|(x_t,\lambda_t)=(\widetilde{x}^1,\overline{\lambda})\right] & \mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^1,\underline{\lambda})\big|(x_t,\lambda_t)=(\widetilde{x}^1,\overline{\lambda})\right] & \dots & \dots & \mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{\mathcal{N}},\underline{\lambda})\big|(x_t,\lambda_t)=(\widetilde{x}^1,\overline{\lambda})\right] \\ \mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^1,\overline{\lambda})\Big|(x_t,\lambda_t)=(\widetilde{x}^1,\underline{\lambda})\right] & \mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^1,\underline{\lambda})\big|(x_t,\lambda_t)=(\widetilde{x}^1,\underline{\lambda})\right] & \dots & \dots & \mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{\mathcal{N}},\underline{\lambda})\big|(x_t,\lambda_t)=(\widetilde{x}^1,\underline{\lambda})\right] \\ \vdots & \vdots & \dots\ \vdots & \dots & \vdots \\ \mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^1,\overline{\lambda})\Big|(x_t,\lambda_t)=(\widetilde{x}^{\mathcal{N}},\underline{\lambda})\right] & \mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^1,\underline{\lambda})\big|(x_t,\lambda_t)=(\widetilde{x}^{\mathcal{N}},\underline{\lambda})\right] & \dots & \dots & \mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{\mathcal{N}},\underline{\lambda})\big|(x_t,\lambda_t)=(\widetilde{x}^{\mathcal{N}},\underline{\lambda})\right] \end{pmatrix} \tag{A.14}$$

where $\widetilde{x}^i$, $i=1,2,\dots,\mathcal{N}$ is a combination of distress or normalcy state for all the trees. The first (second) row reports transition probabilities conditional on no distress for any tree and a 'high' ('low') state of the economy. It is immediate to see, according to the proof of Proposition 2, that

$$(\mathbf{a}+\mathbf{A}^H)^{-1}=\int_t^\infty e^{-a(s-t)}\mathbf{B}_{s-t}ds \tag{A.15}$$

So the matrix $(\mathbf{a}+\mathbf{A}^H)^{-1}$ appearing in the price formula for V^i is just the Laplace transform evaluated at a of the transition probability matrix. This positive transformation preserves relative magnitudes between transition probabilities.

Consider any state $x_t=\widetilde{x}^k$ for the trees' multipliers. We have

$$\begin{aligned}\overline{P}^i(\widetilde{x}^k)-\underline{P}^i(\widetilde{x}^k)=(\sum\widetilde{x}^k)^\gamma\int_t^\infty e^{-a(s-t)}&\left[\sum_{\widetilde{x}^j}\left(\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^j,\overline{\lambda})\Big|(x_t,\lambda_t)=(\widetilde{x}^k,\overline{\lambda})\right]\right.\right.\\ &+\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^j,\underline{\lambda})\big|(x_t,\lambda_t)=(\widetilde{x}^k,\overline{\lambda})\right]-\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^j,\underline{\lambda})\big|(x_t,\lambda_t)=(\widetilde{x}^k,\underline{\lambda})\right]\\ &\left.\left.-\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^j,\overline{\lambda})\Big|(x_t,\lambda_t)=(\widetilde{x}^k,\underline{\lambda})\right]\right)x^i(\sum\widetilde{x}^j)^{-\gamma}\right]ds\end{aligned} \tag{A.16}$$

If the combination of normalcy and distress for the tree $\widetilde{x}^j$ comprises more trees in distress than $\widetilde{x}^k$, then the difference in square bracket is negative, because the overall propensity to observe distress events (recoveries) is higher (lower) conditional on the 'low' state of the economy. When $\gamma>1$, the cash-flow of the security discounted by the marginal utility, i.e. $x^i(\sum\widetilde{x}^j)^{-\gamma}$, is greater when more trees are in distress. Hence the difference $\overline{P}^i(\widetilde{x}^k)-\underline{P}^i(\widetilde{x}^k)$ is negative.

This ends the proof of the Proposition.

Proof of Proposition 2

In this proof we use the notation of the proof of Proposition 1, and we denote for convenience $\overline{\lambda}$ by $\overline{\lambda}^h$ and $\underline{\lambda}$ by $\overline{\lambda}^l$. Consider any state $x_t=\widetilde{x}^k$ for the trees' multipliers. Assume that one of the trees in normalcy state, say tree j experiences a distress and the dividend multipliers jump to a state $x_t=\widetilde{x}^{k-j}$. Let $\mathcal{A}^{k-j}$ denote the set of all states where the collection of trees in normalcy is a subset of the collection of trees in normalcy for state $\widetilde{x}^{k-j}$. It is easy to see that for each state $\widetilde{x}^l\in\mathcal{A}^{k-j}$ there is a state $\widetilde{x}^{l+j}\in\overline{\mathcal{A}}^{k-j}$, the complement of $\mathcal{A}^{k-j}$, that is identical to $\widetilde{x}^l$ with the exception of tree j being in normalcy rather than distress state. It is also easy to see that, for any such pair of states $(\widetilde{x}^l,\widetilde{x}^{l+j})$, we have

$$\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{l+j},\overline{\lambda}_w)\Big|(x_t,\lambda_t)=(\widetilde{x}^k,\overline{\lambda}_w)\right]\geq\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{l+j},\overline{\lambda}_w)\Big|(x_t,\lambda_t)=(\widetilde{x}^{k-j},\overline{\lambda}_w)\right] \tag{A.17}$$

$$\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{l},\overline{\lambda}_w)\Big|(x_t,\lambda_t)=(\widetilde{x}^k,\overline{\lambda}_w)\right]\leq\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{l},\overline{\lambda}_w)\Big|(x_t,\lambda_t)=(\widetilde{x}^{k-j},\overline{\lambda}_w)\right]\quad w=h,l \tag{A.18}$$

and the the following also holds:

$$\begin{aligned}&\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{l},\overline{\lambda}_w)\Big|(x_t,\lambda_t)=(\widetilde{x}^k,\overline{\lambda}_w)\right]-\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{l},\overline{\lambda}_w)\Big|(x_t,\lambda_t)=(\widetilde{x}^{k-j},\overline{\lambda}_w)\right]\\ &=-\left(\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{l+j},\overline{\lambda}_w)\Big|(x_t,\lambda_t)=(\widetilde{x}^k,\overline{\lambda}_w)\right]-\mathbb{P}\left[(x_s,\lambda_s)=(\widetilde{x}^{l+j},\overline{\lambda}_w)\Big|(x_t,\lambda_t)=(\widetilde{x}^{k-j},\overline{\lambda}_w)\right]\right)\quad w=h,l\end{aligned} \tag{A.19}$$

Let $P^i(\widetilde{x}^k)$ and $P^i(\widetilde{x}^{k-j})$ denote the pre- and post-distress prices of the security. Then, by means of (A.13):

$$
\begin{aligned}
&P^i(\widetilde{x}^k) - P^i(\widetilde{x}^{k-j}) = \\
&Y_t \int_t^{\infty} e^{-a(s-t)} \Bigg\{ p_t^h \Bigg[\sum_{\widetilde{x}^{l+j} \in \overline{\mathcal{A}}^{k-j}} \Big(\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^k)^{\gamma} \\
&-\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \Big) x_s^i (\textstyle\sum \widetilde{x}^{l+j})^{-\gamma} + \\
&\sum_{\widetilde{x}^l \in \mathcal{A}^{k-j}} \Big(\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k},\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^k)^{\gamma} - \mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \Big) \times \\
&\times x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} \Bigg] + (1-p_t^h) \Bigg[\sum_{\widetilde{x}^{l+j} \in \overline{\mathcal{A}}^{k-j}} \Big(\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\underline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^k,\underline{\lambda}) \right] (\textstyle\sum \widetilde{x}^k)^{\gamma} \\
&-\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\underline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\underline{\lambda}) \right] (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \Big) x_s^i (\textstyle\sum \widetilde{x}^{l+j})^{-\gamma} + \\
&\sum_{\widetilde{x}^l \in \mathcal{A}^{k-j}} \Big(\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\underline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k},\underline{\lambda}) \right] (\textstyle\sum \widetilde{x}^k)^{\gamma} - \mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\underline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\underline{\lambda}) \right] (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \Big) \times \\
&\times x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} \Bigg] \Bigg\}
\end{aligned} \tag{A.20}
$$

Now consider any paired state $(\widetilde{x}^l, \widetilde{x}^{l+j})$. Taking into account (A.17),(A.18), and (A.19), given for instance a 'high' economic state, the following holds:

$$\Big[\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^k)^{\gamma} \tag{A.21}$$

$$-\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \Big] x_s^i (\textstyle\sum \widetilde{x}^{l+j})^{-\gamma} \tag{A.22}$$

$$+\Big[\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^k)^{\gamma} \tag{A.23}$$

$$-\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \Big] x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} \qquad = \tag{A.24}$$

$$\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^k)^{\gamma} \left[x_s^i (\textstyle\sum \widetilde{x}^{l+j})^{-\gamma} - x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} \right] \tag{A.25}$$

$$+\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] \left[x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} (\textstyle\sum \widetilde{x}^{k})^{\gamma} - x_s^i (\textstyle\sum \widetilde{x}^{l+j})^{-\gamma} (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \right] \tag{A.26}$$

$$+\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} \left[(\textstyle\sum \widetilde{x}^{k})^{\gamma} - (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \right] \qquad \geq \tag{A.27}$$

$$\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] (\textstyle\sum \widetilde{x}^k)^{\gamma} \left[x_s^i (\textstyle\sum \widetilde{x}^{l+j})^{-\gamma} - x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} \right] \tag{A.28}$$

$$+\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] \left[x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} (\textstyle\sum \widetilde{x}^{k})^{\gamma} - x_s^i (\textstyle\sum \widetilde{x}^{l+j})^{-\gamma} (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \right] \tag{A.29}$$

$$+\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} \left[(\textstyle\sum \widetilde{x}^{k})^{\gamma} - (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \right] \qquad \geq \tag{A.30}$$

$$\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+j},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda}) \right] x_s^i (\textstyle\sum \widetilde{x}^{l+j})^{-\gamma} \left[(\textstyle\sum \widetilde{x}^{k})^{\gamma} - (\textstyle\sum \widetilde{x}^{k-k})^{\gamma} \right] \tag{A.31}$$

$$+\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda}) \Big| (x_t,\lambda_t) = (\widetilde{x}^{k-j},\overline{\lambda}) \right] x_s^i (\textstyle\sum \widetilde{x}^{l})^{-\gamma} \left[(\textstyle\sum \widetilde{x}^{k})^{\gamma} - (\textstyle\sum \widetilde{x}^{k-j})^{\gamma} \right] \qquad \geq 0 \tag{A.32}$$

where we have used the fact that

$$(\textstyle\sum \widetilde{x}^{k})^{\gamma} - (\textstyle\sum \widetilde{x}^{k-k})^{\gamma} \geq 0 \tag{A.33}$$

The same conclusion holds for a 'low' state of the economy. Since the integrand function in (A.20) is nonnegative, we conclude that $P^i(\widetilde{x}^k) - P^i(\widetilde{x}^{k-j}) \geq 0$.

Note that

$$
\begin{aligned}
R^i(x_t - j) &= \left(\frac{\underline{x}^j + \sum_{z \neq j} x_t^z}{\overline{x}^j + \sum_{z \neq j} x_t^z} \right)^{\gamma} \frac{\left(p_t^h \frac{\overline{\lambda}^j}{\widehat{\lambda}_t^j}, (1-p_t^h) \frac{\underline{\lambda}^j}{\widehat{\lambda}_t^j} \right) \cdot \overline{C}^i(x_t - j)}{(p_t^h, 1-p_t^h) \cdot \overline{C}^i(x_t)} - 1 = \frac{\left(p_t^h \frac{\overline{\lambda}^j}{\widehat{\lambda}_t^j}, (1-p_t^h) \frac{\underline{\lambda}^j}{\widehat{\lambda}_t^j} \right) \cdot \tilde{P}^i(Y_t, x_t - j)}{(p_t^h, 1-p_t^h) \cdot \tilde{P}^i(Y_t, x_t)} - 1 \\
R^i(x_t + j) &= \left(\frac{\overline{x}^j + \sum_{z \neq j} x_t^z}{\underline{x}^j + \sum_{z \neq j} x_t^z} \right)^{\gamma} \frac{\left(p_t^h \frac{\overline{\eta}^j}{\widehat{\eta}_t^j}, (1-p_t^h) \frac{\underline{\eta}^j}{\widehat{\eta}_t^j} \right) \cdot \overline{C}^i(x_t + j)}{(p_t^h, 1-p_t^h) \cdot \overline{C}^i(x_t)} - 1 = \frac{\left(p_t^h \frac{\overline{\eta}^j}{\widehat{\eta}_t^j}, (1-p_t^h) \frac{\underline{\eta}^j}{\widehat{\eta}_t^j} \right) \cdot \tilde{P}^i(Y_t, x_t + j)}{(p_t^h, 1-p_t^h) \cdot \tilde{P}^i(Y_t, x_t)} - 1
\end{aligned}
$$

are the returns on security i following, respectively, a distress or a recovery for tree j, where $\tilde{P}^i(Y_t, x_t) = [\overline{P}^i(Y_t, x_t), \underline{P}^i(Y_t, x_t)]'$. Conditions (27) and (26) follow after simple manipulations.

This ends the proof of the Proposition.

Proof of Proposition 3

The risk premium of the security reads:

$$\mu_t^i = \mathbb{E}\left[\frac{dP_t^i}{P_t^i} \middle| \mathcal{F}_t^{x,Y} \right] + \frac{D_t^i}{P_t^i} - r_t \tag{A.34}$$

Applying Ito's lemma to the formula for the price process, taking expectations and taking into account the expression for the equilibrium interest rate, we end up with the following expression:

$$\begin{aligned}\mu_t^i = \gamma\sigma_Y^2 - \sum_{j=1}^N \Bigg\{ &(1-H_t^j)(\theta_t^j - 1)\left[\left(\frac{\underline{x}^j + \sum_{z\neq j} x_t^z}{\overline{x}^j + \sum_{z\neq j} x_t^z} \right)^\gamma \left(\frac{\left(p_t^h \frac{\overline{\lambda}^j}{\widehat{\lambda}_t^j}, (1-p_t^h)\frac{\underline{\lambda}^j}{\widehat{\lambda}_t^j} \right) \cdot \overline{C}^i(x_t - j)}{(p_t^h, 1-p_t^h)\cdot \overline{C}^i(x_t)} \right) - 1 \right] \widehat{\lambda}_t^j \\ &+ H_t^j(\theta_t^j - 1)\left[\left(\frac{\overline{x}^j + \sum_{z\neq j} x_t^z}{\underline{x}^j + \sum_{z\neq j} x_t^z} \right)^\gamma \left(\frac{\left(p_t^h \frac{\overline{\eta}^j}{\widehat{\eta}_t^j}, (1-p_t^h)\frac{\underline{\eta}^j}{\widehat{\eta}_t^j} \right) \cdot \overline{C}^i(x_t + j)}{(p_t^h, 1-p_t^h)\cdot \overline{C}^i(x_t)} \right) - 1 \right] \widehat{\eta}_t^j \Bigg\}\end{aligned} \tag{A.35}$$

We remind that $\overline{C}^i(x_t) = (\overline{C}_h^i(x_t), \overline{C}_l^i(x_t))$, where $\overline{C}_h^i(x_t)$ and $\overline{C}_l^i(x_t)$ are the expected discounted cash-flows conditional on the current multiplier x_t and a 'high' and, respectively, 'low' state of the world. These are the two contiguous entries of $(\mathbf{a} + \mathbf{A}^H)^{-1}\mathbf{C}^i$ corresponding to the specific combination of trees in distress and normalcy state contained in x_t. Expression (A.35) coincides with the well known representation:

$$\mu_t^i = -\mathbb{E}_t\left[\frac{d\xi_t}{\xi_t}\frac{dP_t^i}{P_t^i} \middle| \mathcal{F}_t^{x,Y} \right]$$

The expression reported in the text follows after straightforward manipulations.

This concludes the proof.

Proof of Proposition 4

i) Since the risk neutral intensity $\widehat{\lambda}_t^j\theta_t^j$ is always positive, the representation (A.35) for the risk premium implies that if the return following a distress (recovery) of tree j is negative (positive), then the contribution to the premium of this event is positive. According to Proposition 2 this is the case when conditions (27) and (26) are satisfied. Proposition 2 also states that full-information conditional returns are always negative (positive) after a distress (recovery), therefore the full-information disaster (recovery) premium is always positive. If we neglect the variation of the price-dividend ratio, then the contribution of the $j-$th event to the premium is

$$-\left(\mathbb{E}^*[\lambda_t^j|\mathcal{F}_t^{x,Y}] - \mathbb{E}[\lambda_t^j|\mathcal{F}_t^{x,Y}] \right)\theta_t^j \quad \text{and} \quad \left(\mathbb{E}^*[\eta_t^j|\mathcal{F}_t^{x,Y}] - \mathbb{E}[\eta_t^j|\mathcal{F}_t^{x,Y}] \right)\theta_t^j, \tag{A.36}$$

for distress and recovery events, respectively. According to *i*) of Proposition 1 we have $\underline{P}^i(x_t) < \overline{P}^i(x_t)$. This implies (see the discussion about expression (30) for the risk premium) that

$$\mathbb{P}^*\left[\lambda_t^j = \underline{\lambda}^j \middle| \mathcal{F}_t^{x,Y} \right] > \mathbb{P}\left[\lambda_t^j = \underline{\lambda}^j \middle| \mathcal{F}_t^{x,Y} \right] \quad \text{and} \quad \mathbb{P}^*\left[\lambda_t^j = \overline{\lambda}^j \middle| \mathcal{F}_t^{x,Y} \right] < \mathbb{P}\left[\lambda_t^j = \overline{\lambda}^j \middle| \mathcal{F}_t^{x,Y} \right]$$

so that, when compared to the true posterior distribution, the value adjusted distribution puts a higher probability mass on the greatest intensity of distress (in the 'low' state of the economy), and a lower probability mass on the greatest intensity of recovery (in the 'high' state of the economy). It follows that both contributions in (A.36) are negative.

ii) Consider the component of the risk premium that compensates for distress risk, i.e. μ_λ^i in expression (30). For each term in the summation, consider the representation given in (A.35). For distress events, i.e. $H_{t-}^j = 0$, we have

$$\lim_{\gamma\to\infty} \theta_t^j = \lim_{\gamma\to\infty} \left(\frac{\underline{x}^j + \sum_{u\neq j} x_t^u}{\overline{x}^j + \sum_{u\neq j} x_t^u} \right)^{-\gamma} = \infty \tag{A.37}$$

We now show that for a given state of the economy, either 'high' or 'low', the post-distress gross return on the security tends to zero as the risk aversion increases to infinity, i.e.

$$\lim_{\gamma\to\infty}\frac{\overline{P}^i(x_t-j)}{\overline{P}^i(x_t)}=0 \qquad and \qquad \lim_{\gamma\to\infty}\frac{\underline{P}^i(x_t-j)}{\underline{P}^i(x_t)}=0. \tag{A.38}$$

According to the Lemma A.3 *ii*) we have $\frac{\partial P^i}{\partial\gamma}>0$ for $\gamma>\gamma^*$, without any sign change, therefore $\lim_{\gamma\to\infty}\overline{P}^i(x_t)=\infty$ for any x_t (similarly for the price conditional on the 'low' state). Applying L'Hopital rule we obtain

$$\lim_{\gamma\to\infty}\frac{\overline{P}^i(x_t-j)}{\overline{P}^i(x_t)}=\lim_{\gamma\to\infty}\frac{\frac{\partial\overline{P}^i(x_t-j)}{\partial\gamma}}{\frac{\partial\overline{P}^i(x_t)}{\partial\gamma}}=$$

$$\lim_{\gamma\to\infty}\frac{\overline{1}_{x_t-j}(\mathbf{a}+\mathbf{A}^H)^{-1}\begin{pmatrix}\cdots\\ x^i(\sum(x_t-j))^{\gamma}(\sum\widetilde{x}^k)^{-\gamma}(-\mu_Y-\frac{\sigma_Y^2}{2}(1-2\gamma)+\log(\sum(x_t-j))-\log(\sum\widetilde{x}^k))\\ \cdots\end{pmatrix}}{\overline{1}_{x_t}(\mathbf{a}+\mathbf{A}^H)^{-1}\begin{pmatrix}\cdots\\ x^i(\sum x_t)^{\gamma}(\sum\widetilde{x}^k)^{-\gamma}(-\mu_Y-\frac{\sigma_Y^2}{2}(1-2\gamma)+\log(\sum x_t)-\log(\sum\widetilde{x}^k))\\ \cdots\end{pmatrix}}=0 \tag{A.39}$$

because

$$\overline{P}^i(x_t-j)=\overline{1}_{x_t-j}(\mathbf{a}+\mathbf{A}^H)^{-1}\begin{pmatrix}\cdots\\ x^i(\sum(x_t-j))^{\gamma}(\sum\widetilde{x}^k)^{-\gamma}\\ \cdots\end{pmatrix}<\overline{1}_{x_t}(\mathbf{a}+\mathbf{A}^H)^{-1}\begin{pmatrix}\cdots\\ x^i(\sum(x_t))^{\gamma}(\sum\widetilde{x}^k)^{-\gamma}\\ \cdots\end{pmatrix}=\overline{P}^i(x_t) \tag{A.40}$$

and

$$(-\mu_Y-\frac{\sigma_Y^2}{2}(1-2\gamma)+\log(\sum x_t)-\log(\sum\widetilde{x}^k))>(-\mu_Y-\frac{\sigma_Y^2}{2}(1-2\gamma)+\log(\sum(x_t-j))-\log(\sum\widetilde{x}^k))>0$$

for γ sufficiently large. Since the full information post-distress gross return converges to zero in any state of the economy, the ratio of any convex combination of post-distress conditional prices over any convex combination of pre-distress conditional prices will also converge to zero. In light of (A.37) and (A.35), and since an identical reasoning holds for the price conditional on the 'low' state of the economy, this implies that $\mu_\lambda^i\to\infty$ as $\gamma\to\infty$. We also have $\mu_\eta^i\to\infty$ as $\gamma\to\infty$. In the event of recovery, we have

$$\lim_{\gamma\to\infty}\theta_t^j=\lim_{\gamma\to\infty}\left(\frac{\overline{x}^j+\sum_{u\neq j}x_t^u}{\underline{x}^j+\sum_{u\neq j}x_t^u}\right)^{-\gamma}=0$$

Applying the methodology above, we can see that

$$\lim_{\gamma\to\infty}\frac{\overline{P}^i(x_t+j)}{\overline{P}^i(x_t)}=\infty \qquad and \qquad \lim_{\gamma\to\infty}\frac{\underline{P}^i(x_t+j)}{\underline{P}^i(x_t)}=\infty. \tag{A.41}$$

We conclude that $\lim_{\gamma\to\infty}\mu_\eta^i=\infty$ and that, overall, $\lim_{\gamma\to\infty}\mu_t^i=\infty$.

This concludes the proof.

Proof of Proposition 5

The full-information risk premium for the $i-$th endowment claim reads:

$$\overline{\mu}_t^i = -\mathbb{E}\left[\frac{d\xi_t}{\xi_t}\frac{d\overline{V}_z^i}{\overline{V}_z^i}\middle|\mathcal{F}_t\right] \tag{A.42}$$

$$= \sigma_Y^2-\sum_{u=1}^{N}\Big[(1-H_t^u)\theta_t^u\overline{R}_z^i(x_t-u)+H_t^z\theta_t^z\overline{R}_z^i(x_t+u)\Big] \qquad z=h,l \tag{A.43}$$

where, as in the proof of Proposition 2, $\overline{R}_z^i(x_t-u)$ ($\overline{R}_z^i(x_t+u)$) denotes the full-information return on the security i following a distress (recovery) of sector u, when the economy is in state z. We will now show that if the price-dividend ratio of the $j-$th endowment claim is currently higher than that of the $i-$th endowment claim, then

$$\overline{R}_z^j(x_t-u)\geq\overline{R}_z^i(x_t-u) \quad \text{and} \quad \overline{R}_z^j(x_t+u)\leq\overline{R}_z^i(x_t+u)$$

Since the market price of distress (recovery) risk θ_t^u is positive (negative), this will imply relation (38) of the Proposition.

Using the notation of the proof of Proposition 1, rewrite the price-dividend ratio of the $i-$th endowment claim as follows:

$$\frac{P_t^i}{D_t^i} = \int_t^\infty e^{-a(s-t)}[p_t^h, 1-p_t^h, \overline{0}_{\mathcal{N}-2}]\mathbf{B}_{s-t}\overline{\mathbf{C}}^i ds \tag{A.44}$$

where $\mathbf{B}_{s-t}$ - full-information conditional joint transition matrix on the horizon $[s,t]$ for the vector of supplying trees multipliers x and the state of the economy - is reported in the proof of Proposition 1, while $\overline{\mathbf{C}}^i$ is the vector of conditional discounted cash-flow variations in all states for security i, divided by the current value of the dividend multiplier for security i.

We show the inequality above for the event of distress of a given sector z. Considering as initial state the distress state ($\widetilde{x}^{k-z}$ in the notation below), this also implies, after the obvious modifications, the inequality corresponding to the recovery event.

Let $\widetilde{x}^k$ denote the $k-$th of the 2^N possible combinations of distress and normalcy for the dividend multipliers of the economy, and assume $x_t = \widetilde{x}^k$. Assume that a distress occurs for one of the trees in normalcy state, say tree z, and the vector of multipliers jumps to a state $x_t = \widetilde{x}^{k-z}$ from the state $x_{t-} = \widetilde{x}^k$. Let $\mathcal{A}^{k-z}$ denote the set of all states where the collection of trees in normalcy is a subset of the collection of trees in normalcy for state $\widetilde{x}^{k-z}$. It is easy to see that for each state $\widetilde{x}^l \in \mathcal{A}^{k-z}$ there is a state $\widetilde{x}^{l+z} \in \overline{\mathcal{A}}^{k-z}$, the complement of $\mathcal{A}^{k-z}$, that is identical to $\widetilde{x}^l$ with the exception of tree z being in normalcy rather than distress state.

Mimicking the Proof of Proposition 3, we consider any such pair of states $(\widetilde{x}^l, \widetilde{x}^{l+z})$. Given for instance a 'high' economic state, the following holds:

$$\begin{aligned}
&\Bigg[\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+z},\overline{\lambda})\Big|(x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda})\right]\frac{(\sum \widetilde{x}^k)^\gamma}{\epsilon_{t-}^i} \\
&-\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+z},\overline{\lambda})\Big|(x_t,\lambda_t) = (\widetilde{x}^{k-z},\overline{\lambda})\right]\frac{(\sum \widetilde{x}^{k-z})^\gamma}{\epsilon_{t-}^i}\Bigg] x_s^i(\sum \widetilde{x}^{l+z})^{-\gamma} \\
&+\Bigg[\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda})\Big|(x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda})\right]\frac{(\sum \widetilde{x}^k)^\gamma}{\epsilon_{t-}^i} \\
&-\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda})\Big|(x_t,\lambda_t) = (\widetilde{x}^{k-z},\overline{\lambda})\right]\frac{(\sum \widetilde{x}^{k-z})^\gamma}{\epsilon_{t}^i}\Bigg] x_s^i(\sum \widetilde{x}^{l})^{-\gamma} \quad \geq \\
&\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+z},\overline{\lambda})\Big|(x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda})\right] x_s^i(\sum \widetilde{x}^{l+z})^{-\gamma}\left[\frac{(\sum \widetilde{x}^k)^\gamma}{\epsilon_{t-}^i} - \frac{(\sum \widetilde{x}^{k-z})^\gamma}{\epsilon_{t}^i}\right] \\
&+\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l},\overline{\lambda})\Big|(x_t,\lambda_t) = (\widetilde{x}^{k-z},\overline{\lambda})\right] x_s^i(\sum \widetilde{x}^{l})^{-\gamma}\left[\frac{(\sum \widetilde{x}^k)^\gamma}{\epsilon_{t-}^i} - \frac{(\sum \widetilde{x}^{k-z})^\gamma}{\epsilon_{t}^i}\right]
\end{aligned}$$

The last expression is always positive if the tree that experiences a distress does not coincide with the underlying tree, that is, $z \neq i$, because in this case $\epsilon_t^i = \epsilon_{t-}^i$ and

$$(\sum \widetilde{x}^k)^\gamma - (\sum \widetilde{x}^{k-z})^\gamma \geq 0 \tag{A.45}$$

However, if $z = i$, then the last expression is positive only if

$$\gamma > \frac{\overline{x}^i + \sum_{u\neq i} x_{t-}}{\overline{x}^i} > 1 \tag{A.46}$$

The same conclusion holds for a 'low' state of the economy. The Proof of Proposition 3 implies that (A.46) provides a sufficient condition for the price dividend ratio to decreases (increase) after any distress (recovery) event.

Now consider the differences between the price-dividend ratios of securities i and j before and after the distress of sector u. Consider, w.l.o.g., prices conditional on the 'high' state of the economy. We have, for any combination of normalcy and distress for the sectors, $\widetilde{x}^l$

$$\begin{aligned}
\frac{\overline{P}^j(\widetilde{x}^k)}{D_{t-}^j} - \frac{\overline{P}^i(\widetilde{x}^k)}{D_t^j} &= \sum_{\widetilde{x}^l}\int_t^\infty e^{-a(t-s)}\left[\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^l,\overline{\lambda})\Big|(x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda})\right]\right](\frac{x_s^j}{D_{t-}^j} - \frac{x_s^i}{D_{t-}^i})\left(\frac{\sum \widetilde{x}^l}{\sum \widetilde{x}^k}\right)^{-\gamma} ds \geq \\
&\sum_{\widetilde{x}^l}\int_t^\infty e^{-a(t-s)}\left[\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^l,\overline{\lambda})\Big|(x_t,\lambda_t) = (\widetilde{x}^{k-u},\overline{\lambda})\right]\right](\frac{x_s^j}{D_{t-}^j} - \frac{x_s^i}{D_{t-}^i})\left(\frac{\sum \widetilde{x}^l}{\sum \widetilde{x}^k}\right)^{-\gamma} ds = \\
&\frac{\overline{P}^j(\widetilde{x}^{k-u})}{D_{t-}^j} - \frac{\overline{P}^i(\widetilde{x}^{k-u})}{D_t^j}
\end{aligned} \tag{A.47}$$

The last inequality follows from the fact that when the current vector of multipliers comprises more trees in distress, as $\widetilde{x}^{k-u}$, the transition probabilities to reach states with a high number of distresses are higher, and to those states correspond the largest positive terms in the last vector on the RHS, when

$$\gamma \leq \min_{i,j} \left[\frac{\overline{x}^i + \sum_{u\neq i} x^u_{t-}}{\overline{x}^i}, \frac{\overline{x}^j + \sum_{u\neq j} x^u_{t-}}{\overline{x}^j} \right] \quad z = i, j$$

We can then write, from (A.47)

$$\frac{\overline{P}^j(\widetilde{x}^k)}{D^j_{t-}} - \frac{\overline{P}^j(\widetilde{x}^{k-u})}{D^j_t} \geq \left(\frac{\overline{P}^i(\widetilde{x}^k)}{D^j_{i-}} - \frac{\overline{P}^i(\widetilde{x}^{k-u})}{D^i_t} \right) \tag{A.48}$$

Since, by assumption

$$\frac{\overline{P}^j(\widetilde{x}^k)/D^j_{t-}}{\overline{P}^i(\widetilde{x}^k)/D^i_t} \geq 1,$$

(A.48) implies that

$$\frac{\overline{P}^j(\widetilde{x}^{k-u})/D^j_{t-}}{\overline{P}^i(\widetilde{x}^{k-u})/D^i_t} \geq \frac{\overline{P}^j(\widetilde{x}^k)/D^j_{t-}}{\overline{P}^i(\widetilde{x}^k)/D^i_{t-}}$$

which, in turn, leads to

$$\overline{R}^j_h(\widetilde{x}^k - u) \geq \overline{R}^i_h(\widetilde{x}^k - u).$$

We conclude that a sufficient condition for a value premium to arise for the whole cross section in the full-information economy is then:

$$\gamma \leq \min_{i=1,2,\ldots,N} \frac{\overline{x}^i + \sum_{u\neq i} x^u_{t-}}{\overline{x}^i} \tag{A.49}$$

(38) of the Proposition then follows.

To show that incomplete information over the state of the economy and learning give rise to a proportionally lower risk premium for growth stocks - the asset with the higher price dividend ratio, i.e. j - note that growth stocks are characterized by the highest difference between the returns caused by a distress in the two states of the economy, when the risk aversion is 'low', that is, it satisfies the upper bound given in (A.49). This can be formally shown along the lines of expression (A.47), however, the intuition is clear. Growth stocks are characterized by scarcely cyclic dividend streams and low unconditional intensity of distress, therefore their price drop due to a consumption shock is modest in the 'low' state of the economy - where expected consumption growth is lower - and it is more pronounced in the 'high' state of the economy - because expected consumption growth is higher and the low risk aversion decreases even further the hedging demand. Since the posterior probability update following a distress implies an higher probability for the 'low' state of the economy, growth stocks are those that benefit of the highest posterior returns following a negative consumption shocks (an external distress). By virtue of expression (A.42), this also implies that they command the lowest risk premia.

This concludes the proof.

Proof of Proposition 6

The exogeneity measure of sector i, reported in Proposition 3, can be explicitly represented applying Lemma A.2 *iii*):

$$\begin{aligned}
ex^i_{s,T} &= \frac{1}{T-s}\left[\int_s^T \mathbb{P}\left[\left(\bigcup_{z\neq i}(x^z_u = \underline{x}^z)\right)\middle|(x^i_s = \underline{x}^i, x^j_i = \overline{x}^j, \forall j \neq i), \mathcal{F}^{x,Y}_s\right] du - \right.\\
&\quad \left.\int_s^T \mathbb{P}\left[\mathbf{1}(x^i_u = \underline{x}^z)\middle|\left(x^i_s = \overline{x}^i, x^z_s = \underline{x}^z, \forall z \neq i\right), \mathcal{F}^{x,Y}_s\right] du\right]\\
&= \frac{1}{T-s}\left[\int_s^T \left(p^h_s \sum_{v\in\overline{\mathcal{D}}^i} \mathbf{B}^{2*u_1-1,v}_{w-s} - (1-p^h_s)\sum_{v\in\overline{\mathcal{D}}^i}\mathbf{B}^{2*u_1,v}_{w-s}\right) dw - \right.\\
&\quad \left.\int_s^T \left(p^h_s \sum_{v\in\mathcal{D}^i} \mathbf{B}^{2*u_2-1,v}_{w-s} - (1-p^h_s)\sum_{v\in\mathcal{D}^i}\mathbf{B}^{2*u_2,v}_{w-s}\right) dw\right]\\
&= (0'_{2*u_1-2}, p^h_s, 1-p^h_s, 0'_{\mathcal{N}-2*u_1})\frac{(\mathbf{A}^H)^{-1}}{T-s}[I_d - \exp(-\mathbf{A}^H(T-s))]\overline{W}_1 - \\
&\quad (0'_{2*u_2-2}, p^h_s, 1-p^h_s, 0'_{\mathcal{N}-2*u_2})\frac{(\mathbf{A}^H)^{-1}}{T-s}[I_d - \exp(-\mathbf{A}^H(T-s))]\overline{W}_2
\end{aligned}$$

where $\mathbf{B}_{T-t}^{u,v}$ is the $(u,v)-$th entry of the $\mathcal{N}\times\mathcal{N}$ transition matrix between times t and T, $\mathbf{B}_{T-t}=\exp(-\mathbf{A}^H(T-t))$. $\mathcal{D}^i$ ($\overline{\mathcal{D}}^i$) denotes the collection of the columns where sector i (some sector but not i) is in distress. u_1 denotes the combination of normalcy or distress states for the sectors where only sector i is in distress, while u_2 denotes the combination where all sectors excluding i are in distress. $\overline{W}_1$ is a $\mathcal{N}$ vector with ones in states where some tree excluding i is in distress and zeros otherwise, while $\overline{W}_2$ has ones where i is in distress and zeros otherwise. Now assume that $\overline{x}_u^i=\overline{x}_u^j$, $u=h,l$, and write the p/d ratio of sector i as the p/d ratio of its dividend strip with infinity maturity:[22]

$$\frac{P_s^i}{D_s^i} = \lim_{T\to\infty}\frac{Y_s}{(\sum_{i=1}^N x_s^i)^{-\gamma}}(p_s^h,1-p_s^h,0'_{\mathcal{N}-2})\int_s^T e^{-a(u-s)}\mathbf{B}_{u-s}\mathbf{C}\,du$$

Comparing the last expression with the definition of the exogeneity measure, we conclude that, if $ex_{s,T}^i > ex_{s,T}^j$, $\forall T$, then $\frac{P_s^i}{D_s^i} > \frac{P_s^i}{D_s^j}$ for any initial distress and recovery state at time s, because:

$$\mathbb{P}\left[\left(\bigcup_{z\neq i}(x_u^z=\underline{x}^z)\right)\middle| x_s^i=\underline{x}^i,\mathcal{F}_s^{x,Y}\right]\le\mathbb{P}\left[\left(\bigcup_{z\neq i}(x_u^z=\underline{x}^z)\right)\middle|\left(\bigcup_{z=1}^N(x_s^i=\underline{x}^z)\right),\mathcal{F}_s^{x,Y}\right]$$

and

$$\mathbb{P}\left[x_u^i=\underline{x}^z|\,(x_s^z=\underline{x}^z,\forall z\neq i),\mathcal{F}_s^{x,Y}\right]\ge\mathbb{P}\left[x_u^i=\underline{x}^z|\left(\bigcup_{z=1}^N x_s^z=\underline{x}^z\right),\mathcal{F}_s^{x,Y}\right]$$

Auxiliary Results

Lemma A.1 *Any $\mathcal{F}_t^{x,Y}-$ martingale X_t admits the representation:*

$$X_t=X_0+\int_0^t\sum_{i=1}^N f_s^{H^i}(dH_s^i-\widehat{\lambda}_s^{H^i}ds)$$

where adapted processes $f_t^{H^i}$ satisfies the integrability conditions in Theorem 19.1 of Lipster and Shyriaev (2001).

Proof. A straightforward adaptation of Theorem 19.1 in Lipster and Shyriaev (2001).

Lemma A.2 *Let $\mathcal{D}:=\{d_1,d_2,\ldots,d_L\}$ denote a collection of $L\le N$ trees in the Lucas orchard and let τ_{d_i},$i=1,2,\ldots,L$ be the first distress time of tree d_i.*

(i) The conditional posterior probability of no distress until time T, out of the trees in set $\mathcal{D}$ and conditional on no distress of any tree at time $s\le T$, is given by:

$$\mathbb{P}\left[\tau_{d_1}>T,\tau_{d_2}>T,\ldots,\tau_{d_L}>T|F_s^{x,Y}\right]=(p_t^h,1-p_t^h,0'_{N-2})\exp(-\mathbf{A}(T-s))\overline{1}_{\mathcal{N}},\tag{A.50}$$

where $\exp(A)$ denotes the matrix exponential of A, $\overline{1}_M$ ($\overline{0}_M$) is a M-dimensional vector of ones (zeros), $\mathcal{N}=2^{N-L}$ and the $\mathcal{N}\times\mathcal{N}$ matrix $\mathbf{A}$ is given explicitly in (A.57) below.

(ii) The expected time left until the first distress of any of the trees in set $\mathcal{D}$, conditional on no distress of any tree at time s, is given by:

$$\mathbb{E}\left[\min_{i\in D}\tau_{d_i}-s|\mathcal{F}_s^{x,Y}\right]=(p_t^h,1-p_t^h,0'_{\mathcal{N}-2})\mathbf{A}^{-1}\overline{1}_{\mathcal{N}}\tag{A.51}$$

(iii) The expected fraction of the time interval $[s,T]$ with no distress for any tree in set $\mathcal{D}$, conditional on no distress of any tree at time s, is given by:

$$(p_t^h,1-p_t^h,0'_{\mathcal{N}-2})\frac{(\mathbf{A}^H)^{-1}}{T-s}[I_d-\exp(-\mathbf{A}^H(T-s))\overline{W}_{\mathcal{N}}\tag{A.52}$$

where the closed-form expressions for the $\mathcal{N}\times\mathcal{N}$ matrix $\mathbf{A}^H$ and $\mathcal{N}-$dimensional vector $\overline{W}_{\mathcal{N}}$ are reported in (A.61) and (A.60) below, respectively.

[22] Assuming, without loss of generality, that none of the trees is initially in distress.

Proof.

(*i*) Let $H_t = [H_t^1, H_t^2, \dots, H_t^N]'$ and assume, without loss of generality, that none of the N trees is in a distress state at time s, so that $H_s = 0'_N$, an $N-$dimensional vector of zeros. The generalization to a different state for H_t is straightforward. Note that 'survival' probabilities for individual trees are obtained as a special case of this methodology, when set $\mathcal{D}$ is a singleton. We can write:

$$\mathbb{P}[\tau_{d_1} > T, \tau_{d_2} > T, \dots \tau_{d_L} > T | \mathcal{F}_s^{x,Y}] = \mathbb{E}\left[\mathbb{P}\left[\tau_{d_1} > T, \tau_{d_2} > T, \dots \tau_{d_L} > T \middle| \mathcal{F}_s\right] | \mathcal{F}_s^{x,Y}\right] \tag{A.53}$$

Assume that the economy is in a 'high' state, so that $\lambda_s = \overline{\lambda}$ and $\eta_s = \overline{\eta}$. By the law of iterated expectations the inner expectation is an $\mathcal{F}_s-$martingale, therefore the 'drift' component of its Ito representation must vanish. By the Markov property we must have

$$\mathbb{P}[\tau_{d_1} > T, \tau_{d_2} > T, \dots \tau_{d_L} > T | \mathcal{F}_s] = \mathbf{1}(\tau_{d_1} > s, \tau_{d_2} > s, \dots \tau_{d_L} > s) V^h(s, H_t) \tag{A.54}$$

Let $\mathcal{S}^D(N-L, K)$ denote the set of combinations of the $N-L$ available trees excluding those in $\mathcal{D}$, into groups of K, and let $\mathcal{S}^D(N-L, K)_h$ denote the $h-$th element of this set. The set $\mathcal{S}^D$ will be used to denote the trees that are not in a distress state. We use the notation $\overline{\mathcal{S}}^D(N-L, K)_h$ to denote the complement of the $h-$th element, that is, the trees that are in a distress state. We apply Ito's lemma to the RHS of (A.54), take conditional expectations and impose the martingale property, according to which the conditional mean of the RHS of (A.54) must vanish. Applying this argument also to the probability conditional on the 'low' state of the economy, we obtain the following system of ordinary differential equations:

$$\frac{\partial}{\partial s}\begin{bmatrix} V^h(s, H_s) \\ V^l(s, H_s) \end{bmatrix} = \left(\begin{bmatrix} \sum_{j=1}^N \overline{\lambda}^j & 0 \\ 0 & \sum_{j=1}^N \underline{\lambda}^j \end{bmatrix} - I\right)\begin{bmatrix} V^h(s, H_s) \\ V^l(s, H_s) \end{bmatrix} - \begin{bmatrix} \sum_{j=1}^{\#(\mathcal{S}^D(N-L,N-L-1))} \overline{\lambda}^j V^h(s, \mathcal{S}^D(N-L, N-L-1)_j) \\ \sum_{j=1}^{\#(\mathcal{S}^D(N-L,N-L-1))} \underline{\lambda}^j V^l(s, \mathcal{S}^D(N-L, N-L-1)_j) \end{bmatrix} \tag{A.55}$$

The system involves all functions V conditional on any combination of normalcy or distress state for all the N trees excluding those L for which we want to compute the probability of no-distress. This system of equations can be written compactly in vector notation.

$$\frac{d}{ds}\begin{bmatrix} \mathbf{V}(s, H_s) \\ \mathbf{V}(s, \mathcal{S}^D(N-L, N-L-1)_1) \\ \vdots \\ \mathbf{V}(s, \mathcal{S}^D(N-L, N-L-1)_{N-L}) \\ \mathbf{V}(s, \mathcal{S}^D(N-L, N-L-2)_1) \\ \vdots \\ \mathbf{V}(s, \mathcal{S}^D(N-L, 1)) \\ \mathbf{V}(s, \mathcal{S}^D) \end{bmatrix} = \mathbf{A}\begin{bmatrix} \mathbf{V}(s, H_s) \\ \mathbf{V}(s, \mathcal{S}^D(N-L, N-L-1)_1) \\ \vdots \\ \mathbf{V}(s, \mathcal{S}^D(N-L, N-L-1)_{N-L}) \\ \mathbf{V}(s, \mathcal{S}^D(N-L, N-L-2)_1) \\ \vdots \\ \mathbf{V}(s, \mathcal{S}^D(N-L, 1)) \\ \mathbf{V}(s, \mathcal{S}^D) \end{bmatrix} \tag{A.56}$$

where $\mathbf{V}(s, \cdot) = [V^h(s, \cdot), V^l(s, \cdot)]'$, and $\mathbf{V}(s, \mathcal{S}^D)$ denotes the function $\mathbf{V}$ conditional on all trees excluding those in $\mathcal{D}$ being in a distress state.

$$\begin{aligned}\mathbf{A} = \operatorname{diag}[&\mathbf{\Upsilon}^{N-L}, \mathbf{\Upsilon}^{\mathcal{S}^D(N-L,N-L-1)_1}, \dots, \mathbf{\Upsilon}^{\mathcal{S}^D(N-L,N-L-1)_{N-L}}, \mathbf{\Upsilon}^{\mathcal{S}^D(N-L,N-L-2)_1}, \\ &\dots, \mathbf{\Upsilon}^{\mathcal{S}^D(N-L,1)_{N-L}}, \mathbf{\Upsilon}^{\mathcal{S}^D}] - \begin{bmatrix} \mathbf{0} & \mathbf{F}^1 & \mathbf{F}^2 & \dots & \mathbf{F}^{N-L} & \mathbf{0} & \dots & \mathbf{0} & \dots \\ \mathbf{G}^1 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{F}^2 & \mathbf{F}^3 & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}\end{aligned} \tag{A.57}$$

with

$$\begin{aligned}
\mathbf{F}^i &= \operatorname{diag}[\overline{\lambda}^i, \underline{\lambda}^i] \qquad \mathbf{G}^i = \operatorname{diag}[\overline{\eta}^i, \underline{\eta}^i] \\
\mathbf{\Upsilon}^{\mathcal{S}^D(K,K-1)_j} &= \operatorname{diag}[\sum_{u\in\mathcal{D}} \overline{\lambda}^u, \sum_{u\in\mathcal{D}} \underline{\lambda}^u] + \operatorname{diag}[\sum_{u\in\mathcal{S}^D(K,K-1)_j} \overline{\lambda}^u, \sum_{u\in\mathcal{S}^D(K,K-1)_j} \underline{\lambda}^u] \\
&\quad + \operatorname{diag}[\sum_{u\in\overline{\mathcal{S}}^D(K,K-1)_j} \overline{\eta}^u, \sum_{u\in\overline{\mathcal{S}}^D(K,K-1)_j} \underline{\eta}^u] - I \\
\mathbf{\Upsilon}^{N-L} &= \operatorname{diag}[\sum_{u=1}^{N} \overline{\lambda}^u, \sum_{u=1}^{N} \underline{\lambda}^u] - I \\
\mathbf{\Upsilon}^{\mathcal{S}^D} &= \operatorname{diag}[\sum_{u=1}^{N-L} \overline{\eta}^u, \sum_{u=1}^{N-L} \underline{\eta}^u] + \operatorname{diag}[\sum_{u\in\mathcal{D}} \overline{\lambda}^u, \sum_{u\in\mathcal{D}} \underline{\lambda}^u] - I,
\end{aligned}$$

and $\mathbf{0} = \text{diag}[0,0]$. The terminal condition is $V(T) = 1$. The solution of this system is immediately characterized in terms of matrix exponential operator, so that:

$$\mathbb{P}\left[\tau_{d_1} > T, \tau_{d_2} > T, \dots \tau_{d_L} > T | \mathcal{F}_s^{x,Y}\right] = \mathbf{1}(\tau_{d_1} > s, \tau_{d_2} > s, \dots \tau_{d_L} > s)(p_t^h, 1-p_t^h, 0_{\mathcal{N}-2}) \cdot \exp\left(-\mathbf{A}(T-s)\right) \overline{1}_{\mathcal{N}} \tag{A.58}$$

where $\mathcal{N} = 2^{N-L}$

(ii) The expected time until the next distress of any of the trees in group $\mathcal{D}$ is given by

$$\begin{aligned}
\mathbb{E}\left[\min_{i\in D} \tau^i - s \middle| \mathcal{F}_s^{x,Y}\right] &= \int_s^\infty u \frac{\partial}{\partial u}\left[1 - \mathbb{P}\left[\tau_{d_1} > u, \tau_{d_2} > u, \dots \tau_{d_L} > u | \mathcal{F}_s^{x,Y}\right]\right] du \\
&= \int_s^\infty -u \frac{\partial}{\partial u} \mathbb{P}\left[\tau_{d_1} > u, \tau_{d_2} > u, \dots \tau_{d_L} > u | \mathcal{F}_s^{x,Y}\right] du - s \\
&= \mathbf{1}(\tau_{d_1} > s, \tau_{d_2} > s, \dots \tau_{d_L} > s) \times \\
&\quad (p_t^h, 1-p_t^h, 0_{\mathcal{N}-2}) \cdot \left[\int_s^\infty u\mathbf{A}\exp\left(-\mathbf{A}(u-s)\right) du\right] \overline{1}_{\mathcal{N}} - s \\
&= (p_t^h, 1-p_t^h, 0_{\mathcal{N}-2}) \cdot \mathbf{A}^{-1}\overline{1}_{\mathcal{N}}
\end{aligned} \tag{A.59}$$

(iii) We apply a similar methodology to compute the expected fraction of time spent in a distress state. Assume without loss of generality that none of the N trees is currently in a distress state. We remind that $H_t^i = \mathbf{1}(x_t^i = 0)$. For a collection $\mathcal{D} = \{d_1, d_2, \dots d_L\}$ of trees, we have

$$\mathbb{P}\left[H_T^{d_1} = 0, H_T^{d_2} = 0, \dots H_T^{d_L} = 0 | \mathcal{F}_s^{x,Y}\right] = (p_t^h, 1-p_t^h, 0_{\mathcal{N}-2}) \cdot \exp\left(-\mathbf{A}^H(T-s)\right) \overline{W}_{\mathcal{N}} \tag{A.60}$$

where $\overline{W}_{\mathcal{N}}$ is the column vector with j−th element $\mathbf{1}(D \in \mathcal{S}(N,K)_j)$ and dimension given by $\mathcal{N} = 2^N$

$$\begin{aligned}
\mathbf{A}^H = \text{diag}[&\mathbf{\Upsilon}_H^N, \mathbf{\Upsilon}_H^{\mathcal{S}(N,N-1)_1}, \dots, \mathbf{\Upsilon}_H^{\mathcal{S}(N,N-1)_{N-1}}, \mathbf{\Upsilon}_H^{\mathcal{S}(N,N-2)_1}, \dots, \mathbf{\Upsilon}_H^{\mathcal{S}(N,1)}, \mathbf{\Upsilon}_H^{\mathcal{S}}] - \\
&\begin{bmatrix}
\mathbf{0} & \mathbf{F}^1 & \mathbf{F}^2 & \dots & \mathbf{F}^N & \mathbf{0} & \dots & \mathbf{0} & \dots \\
\mathbf{G}^1 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{F}^2 & \mathbf{F}^3 & \dots & \dots \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots
\end{bmatrix}
\end{aligned} \tag{A.61}$$

with

$$\begin{aligned}
\mathbf{F}^i &= \text{diag}[\overline{\lambda}^i, \underline{\lambda}^i] \\
\mathbf{G}^i &= \text{diag}[\overline{\eta}^i, \underline{\eta}^i] \\
\mathbf{\Upsilon}_H^{\mathcal{S}(N,K)_j} &= \text{diag}[\sum_{u\in\mathcal{S}(N,K)_j} \overline{\lambda}^u, \sum_{u\in\mathcal{S}(N,K)_j} \underline{\lambda}^u] + \text{diag}[\sum_{u\in\overline{\mathcal{S}}(N,K)_j} \overline{\eta}^u, \sum_{u\in\overline{\mathcal{S}}(N,K)_j} \underline{\eta}^u] - I \\
\mathbf{\Upsilon}_H^N &= \sum_{i=1}^N \text{diag}[\overline{\lambda}^i, \underline{\lambda}^i] - I \\
\mathbf{\Upsilon}_H^{\mathcal{S}} &= \sum_{i=1}^N \text{diag}[\overline{\eta}^i, \underline{\eta}^i] - I,
\end{aligned}$$

and $\mathbf{0} = \text{diag}[0,0]$. $\mathcal{S}(N,K)$ now denotes the set of combinations of all the N trees in groups of K. The expected fraction of time on the horizon $[s,T]$ with no distress state for members of group D is then:

$$\begin{aligned}
&\frac{1}{T-s}\mathbb{E}\left[\int_s^T \mathbf{1}(H_u^{d_1} = 0, H_u^{d_2} = 0, \dots H_u^{d_2} = 0) \middle| \mathcal{F}_s^{x,Y}\right] du = \frac{1}{T-s}\int_s^T \mathbb{P}\left[H_u^{d_1} = 0, H_u^{d_2} = 0, \dots H_u^{d_2} = 0 | \mathcal{F}_s^{x,Y}\right] du \\
&= \frac{(p_t^h, 1-p_t^h, 0_{\mathcal{N}-2})}{T-s} \cdot \left[\int_0^{T-s} \exp\left(-\mathbf{A}^H\tau\right) d\tau\right] \overline{W}_{\mathcal{N}} \qquad = [p_t^h, 1-p_t^h, 0_{\mathcal{N}-2}] \cdot \frac{(\mathbf{A}^H)^{-1}}{T-s}\left[I_d - \exp(-\mathbf{A}^H(T-s))\right] \overline{W}_{\mathcal{N}}
\end{aligned} \tag{A.62}$$

This ends the proof of the Lemma.

Lemma A.3 *The following properties hold for the $i-th$ security price, $i = 1, 2, \dots, N$:*

i) Leaving market share unaltered, the price increases (decreases) when the 'high' or the 'low'-state intensity of distress (recovery) of a tree $j \neq i$ increases. There exists a critical level of risk aversion γ^* below which the price decreases (increases) if the intensity of distress (recovery) of tree i increases and above which the opposite occurs.

ii) The price is a U-shaped function of the Relative Risk Aversion parameter γ, that is, there is a critical level of risk aversion γ^* after which the price increases when the risk aversion increases.

Proof.

i) For any state of the economy, $w = h, l$, applying the rules of matrix calculus we obtain:

$$\frac{dP_t^i}{d\overline{\lambda}_w^j} = Y_t(\sum_{i=1}^N x_t)^\gamma(p_t, 1-p_t, \overline{0}_{\mathcal{N}-2})(\mathbf{a}+\mathbf{A}^H)^{-1}\left(-\frac{d\mathbf{A}^H}{d\overline{\lambda}_w^j}\right)\overline{V}^i \tag{A.63}$$

where $\overline{V}^i = (\mathbf{a}+\mathbf{A}^H)^{-1}\mathbf{C}^i$. For convenience of notation, we have denoted $\underline{\lambda}$ by $\overline{\lambda}_l$, and $\overline{\lambda}$ by $\overline{\lambda}_h$. We also denote by $\overline{V}_w^i(\widetilde{x}^k)$ the entry of the matrix $\overline{V}^i$ conditional on the persistent dividend state $\widetilde{x}^i$ and on the state of the common factor w, h, l.

Let $j \neq i$. Following the methodology of the proof of i), and, in particular, taking (A.17),(A.18), and (A.19) into account, it is easily seen that for any state of the vector dividend multiplier $\widetilde{x}^k$, we have $\overline{V}_w^i(\widetilde{x}^k) < \overline{V}_w^i(\widetilde{x}^{k-j})$. Using the latter inequality we have

$$\left(-\frac{d\mathbf{A}^H}{d\overline{\lambda}_w^j}\right)\overline{V}^i = \begin{pmatrix} 0 \\ \dots \\ \overline{V}_w^i(\widetilde{x}^{k-j}) - \overline{V}_w^i(\widetilde{x}^k) \\ \dots \end{pmatrix} \tag{A.64}$$

where the elements of the RHS are zero if in the state $\widetilde{x}^k$ the $j-$th tree is in distress and nonnegative otherwise. It then follows from (A.63) that $\frac{dV^i}{d\overline{\lambda}_w^j} \geq 0$, because the elements of the matrix $(\mathbf{a}+\mathbf{A}^H)^{-1}$, being Laplace transforms of transition probabilities, are all nonnegative.

Let $j = i$. Repeating the arguments in the proof of ii) and using (A.17),(A.18), and (A.19), we find that:

$$\overline{V}^i(\widetilde{x}^k) - \overline{V}^i(\widetilde{x}^{k-i}) = \int_t^\infty e^{-a(s-t)}\left[\sum_{\widetilde{x}^{l+i}\in\overline{\mathcal{A}}^{k-i}}\left(\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+i},\overline{\lambda}_w)\middle|(x_t,\lambda_t) = (\widetilde{x}^k,\overline{\lambda}_w)\right]\right.\right.$$
$$\left.\left.-\mathbb{P}\left[(x_s,\lambda_s) = (\widetilde{x}^{l+i},\overline{\lambda}_w)\middle|(x_t,\lambda_t) = (\widetilde{x}^{k-i},\overline{\lambda}_w)\right]\right)[\overline{x}^i(\sum\widetilde{x}^{l+i})^{-\gamma} - \underline{x}^i(\sum\widetilde{x}^l)^{-\gamma}]\right]ds \tag{A.65}$$

It is easy to see that for each term in the sum above the following holds:

$$\overline{x}^i(\sum\widetilde{x}^{l+i})^{-\gamma} - \underline{x}^i(\sum\widetilde{x}^l)^{-\gamma} \gtreqless 0 \iff \gamma \lesseqgtr \frac{\log(\overline{x}^i/\underline{x}^i)}{\log(\overline{x}^i + \sum\widetilde{x}^l) - \log(\underline{x}^i + \sum\widetilde{x}^l)} \tag{A.66}$$

Then, letting $\gamma^{\min} = \min\gamma_l^*$ and $\gamma^{\max} = \max\gamma_l^*$, and taking (A.17) and (A.18) into account, we conclude that

$$\overline{V}^i(\widetilde{x}^k) > \overline{V}^i(\widetilde{x}^{k-j}) \quad \text{if} \quad \gamma < \gamma^{\min} \tag{A.67}$$
$$\overline{V}^i(\widetilde{x}^k) < \overline{V}^i(\widetilde{x}^{k-j}) \quad \text{if} \quad \gamma > \gamma^{\max} \tag{A.68}$$

In light of (A.64) this implies that $\frac{dV^i}{d\overline{\lambda}_w^i} > 0$ for $\gamma > \gamma^{\max}$, and $\frac{dV^i}{d\overline{\lambda}_w^i} < 0$ for $\gamma < \gamma^{\min}$ because the elements of the matrix $(\mathbf{a}+\mathbf{A}^H)^{-1}$ are all positive.

It is immediate to prove the opposite inequalities when we consider a variation in the intensities of recovery η.

*ii)*We assume without loss of generality that none of trees in currently in distress, the intuition being identical for any other combination of normalcy or distress for the trees. Using the notation of point ii), the derivative with respect to γ of the price of the claim to the $i-th$ endowment is:

$$\frac{\partial P_t^i}{\partial\gamma} = Y_t(p_t, 1-p_t, \overline{0}_{\mathcal{N}-2})(\mathbf{a}+\mathbf{A}^H)^{-1}\begin{pmatrix} \dots \\ x^i(\sum_{i=1}^N x_t)^\gamma(\sum\widetilde{x}^k)^{-\gamma}(-\mu_Y - \frac{\sigma_Y^2}{2}(1-2\gamma) + \log(\sum_{i=1}^N x_t) - \log(\sum\widetilde{x}^k)) \\ \dots \end{pmatrix} \tag{A.69}$$

for all of the 2^N combinations of multiplier is normalcy or distress. Since

$$-\mu_Y - \frac{\sigma_Y^2}{2}(1-2\gamma) + \log(\sum_{i=1}^{N} x_t) - \log(\sum \widetilde{x}^k) \gtreqless 0 \quad \Longleftrightarrow \quad \gamma \gtreqless \left(\log\left(\frac{\sum \widetilde{x}^k}{\sum_{i=1}^N x_t}\right) + \mu_Y + \frac{\sigma_Y^2}{2}\right)\frac{1}{\sigma_Y^2} = \gamma^k \tag{A.70}$$

and $\frac{\partial P_t^i}{\partial \gamma}$ is a convex combination of the terms $-\mu_Y - \frac{\sigma_Y^2}{2}(1-2\gamma) + \log(\sum_{i=1}^N x_t) - \log(\sum \widetilde{x}^k)$, there exists a $\gamma^* \leq \max_{\widetilde{x}^k} \gamma^k$ such that $\frac{\partial P_t^i}{\partial \gamma} < 0$ for $\gamma < \gamma^*$ and $\frac{\partial P_t^i}{\partial \gamma} \geq 0$ for $\gamma \geq \gamma^*$.

This ends the proof of the Lemma.

Lemma A.4 *The equilibrium interest rate, r_t, the market price of diffusive risk, κ_t, and the market price of event risk for each tree, θ_t^i, $i = 1, 2, \ldots, N$, are given by the following expressions:*

$$\begin{aligned} r_t &= \delta + \gamma\mu_Y - \frac{1}{2}\gamma(\gamma+1)\sigma_Y^2 + \sum_{i=1}^N \left\{ H_t^i \left[1 - \left(\frac{\overline{x}^i + \sum x_{t-}}{\underline{x}^i + \sum x_{t-}}\right)^{-\gamma}\right]\widehat{\eta}_t^i + \right. && \text{(A.71)} \\ & \qquad \left. (1-H_t^i)\left[1 - \left(\frac{\underline{x}^i + \sum x_{t-}}{\overline{x}^i + \sum x_{t-}}\right)^{-\gamma}\right]\widehat{\lambda}_t^i \right\} && \text{(A.72)} \\ \kappa_t &= \gamma\sigma_Y && \text{(A.73)} \\ \theta_t^i &= H_t^i \left(\frac{\overline{x}^i + \sum x_{t-}}{\underline{x}^i + \sum x_{t-}}\right)^{-\gamma} + (1-H_t^i)\left(\frac{\underline{x}^i + \sum x_{t-}}{\overline{x}^i + \sum x_{t-}}\right)^{-\gamma} \qquad i = 1, 2, \ldots N && \text{(A.74)} \end{aligned}$$

Proof. We remind that $H_t^i = \mathbf{1}(x_t^i = 0)$, therefore $dH_t^i = 1$ if a distress occurs, $H_t^i = 0$, and $dH_t^i = -1$ if a recovery occurs. $dH_t^i = 0$ if the tree persists in its distress or normalcy state.

According to the optimality conditions for the representative agent, the equilibrium state price density ξ_t is:

$$\xi_t = e^{-\delta t} Y_t^{-\gamma}\left(\sum_{i=1}^N x_t^i\right)^{-\gamma} \tag{A.75}$$

On the other hand, the state-price density must also obey:

$$\xi_t = \exp\left(-\int_0^t (r_s + \frac{\kappa_s^2}{2})ds - \int_0^t \kappa_s dZ_s + \int_0^t \sum_{i=1}^N \widehat{\lambda}_s^{H^i}(1-\theta_s^i)ds + \int_0^t \sum_{i=1}^N -\log(\theta_s^i)\mathrm{sgn}(H_t^i)dH_s^i\right) \tag{A.76}$$

where $\mathrm{sgn}(H_t^i) = -1$ if $H_t^i \leq 0$ and $\mathrm{sgn}(H_t^i) = 1$ if $H_t^i > 0$. Furthermore, θ_t^i is the market price of event risk for tree i - distress risk, if tree i is in normalcy, i.e. $H_t^i = 0$, recovery risk if tree i is in distress, i.e. $H_t^i = 0$ - κ_t is the market price of diffusive risk, and

$$\widehat{\lambda}_t^{H^i} = H_t^i\widehat{\eta}^i + (1-H_t^i)\widehat{\lambda}_t^i.$$

By applying Ito's lemma to (A.76) we obtain:

$$d\xi_t = -\xi_t r_t dt - \xi_t \kappa dZ_t + \xi_t\left[\sum_{i=1}^N (\theta_s^i - 1)(-\mathrm{sgn}(H_t^i)dH_t^i - \widehat{\lambda}_t^{H^i})\right] \tag{A.77}$$

By Ito's lemma applied to (A.75) we obtain the alternative representation:

$$\begin{aligned} d\xi_t = -\delta\xi_t - \gamma\mu_Y\xi_t dt + \frac{1}{2}\gamma(\gamma+1)\sigma_Y^2\xi_t dt + \xi_t \sum_{i=1}^N & \left[(1-H_t)\frac{\left[(\underline{x}^i + \sum x_{t-})^{-\gamma} - (\overline{x}^i + \sum x_{t-})^{-\gamma}\right]}{(\overline{x}^i + \sum x_{t-})^{-\gamma}}\widehat{\lambda}_t^i \right. \\ & \left. + H_t \frac{\left[(\overline{x}^i + \sum x_{t-})^{-\gamma} - (\underline{x}^i + \sum x_{t-})^{-\gamma}\right]}{(\underline{x}^i + \sum x_{t-})^{-\gamma}}\widehat{\eta}_t^i\right] - \gamma\xi_t\sigma_Y dZ_t + \\ \xi_t \sum_{i=1}^N \left[(1-H_t)\frac{\left[(\underline{x}^i + \sum x_{t-})^{-\gamma} - (\overline{x}^i + \sum x_{t-})^{-\gamma}\right]}{(\overline{x}^i + \sum x_{t-})^{-\gamma}}(dH_t^i - \widehat{\lambda}_t^i) \right. & \left. + H_t \frac{\left[(\overline{x}^i + \sum x_{t-})^{-\gamma} - (\underline{x}^i + \sum x_{t-})^{-\gamma}\right]}{(\underline{x}^i + \sum x_{t-})^{-\gamma}}(-dH_t^i - \widehat{\eta}_t^i)\right] \end{aligned}$$

which, compared to (A.76) yields the reported expressions for the equilibrium interest rates and market prices of risk. This ends the proof of the Lemma.

Lemma A.5 *Assume, without loss of generality, that no sector is in distress at time s. The price of the $T-$maturity dividend strip of the $i-$th sector/tree, as evaluated at time s, is given by the following expression:*

$$P^{D^i}_{s,T} = \frac{Y_s}{(\sum_{i=1}^N x^i_s)^{-\gamma}} (p^h_s, 1-p^h_s, \overline{0}_{\mathcal{N}-2})(\mathbf{a}+\mathbf{A}^H)^{-1}\left(I_d - e^{-(\mathbf{a}+\mathbf{A}^H)(T-s)}\right)\mathbf{C}^i \tag{A.78}$$

where the Markovian matrix $\mathbf{A}^H$, as well as the vector $\mathbf{C}^i$, are reported in the proof of Proposition 1. The conditional risk premium of this dividend strip is:

$$\begin{aligned}\mu^{D^i}_{s,T} = \gamma\sigma^2_Y - \sum_{j=1}^N \Bigg\{ &(1-H^j_t)(\theta^j_t - 1)\left[\left(\frac{\left(p^h_t \frac{\overline{\lambda}^j}{\widehat{\lambda}^j_t}, (1-p^h_t)\frac{\underline{\lambda}^j}{\widehat{\lambda}^j_t}\right)\cdot \overline{P}^{D^i}_{s,T}(x_s - j)}{(p^h_t, 1-p^h_t)\cdot \overline{P}^{D^i}_{s,T}(x_s)}\right) - 1\right]\widehat{\lambda}^j_t \\ &+ H^j_t(\theta^j_t - 1)\left[\left(\frac{\left(p^h_t \frac{\overline{\eta}^j}{\widehat{\eta}^j_t}, (1-p^h_t)\frac{\underline{\eta}^j}{\widehat{\eta}^j_t}\right)\cdot \overline{P}^{D^i}_{s,T}(x_s + j)}{(p^h_t, 1-p^h_t)\cdot \overline{P}^{D^i}_{s,T}(x_s)}\right) - 1\right]\widehat{\eta}^j_t \Bigg\}\end{aligned} \tag{A.79}$$

where $\overline{P}^{D^i}_{s,T}(x_s - j)$ $(\overline{P}^{D^i}_{s,T}(x_s + j))$ denotes the full-information price vector of the dividend strip at time s immediately after a distress (recovery) of tree j.

Proof. The $T-$maturity dividend strip of the $i-$th sector/tree, as evaluated at time s, is the claim to the $i-$th sector's dividend stream paid from s to T. Its price is given by the following expression:

$$P^{D^i}_{s,T} = \frac{1}{\xi_s}\mathbb{E}\left[\int_s^T \xi_s Y_s x^i_s ds \middle| \mathcal{F}^{x,Y}_s\right] \tag{A.80}$$

This expression is computed in closed-form using the same methodology of the proof of Proposition 1:

$$\begin{aligned} P^{D^i}_{s,T} &= \frac{Y_s}{(\sum_{i=1}^N x^i_s)^{-\gamma}} \int_s^T e^{-a(u-s)}\mathbb{E}\left[x^i_u \left(\sum_{j=1}^N x^i_u\right)^{-\gamma} \middle| \mathcal{F}^{x,Y}_s\right] du &\text{(A.81)} \\ &= \frac{Y_s}{(\sum_{i=1}^N x^i_s)^{-\gamma}} \int_s^T e^{-a(u-s)}(p^h_t, 1-p^h_t, \overline{0}_{\mathcal{N}-2})\mathbf{B}_{u-s}\mathbf{C}^i du &\text{(A.82)} \\ &= \frac{Y_s}{(\sum_{i=1}^N x^i_s)^{-\gamma}} (p^h_s, 1-p^h_s, \overline{0}_{\mathcal{N}-2})(\mathbf{a}+\mathbf{A}^H)^{-1}\left(I_d - e^{-(\mathbf{a}+\mathbf{A}^H)(T-s)}\right)\mathbf{C}^i &\text{(A.83)} \end{aligned}$$

$\mathbf{B}_{u-s}$ is the $(\mathcal{N}+1)\times(\mathcal{N}+1)$ full-information conditional joint transition probability matrix of the vector of supplying trees multipliers x and of the state of the latent factor from time s to time u:

$$\mathbf{B}_{u-s} = e^{-\mathbf{A}^H(u-s)}$$

Conditional risk premia of dividend strips are computed in the same fashion of conditional risk premia of the infinitely-lived securities:

$$\mu^{D^i}_{s,T} = \mathbb{E}\left[\frac{dV^{D^i}_{s,T}}{V^{D^i}_{s,T}} \middle| \mathcal{F}^{x,Y}_s\right] + \frac{D^i_s}{V^{D^i}_{s,T}} - r_s. \tag{A.84}$$

From the explicit expression of the conditional risk premium, we compute the annualized unconditional risk premium numerically as the time-series average of the conditional premium evaluated over a simulated time series of dividend persistent components x_t, posterior probability p^h_t, and state of the common factor, keeping time to maturity $T-s$ fixed. The length of the time-series is 2500 years, using 100 discretization steps per year:

$$\overline{\mu}^{D^i}_{s,T} = \left(\frac{1}{2500*100}\sum_{j=1}^{2500*100}\mu^{D^i}_{j,j+T-s}\right)100$$

The term structure of risk premia is obtained letting the maturity T of the dividend strip vary. The full-information term structure in computed similarly, assuming observability of the state of the common factor. This ends the proof of the Lemma.

Appendix B: Calibration Precedure

In this Appendix we outline the methodology used to calibrate parameter values in the empirical application on US industry portfolio data discussed in Section IV. We split the parameter set into 5 subsets – $\theta_1 = (\mu_y, \sigma_Y)$, $\theta_2 = (k_h, k_l)$, $\theta_3 = (\overline{\lambda}^i, \underline{\lambda}^i, \overline{\eta}^i, \underline{\eta}^i)$, $i = 1, 2, \ldots, 12$, $\theta_4 = (\overline{x}^i/\underline{x}^i, \overline{x}^i)$, $i = 1, 2, \ldots, 12$, and γ – and we apply a sequential calibration procedure to identify each subset in successive steps. Each step takes as given the parameters estimated at the previous step.

1) *Calibration of* θ_1. The conditional mean, μ_Y, and the standard deviation σ_Y of the diffusive component Y common to all dividend are the mean and standard deviation of aggregate consumption growth absent any sector's transition to distress. Using the empirical analysis of Barro and Ursua (2008), we identify those years of aggregate US consumption expenditures characterized by a drop-off in consumption of more than 10% - years of distress - and we obtain parameters μ_Y and σ_Y fitting the mean and the variance of logarithmic consumption growth conditional on no distress with a lognormal IID consumption growth model.

2) *Calibration of* θ_2. To obtain the regime switch parameters for the latent business cycle factor, k_h, k_l, we follow Ribeiro and Veronesi (2002). In the absence of feed-backs between sectors' distress states and the business cycle, they estimate a quarterly probability of 0.0501 of switching from 'Peak' ('high' state in our terminology) to 'Trough' ('low' state), and a quarterly probability of 0.2716 for the opposite transition. This means that:

$$\exp\left(\frac{1}{4}\begin{bmatrix} -k_h & k_h \\ k_l & -k_l \end{bmatrix}\right) = \begin{bmatrix} 1-0.0501 & 0.0501 \\ 0.2716 & 1-0.2716 \end{bmatrix}, \tag{B.1}$$

where the LHS is the quarterly transition probability matrix of the Markov chain followed by the business cycle. It follows that $k_h = 0.2418$ and $k_l = 1.3109$.

3) *Calibration of* θ_3*, taking* θ_2 *as given.* We calibrate parameters that govern event risk, namely $\overline{\lambda}^i$, $\underline{\lambda}^i$, $\overline{\eta}^i$, $\underline{\eta}^i$, i being the Industry identifier, using statistics on historical corporate bonds distress/default events by Industry. We extract from debt defaults the conditioning information on the distress events that is not possible to identify solely from dividend pay-out series. To match both empirical frequency and persistence features of the events, we combine information from the *Average Cumulative Issuer-Weighted Global Default Rates by Broad Industry Group*, as reported in Exhibit 38 of Moody's *Corporate Default and Recovery Rates, 1920-2009*, with information from the *Global Default Rates By Industry*, as reported in Table 19 of S&P's *Default, Transition, and Recovery: 2009 Annual Global Corporate Default Study And Rating Transitions.* To simplify our fitting procedure, we assume that $\overline{\eta} = \underline{\eta} = \eta$. For each sector, we match the empirical default rate within $\tau-$years to the steady-state $\tau-$years probability of a sector's distress conditional on not being in distress, where $\tau = 1, 3, 10$. 'Steady-state' means that we are using the unconditional posterior probability of a 'high' state to compute the distress probability, while the only conditioning variable is the sector current non-distress status. Note that, due to cross-sectional learning, this steady-state posterior probability depends on the intensities of all sectors. This means that event risk parameters of all sectors have to be jointly determined. In other words, we use the following set of 3×12 moment conditions to calibrate the parameter set $(\overline{\lambda}^i, \underline{\lambda}^i, \eta^i)$, $i = 1, 2, \ldots 12$:

$$\mathbb{E}\left[\mathbb{P}\left[x^i_{t+\tau} = \underline{x}^i \middle| \mathcal{F}^{x,Y}_t, x^i_t = \overline{x}^i\right]\right] - d^i_\tau \qquad \tau = 1y, 3y, 10y \quad i = 1, 2, \ldots 12 \tag{B.2}$$

d^i_τ denotes the historical default rate on sector's i issued debt over an holding period of τ years. The next table reports these empirical default rates for all sectors.

Insert Table 4 about here

We use a simple simulation method to compute numerically the unconditional expectations appearing in (B.2). We need to compute

$$\mathbb{E}\left[\mathbb{P}\left[x^i_{t+\tau} = \underline{x}^i \middle| \mathcal{F}^{x,Y}_t, x^i_t = \overline{x}^i\right]\right] = \int_0^1 \mathbb{P}\left[x^i_{t+\tau} = \underline{x}^i \middle| \mathcal{F}^{x,Y}_t, x^i_t = \overline{x}^i\right] d\pi(p^h) \tag{B.3}$$

where $\pi(p^h)$ is the stationary distribution of the posterior probability of an 'high' state, and $\mathbb{P}\left[x^i_T = \underline{x}^i \middle| \mathcal{F}^{x,Y}_t, x^i_t = \overline{x}^i\right]$ is computed from Lemma A.2 *i*). We simulate M T-quarters trajectories of dividend realizations and posterior beliefs under the observation filtration $\mathcal{F}^{x,Y}_t$, initializing each dividend path at the empirically observed initial dividend of the sample and initializing each belief path at the steady-state expected posterior belief, $k_l/(k_h + k_l)$. We simulate log dividends and posterior beliefs using and Euler discretization scheme for jump-diffusions,[23] discretizing over a daily grid and then sampling quarterly. We then use the following approximation:

$$\mathbb{E}\left[\mathbb{P}\left[x^i_{t+\tau}=\underline{x}^i \middle| \mathcal{F}^{x,Y}_t, x^i_t=\overline{x}^i\right]\right] \approx \frac{1}{T}\sum_{t=1}^{T}\frac{1}{M}\sum_{s=1}^{M}\mathbb{P}\left[x^i_{t+\tau}=\underline{x}^i \middle| \mathcal{F}^{x,Y}_t, x^i_{t,s}=\overline{x}^i\right] \tag{B.4}$$

where $x^i_{t,s}$ denotes the realization at time t and along the $s-$th path of the $i-$th sector persistent component. $\mathbb{P}\left[x^i_{t+\tau}=\underline{x}^i \middle| \mathcal{F}^{x,Y}_t, x^i_{t,s}=\overline{x}^i\right]$ is computed using Lemma A.2 *i*) using as posterior probability of an 'high' state the simulated realization at time t across the $s-$th path. Summarizing, parameters $\overline{\lambda}^i$, $\underline{\lambda}^i$, η^i are calibrated using an exactly identified method $(\overline{\lambda}^i, \underline{\lambda}^i, \eta^i)$, $i=1,2,\ldots,12$ of simulated moment procedure, with the following moment conditions:[24]

$$\frac{1}{T}\sum_{t=1}^{T}\frac{1}{M}\sum_{s=1}^{M}\mathbb{P}\left[x^i_{t+\tau}=\underline{x}^i \middle| \mathcal{F}^{x,Y}_t, x^i_{t,s}=\overline{x}^i\right]-d^i_\tau \qquad i=1,2,\ldots N \quad \tau=1,3,10.$$

This system of nonlinear equations is solved numerically with a Newton-algorithm.

4) *Calibration of θ_4, taking θ_1, θ_2, and θ_3 as given.* We find the dividend loss upon distress for each sector, $\overline{x}^i/\underline{x}^i$, matching model-implied to historical expected dividend growths, given the intesities of distress and recovery calibrated at the previous step. To compute the unconditional model-implied dividend growth, we apply the same numerical procedure outlined above. Let $\widetilde{d}^i_\tau$ denote the logarithmic expected unconditional quarterly ($\tau = 1\,quarter$) dividend growth for the $i-$th industry portfolio:

$$\widetilde{d}^i_\tau = \int_0^1 \mathbb{E}\left[\log D^i_{t+\tau}-\log D^i_t \middle| \mathcal{F}^{x,Y}_t\right] d\pi(p^h) \tag{B.5}$$

where $\pi(p^h)$ is the stationary distribution of the posterior probability of an 'high' state, $\mathbb{E}\left[\log D^i_{t+\tau}-\log D^i_t \middle| \mathcal{F}^{x,Y}_t\right]$ is the posterior expected log dividend growth. We then approximate $\widetilde{d}^i_\tau$ as follows:

$$\widetilde{d}^i_\tau \approx \frac{1}{T}\sum_{t=1}^{T}\frac{1}{M}\sum_{s=1}^{M}(\log \widehat{D}^{i,s}_{t+\tau}-\log \widehat{D}^{i,s}_t) \tag{B.6}$$

where $\widehat{D}^{i,s}_t$ denotes the simulated realization for the $i-$th industry dividend at quarter t and along the $s-$th path. Note that under the observation filtration $\mathcal{F}^{x,Y}_t$ the log dividend follows the dynamics:

$$d\log D^i_t = (\mu_Y - \frac{1}{2}\sigma^2_Y)dt + H^i_{t-}\log(\overline{x}^i/\underline{x}^i)dH^i_t + (1-H^i_{t-})\log(\overline{x}^i/\underline{x}^i)dH^i_t + \sigma_Y dZ_t \tag{B.7}$$

where the posterior event intensity is $(1-H^i_t)\widehat{\lambda}^i_t + H^i_t\widehat{\eta}^i_t$. We calibrate $\overline{x}^i/\underline{x}^i$ by matching (B.6) to the empirical average quarterly dividend growth of each sector, taking as given parameters $(\overline{\lambda}^i, \underline{\lambda}^i, \eta^i)$, $i=1,2,\ldots,12$, calibrated at the previous step. We then retrieve individual parameters $\overline{x}^i/\underline{x}^i$ by matching the empirical average sectors' dividend levels. More specifically, since the common dividend component Y_t is unobservable, we apply without loss of generality the normalization $Y_0 = 1$.

5) *Calibration of γ, taking all the rest of parameters as given.* Finally we obtain the relative risk aversion parameter γ that minimizes the mean squared error between the unconditional expected theoretical p/d ratio of the equally weighted market portfolio and the sample average of this quantity. To compute the unconditional expected p/d ratio we use the closed-form solution for the conditional p/d ratio and we apply the familiar procedure implemented at steps 3 and 4.

[23] see Glasserman (2004)

[24] Indeed a Simulated Maximum Likelihood estimation approach would provide a more rigorous assessment of the goodness of fit of the model. We have chosen this simpler calibration strategy for reasons related to the computational complexity involved in the SML procedure.

Appendix C: When $N \to \infty$

This Appendix is devoted to a discussion about the case in which the economy is populated by infinitely many trees. For the aggregate endowment C_t to be finite, the consumption share of each firm/sector must become arbitrarily small, which implies $x_s^i \to 0$, both in distress and normal state. A distress (recovery) might still be considered an 'event' at the individual tree level, but not at the aggregate level, because the consumption fall (rise) upon distress (recovery) is a vanishing fraction of the aggregate. There is probability one that some sector/firm experiences a distress at each small time interval ds, and since the instantaneous aggregate dividend growth has an order of magnitude of $\sim k_N\sqrt{ds}$, where $\lim_{N\to\infty} k_N = 0$, the full information return innovation of the $i-$th equity asset when the $j-$th tree falls in distress has the order of magnitude of a return in response to a Brownian dividend shock. As a consequence, the equity premium component deriving from a potential distress of tree j is $\approx -\gamma h_N \lambda^i\theta^i$, where $\lim_{N\to\infty} h_N = 0$.

The market price of event risk for tree i converges to zero, in accordance with the intuition that the covariance of aggregate consumption and individual dividend shocks is arbitrarily small. Accordingly, the risk neutral distress intensity converges to the objective intensity, $\lambda^i\theta^i \to \lambda^i$. The vanishing dividend growth upon distress of tree j implies that the $i-$th sector premium for this event scales linearly in risk aversion. The partial information premium has the same order of magnitude. There is now an infinite number of imperfectly correlated consumption share processes, each being a priced source of risk. The risk premium then comprises compensation for an infinite number of potential sources of distress:

$$-Cov[dU(C)'/U'(C), dR^i] = \lim_{N\to\infty} \sum_{i=1}^{N} -\gamma h_N^i \widehat{\lambda}_t^i \theta^i \tag{C.1}$$

The ability of the model to cope with the equity premium puzzle may be compromised, as apparent from expression (C.1), because of the small variation in the marginal utility of consumption upon distress and the fact that the agent expects at most one distress of small amplitude, relative to aggregate consumption, during the next time instant ds.[25] Note that with an infinite number of firms, a Brownian model for dividends is more immune to this issue, because infinitely many small consumption-share shocks take place contemporaneously at each time instant, while in the limiting case of our model at most one shock occurs. When the number of dividend supplying trees is infinite and only events of negligible aggregate importance take place, our modeling framework becomes inappropriate, because in this case the notion of 'event' would require a set-up that allows for contemporaneous individual distress and recovery events. For instance, as the number of trees increases, it is likely that their cash-flows group-wise display similar dependence on economic fundamentals, and differ within groups because of idiosyncratic factors. In other words, the events pertaining tree j of group i, $dH_t^{i,j}$, may be decomposed into group-wise (systematic) events and element-wise (idiosyncratic) events:

$$dH_t^{i,j} = dH_t^i + dH_t^{e,j}$$

The intensity of systematic events, $(1-H_t^i)(1-H_t^{e,j})\lambda_t^i + [H_t^{e,j}(1-H_t^i) + H_t^i(1-H_t^{e,j})]\eta_t^i$, is the only one that depends on the latent systematic factor. As firms belonging to a given industrial sector, each tree's distress can be either a group (systematic) or an elemental (idiosyncratic) distress. We don't explore the details of this framework, but we note that the case of arbitrarily many trees can be accommodated assuming a sufficiently high, but fixed number of groups, whose consumption share remains finite as $N = \#groups \times \#(trees\, for\, each\, group) \to \infty$, while the consumption share of individual trees vanishes. The distress and recovery 'events' to which we refer in the paper are the events that are common to all elements of a group, systematic events, while the impact of idiosyncratic events is asymptotically negligible.

To summarize: *i)* To any practical means the number of sectors/firms of the economies to which our model can be applied is finite, and at least some of those provide a nonnegligible share of aggregate output. This is all we require for the predictions of the paper to have practical relevance. *ii)* In the theoretical case of an infinite number of trees, it is unreasonable to assume that each divided process displays a different dependence of market fundamentals. Allowing for a finite number of asset classes with common exposition to fundamentals and defining distress events as those common to the trees/firms of each class, implies that a nonnegligible consumption share is exposed to event risk at each time-instant, regardless of the number of trees.

[25] Even though the point processes (Markov chains) regulating individual sectors are correlated, we are assuming that their sum – which results in the point processes that regulates aggregate consumption – converges to a Poisson point process, for which the instantaneous probability of two or more events is zero.

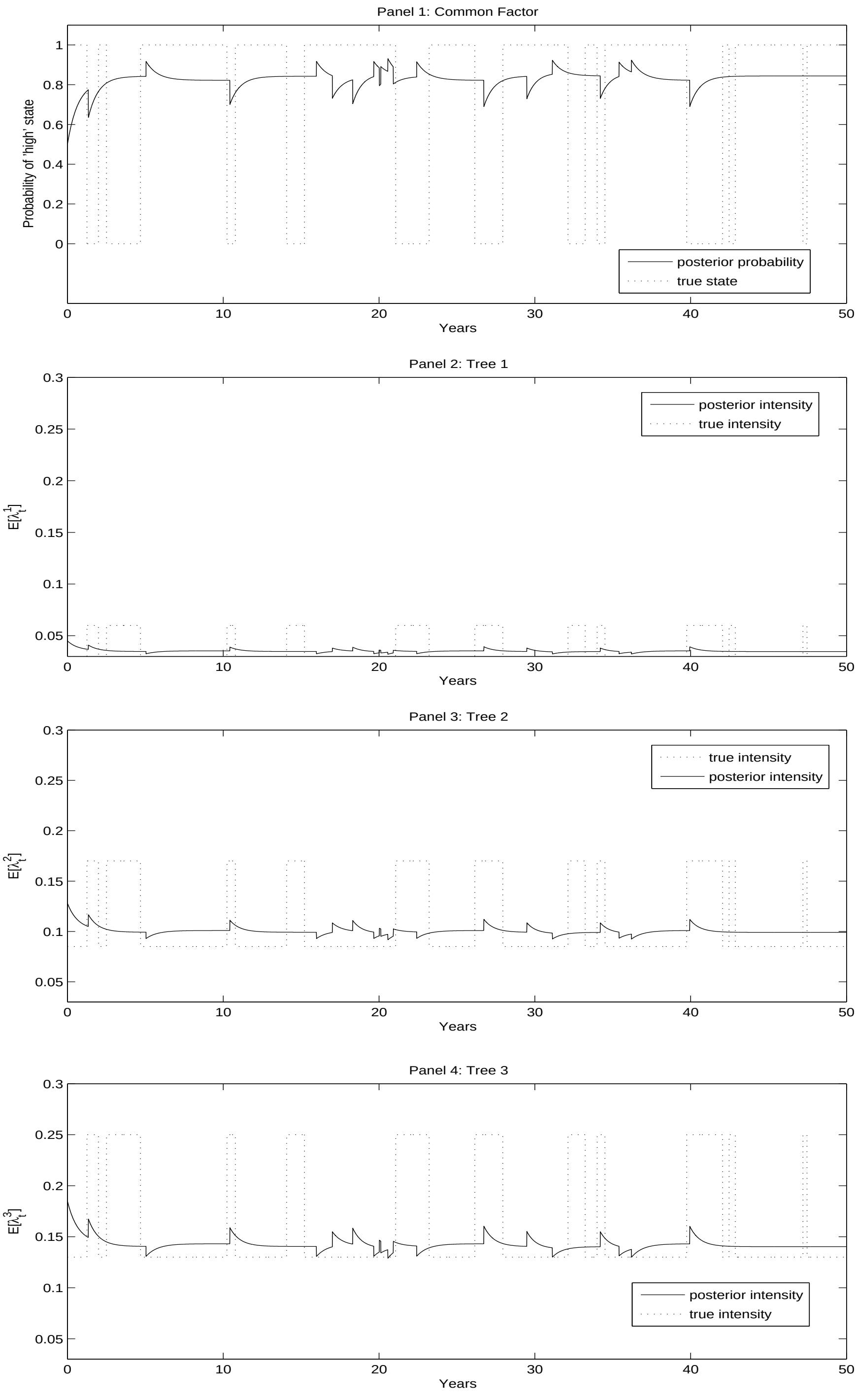


Fig. 1.– Simulated Posterior Intensities: Case of High Estimation Error. This Figure considers an economy populated by three trees, where the learning mechanism is not effective at estimating the true state of the common factor: distress and recovery events are scarcely informative signals, because trees' intensities comove weakly with the common factor, that is, they are similar across its states. As a result, the standard errors of the Bayesian estimates of the true state and true intensities are large. The parameter configuration is: $\overline{\lambda} = (0.03, 0.08, 0.12)$, $\underline{\lambda} = (0.06, 0.15, 0.23)$, $\overline{\eta} = (0.10, 0.18, 0.30)$, $\underline{\eta} = (0.06, 0.12, 0.2)$. Transition intensities from the 'high' to the 'low' state of the common factor, and conversely, have been set to $k_h = 0.24$ and $k_l = 1.31$. *Panel 1* shows a simulated trajectory of the posterior probability of 'high' state (solid line) arising in this economy, together with the unobservable true state of the common factor (dotted line). 0 corresponds to a 'low' state and 1 corresponds to a 'high' state. *Panels 2, 3, and 4* show the corresponding posterior intensities of distress for tree 1, 2, and 3, respectively (solid lines), together with the true, unobservable intensities (dotted lines).

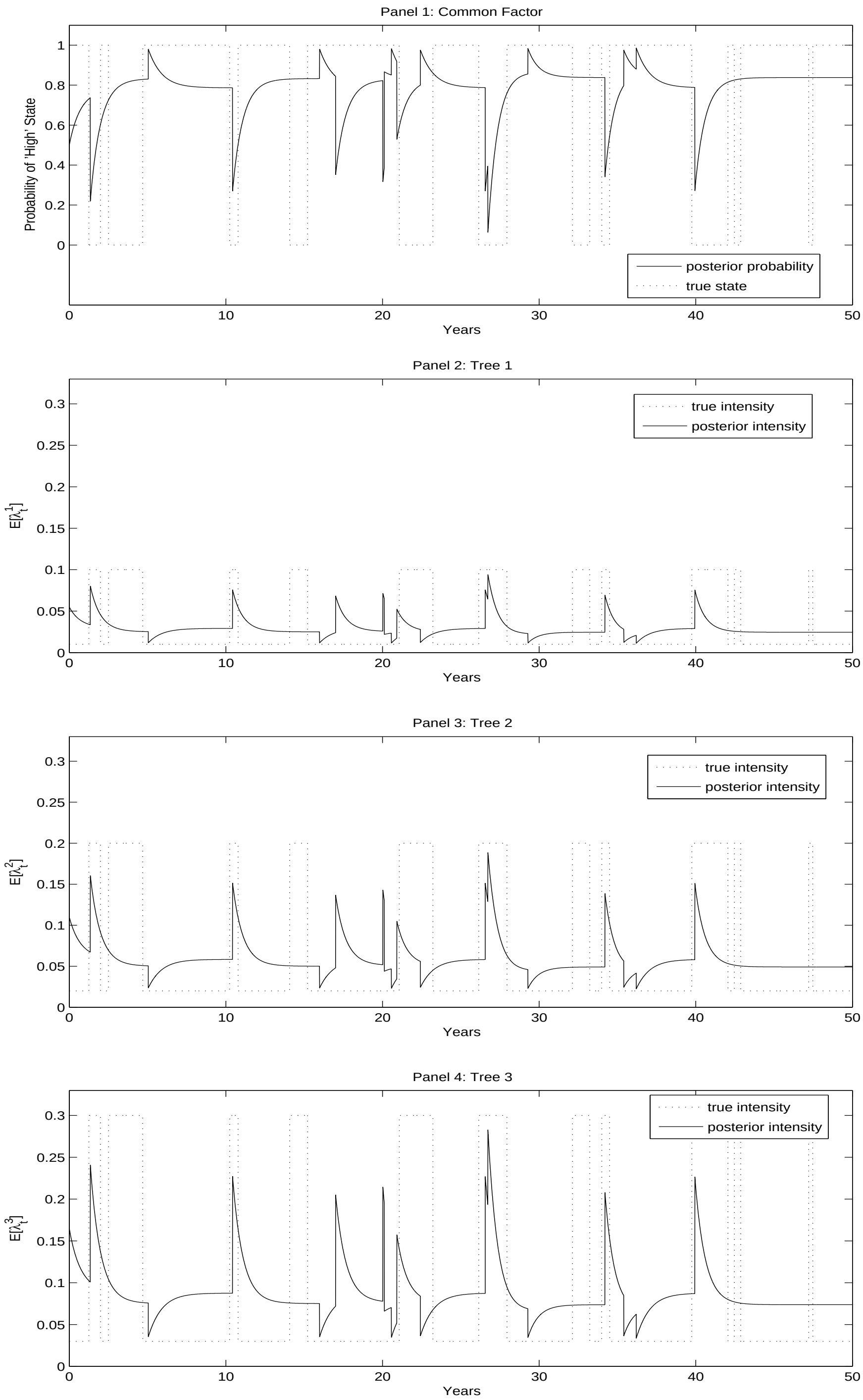


Fig. 2.– Simulated Posterior Intensities: Case of Low Estimation Error. This Figure considers an economy populated by three trees, where the learning mechanism is effective at estimating the true state of the common factor: distress and recovery events are highly precise signals, because trees' intensities strongly comove with the common factor, that is, they are very different across its states. As a result, the standard errors of the Bayesian estimates of the true state and true intensities are low. The parameter configuration is: $\overline{\lambda} = (0.01, 0.02, 0.03)$, $\underline{\lambda} = (0.10, 0.20, 0.30)$, $\overline{\eta} = (0.30, 0.25, 0.21)$, $\underline{\eta} = (0.03, 0.025, 0.021)$.. Transition intensities from the 'high' to the 'low' state of the common factor, and conversely, have been set to $k_h = 0.24$ and $k_l = 1.31$. *Panel 1* shows a simulated trajectory of the posterior probability of 'high' state (solid line) arising in this economy, together with the unobservable true state of the common factor (dotted line). 0 corresponds to a 'low' state and 1 corresponds to a 'high' state. *Panels 2, 3, and 4,* show the corresponding posterior intensities of distress three 1, 2, and 3, respectively (solid lines), together with the true, unobservable intensities (dotted lines).

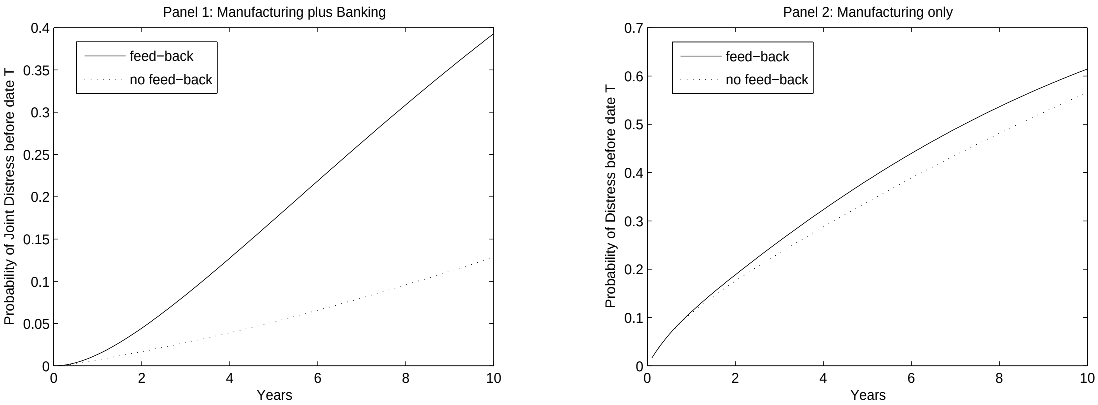


Fig. 3.– Probability of (Joint) Distress Before a Given Date. This Figure considers the three-sectors economy depicted in (5), where the Housing, Banking, and Manufacturing sectors, respectively, have the following intensities $\overline{\lambda} = (0.02, 0.01, 0.03)$, $\underline{\lambda} = (0.20, 0.10, 0.30)$, $\overline{\eta} = (0.25, 0.30, 0.21)$, $\underline{\eta} = (0.025, 0.030, 0.021)$. *Panel 1* reports the term structure of the probability that both the Manufacturing and the Banking sector experience a distress before date T. Date T is reported in years on the x axis. The solid line corresponds to the case with feed-back effects (an *asymmetric* network), where instantaneous probabilities of transition from the 'high' to 'low' state of the common factor, and conversely, have been set as in (6), with $\overline{k}_h = 0.24$, $\overline{k}_l = 1.31$ $a_1 = 2$, $a_2 = 5$, $b_1 = 0.2$, and $b_2 = 0.5$. The dotted line shows the probability obtained in the no-feed-back case, that is, setting $a_1 = a_2 = b_1 = b_2 = 0$ (*symmetric* network). In the same modeling context, *Panel 2* reports the marginal probability of distress before T for the Manufacturing sector.

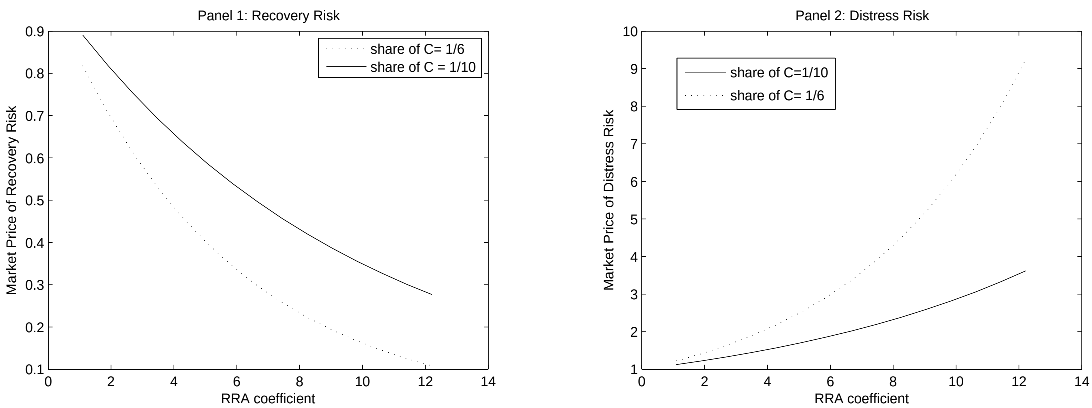


Fig. 4.– Market Price of Event Risk. This Figure considers a three-sectors economy. Assuming that one of the trees is in distress, *Panel 1* reports its equilibrium market price of *recovery risk*, plotted as a function of the Relative Risk Aversion coefficient, when this tree supplies $1/6\,th$ of the aggregate consumption (dashed line) and when it supplies $1/10\,th$ (solid line). All remaining trees are not experiencing a distress. In *Panel 2* the tree is not in distress, and its equilibrium market price of *distress* risk is plotted, everything else being as in Panel 1.

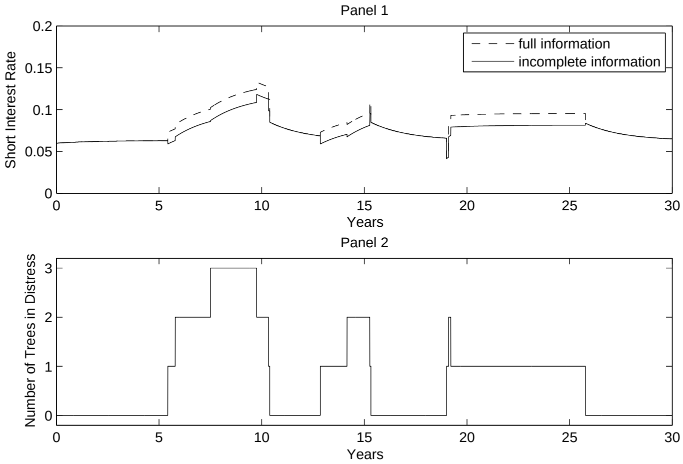


Fig. 5.– Behavior of Equilibrium Interest Rate. This Figure considers an economy with three trees, supplying the same dividend process both in distress and non in distress, that is $\overline{x}^i = \underline{x}^j$, $i, j = 1, 2, 3$, $i \neq j$. Trees' distress and recovery intensities are: $\overline{\lambda} = (0.02, 0.01, 0.03)$, $\underline{\lambda} = (0.20, 0.10, 0.30)$, $\overline{\eta} = (0.25, 0.30, 0.21)$, $\underline{\eta} = (0.025, 0.030, 0.021)$. *Panel 1*: The solid line reports a simulated trajectory of the equilibrium interest rate with incomplete information and learning, while the dashed line reports the corresponding path that would arise with full-information. *Panel 2*: Total number of trees in distress for the same simulated history of Panel 1.

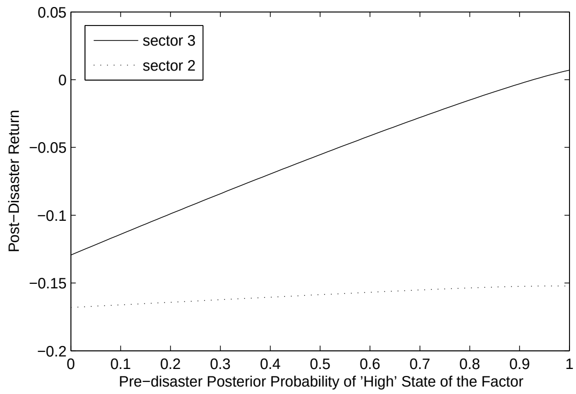


Fig. 6.– Post-Distress Return and Incomplete Information. This Figure considers a symmetric 3-sectors economy in which trees differ only in terms of their distress intensity. Sector 1 has the highest distress intensity and sector 3 the lowest: $\overline{\lambda} = (0.1, 0.01, 0.003)$, $\underline{\lambda} = (0.6, 0.1, 0.003)$, $\overline{\eta} = \underline{\eta} = (0.25, 0.25, 0.25)$. Cash flow shocks are symmetric both in distress and normalcy: $\overline{x} = (1, 1, 1)$, $\underline{x} = (0.6, 0.6, 0.6)$. Transition intensities from the 'high' to the 'low' state, and conversely, are, respectively, $k_h = 0.24$, $k_l = 1.31$. We consider a dividend shock to sector 1 and compute its impact on sector 2 and 3. The solid line shows the impulse response (post-distress return) of sector 3's equity to the observation of a negative dividend shock (distress) of sector 1, plotted as a function of the ex-ante (i.e. pre-update) posterior probability of 'high' state. The dotted line shows the impulse response of Sector 2.

TABLE 1

Comparison of models' predictions

Article	Lucas (1978)	Veronesi (2000)	Santos and Veronesi (2009)	Buraschi, Porchia, and Trojani (2010)
One Tree vs Orchard	One Tree	One Tree	Orchard	Orchard
Full vs Incomplete Info and Learning	Full Info	Inc Info and Learning	Full Info	Inc Info and Learning
1) Response of Interest Rate to a Distress	+	−		+ or −
2) Response of Price-Dividend Ratio to a Distress	−	+		$\begin{cases} - \text{ if } (27) \\ + \text{ if not } (27) \end{cases}$
3) Risk Premium for Event Risk	+	+ or −		$\begin{cases} + \text{ if } (27) \\ - \text{ if not } (27) \end{cases}$
4) Value Premium: $\frac{\Delta\, E[R]}{\Delta \text{ p/d ratio}} < 0$			$\begin{cases} \text{homog. cash-flow risk: NO} \\ \text{heterog. cash-flow risk: YES} \end{cases}$	$\begin{cases} \text{FI: } \begin{cases} \text{YES if } \gamma < \gamma^* \\ \text{NO if } \gamma > \gamma^* \end{cases} \\ \text{II: YES} \end{cases}$

Note.– This table summarizes the predictions of our model in terms of the behavior of the interest rate, the price-dividend ratio, the risk premium and the cross-sectional dispersion of returns, with respect to the predictions for the same quantities in related literature. We consider the following models: *i*) a one-tree Lucas (1978) version of our model with full information, *ii*) the event-risk version of Veronesi (2000), with one tree subject to distress events in the form of a Poisson jump shocks with time varying and unobservable intensity. In this extension, the distress state is temporary differently than in our model; *iii*) Santos and Veronesi (2009), based on a diffusion process specification of an orchard economy.
The Table first analyzes the response of the interest rate to a distress, '+' denotes an increase, while'−' denotes a decrease. It then analyzes the response price-dividend ratio of the market portfolio to a distress. We report for convenience condition (27): $p_t^h(1-p_t^h)\frac{\overline{\lambda}^j-\underline{\lambda}^j}{\widehat{\lambda}_t^j}(\overline{P}^i(Y_t,x_t-j)-\underline{P}^i(Y_t,x_t-j)) < p_t^h\overline{\Delta P}+(1-p_t^h)\underline{\Delta P}$, where $\overline{\Delta P}=\overline{P}^i(Y_t,x_t)-\overline{P}^i(Y_t,x_t+j)$, and $\underline{\Delta P}=\underline{P}_t^i(Y_t,x_t)-\underline{P}_t^i(Y_t,x_t+j)$. The Table then considers the theoretical sign of market risk premia for event risk, for the same level of risk aversion. Finally, it deals with models' prediction concerning the relation between price-dividend ratios and risk premia. If this relation is negative, the model is consistent with the 'value premium' empirical regularity. In Santos and Veronesi (2009), homogeneous cash-flow risk means that all sectors' dividends have the same covariance with aggregate consumption. FI (II) means full (incomplete) information. We report γ^* from Proposition 5: $\gamma^* : \min_i \left[\frac{\overline{x}^i+\sum_{u\neq i} x_{t-}^u}{\overline{x}^i}\right] \quad i=1,2,\ldots,N.$

TABLE 2

CALIBRATED PARAMETER VALUES AND IMPLIED EVENT RISK INDICATORS

	CALIBRATED PARAMETER VALUES					IMPLIED INDICATORS	
INDUSTRY	$\overline{\lambda}$	$\underline{\lambda}$	η	$\overline{x}$	$\underline{x}$	$\overline{\tau}$	$FTD\%$
Automotive	0.0197	0.0602	0.2102	5609.90	4976.90	4.72	11.68
Chemical	0.0085	0.0602	0.1699	3266.70	2507.10	5.07	7.770
Construction and Constr. Materials	0.0101	0.0803	0.1227	2019.70	1790.50	6.31	11.54
Consumer Durables	0.0124	0.0553	0.1777	2024.50	1678.00	5.06	9.260
Fabricated Products	0.0206	0.0602	0.1631	326.700	306.000	6.22	15.11
Financials	0.0005	0.0201	0.1120	27674.9	8986.30	6.75	2.500
Machinery and Business Equipment	0.0039	0.0602	0.1832	8801.90	6114.50	5.42	5.910
Oil and Petr. Products	0.0082	0.0302	0.1575	10339.7	7916.30	6.55	6.820
Retail Stores	0.0213	0.0502	0.2128	3953.50	3299.70	4.67	10.32
Transportation	0.0100	0.0397	0.2144	3475.80	2607.80	4.44	6.400
Utilities	0.0001	0.0151	0.2873	8633.60	1131.10	4.24	0.930
Other	0.0254	0.0631	0.1575	29693.3	25367.4	6.58	16.79
COMMON PARAMETERS	$k_h = 0.2418$	$k_l = 1.3109$	$\mu_Y = 0.023$	$\sigma_Y = 0.03$	$\gamma = 3.59$		

NOTE.– The table reports parameter values for the empirical application of Section VI, obtained according to the calibration procedure described in Appendix B. The sample comprises quarterly dividend distributions on 12 US Industry portfolios, from 1947 to 2007, and historical default rates on corporate debt grouped by Industry, as published by *Moody's* and *Standard & Poor's*. $\overline{\lambda}$ and $\underline{\lambda}$ denote the sector's intensities of distress in the 'high' and in the 'low' state, respectively, while $\eta = \overline{\eta} = \underline{\eta}$ denotes the (state independent, for simplicity) intensity of recovery. $\overline{x}$ ($\underline{x}$) is the persistent dividend component in the 'high' ('low') state. k_h (k_l) is the transition intensity to the 'low' ('high') state of the common factor. μ_Y and σ_Y denote the mean and volatility of the diffusive dividend growth component. The last two columns report, for each sector, the unconditional expected duration of each distress event ($\overline{\tau}$) and the percentage number of years over 100 spent in distress in the long-run (FTD, Fraction of Time in Distress).

TABLE 3

SECTOR'S CHARACTERISTICS IN THE CALIBRATED ASYMMETRIC NETWORK

INDUSTRY	a	$\overline{ex}^i$ *symmetric*	$\overline{ex}^i$ *asymmetric*
Automotive	0.0417	-0.0921	-0.1041
Chemicals	0.0750	-0.0071	-0.0310
Construction and Constr. Materials	0.0330	-0.1030	-0.1104
Consumer Durables	0.0583	-0.0670	-0.0720
Fabricated Products	0.0250	-0.1156	-0.1244
Financials	0.0917	0.0331	0.0390
Machinery and Business Equipmnt.	0.0667	-0.0450	-0.0410
Oil and Petr. Products	0.0833	0.0173	0.0210
Retail Stores	0.0167	-0.1513	-0.1721
Transportation	0.050	-0.0741	-0.0761
Utilities	0.1000	0.0651	0.0813
Other	0.0083	-0.1820	-0.2309

NOTE.– Starting from the calibrated parameters of the symmetric network, we have created an asymmetric network introducing feed-back effects between sectors' distress events and the state of the common factor:

$$k_h = 0.2418 \prod_{i=1}^{12} (1 + a_i \mathbf{1}(x_t^i = \underline{x}^i)) \qquad k_l = 1.3109 / \prod_{i=1}^{12} (1 + a_i \mathbf{1}(x_t^i = \underline{x}^i))$$

We are not considering direct feed-backs among sectors. The rest of the parameters are those of the symmetric network, reported in Table 2. For each sector, the Table reports the feed-back coefficients a_i that we have used, and the sector's average exogeneity $\overline{ex}^i$ (over a 30-years horizon), as defined in Definition 3, both in the asymmetric network and in the base symmetric case (corresponding to $a^i = 0$, $i = 1, 2, \ldots, 12$).

TABLE 4

AVERAGE $\tau-$YEARS CUMULATIVE DEFAULT RATES BY INDUSTRY GROUP

INDUSTRY	d_{1y}	d_{3y}	d_{10y}
Automotive	2.78	8.12	19.25
Chemical	1.73	6.05	15.41
Construction and Constr. Materials	2.83	8.95	19.23
Consumer Durables	2.54	7.70	17.63
Fabricated Products	2.13	6.78	17.72
Financials	0.53	1.69	4.74
Machinery and Business Equipmnt.	1.87	5.43	10.11
Oil and Petr. Products	1.54	4.32	9.53
Retail Stores	2.80	8.43	19.79
Transportation	2.28	5.95	13.49
Utilities	0.17	0.50	1.50
Other	3.83	9.10	19.50

NOTE.– The table reports percentage default rates (d_τ) per industry group for bonds' holding periods of 1, 3 and 10 years. This table is adapted from Exhibit 38 of Moody's *Corporate Default and Recovery Rates, 1920-2009* and from Table 19 of S&P's *Default, Transition, and Recovery: 2009 Annual Global Corporate Default Study And Rating Transitions*.

TABLE 5

RESULTS OF FAMA-MCBETH REGRESSIONS FOR P/D RATIOS

PANEL 1: INCOMPLETE INFORMATION

α	β	se(α)	se(β)	R^2
63.66	0.494	45.12	0.195	0.2412

PANEL 2: FULL INFORMATION

α	β	se(α)	se(β)	R^2
51.66	0.362	43.13	0.165	0.155

NOTE.– We consider the linear model

$$\frac{P_s^i}{D_s^i} = \alpha^i + \beta_i \left(\frac{P_s^i}{D_s^i}\right)^* + \varepsilon_s^i \quad i = 1, 2, \dots, 12,$$

that regresses observed price-dividend ratios onto model-implied ones. To address the presence of the unobservable components p_t^h and x_t, we simulate 2500 sample paths of these quantities. For each sample, at each point in time we estimate the coefficients of the cross-sectional regression. Coefficients reported are time-series averages and then averages across simulated samples. Their standard errors are Newey-West adjusted for autocorrelation and heteroskedasticity. In *Panel 1* model-implied price-dividend ratios are obtained under incomplete information, while in *Panel 2* they are obtained assuming full information.

TABLE 6

RESULTS OF FAMA-MCBETH REGRESSIONS FOR RISK PREMIA

PANEL 1: INCOMPLETE INFORMATION

α	ω	se(α)	se(ω)	R^2	Var[*Cons. Beta*]/Var[R^e]
0.0092	0.344	0.013	0.129	0.1962	0.2143

PANEL 2: FULL INFORMATION

α	ω	se(α)	se(ω)	R^2	Var[*Cons. Beta*]/Var[R^e]
0.0083	0.263	0.015	0.553	0.062	0.084

NOTE.– We consider the linear model

$$R_s^{i,e} = \alpha^i + \omega^i \frac{-\mathrm{Cov}\left(\frac{U'(C_s)}{U'(C_{s-1})}, \frac{dP_s^i}{P_s^i}\right)}{\mathbb{E}\left[\frac{U'(C_s)}{U'(C_{s-1})}\right]} + \varepsilon_s^i \quad i = 1, 2, \dots, 12$$

that regresses observed returns on the industry portfolios onto consumption betas implied by our model, that is, the quarterly equity premia that would be observed if the model held true.To address the presence of the unobservable components p_t^h and x_t, we simulate 2500 sample paths of these quantities. For each sample, at each point in time we estimate the coefficients of the cross-sectional regression. Coefficients reported are time-series averages and then averages across simulated samples. Their standard errors are Newey-West adjusted for autocorrelation and heteroskedasticity. *Panel 1* reports results arising when consumption betas are generated by our model with incomplete information. Results in *Panel 2* are obtained using theoretical consumption betas under full-information.

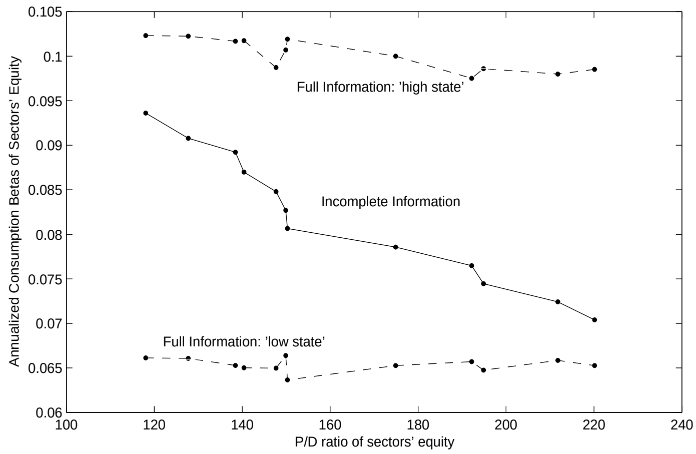


FIG. 7.– CROSS-SECTION OF CONSUMPTION BETAS. Model implied consumption betas (theoretical risk premia, $-\mathrm{Cov}\left(\frac{dU'(C_s)}{U'(C_s)}, \frac{dP_s^i}{P_s^i}\right)/\mathbb{E}\left[\frac{dU'(C_s)}{U'(C_s)}\right]$) as a function of price-dividend ratios for the 12-sectors economy in our empirical investigation. Calibrated model parameters used are reported in Table 2. The solid line plots partial information consumption betas, while dashed lines plot full-information consumption betas conditional on the 'low' and the 'high' state of the latent economic factor. Figures reported are conditional to no sector being in distress. The posterior probability of high economic state is fixed at its steady-state level.

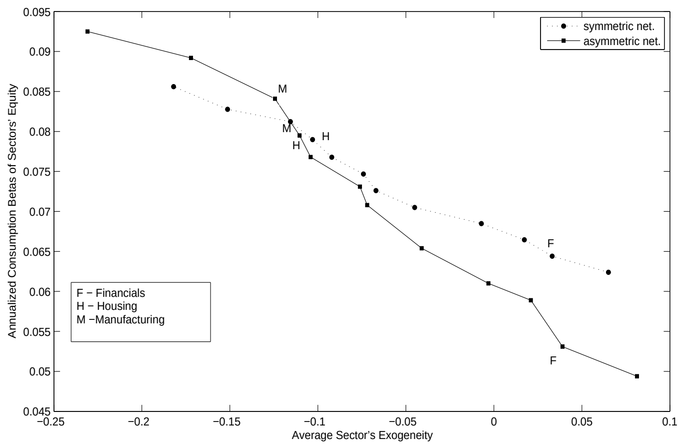


FIG. 8.– CROSS-SECTION OF CONSUMPTION BETAS IN THE ASYMMETRIC NETWORK. Model implied consumption betas (theoretical risk premia, $-\mathrm{Cov}\left(\frac{dU'(C_s)}{U'(C_s)}, \frac{dP_s^i}{P_s^i}\right)/\mathbb{E}\left[\frac{dU'(C_s)}{U'(C_s)}\right]$) as a function of the average exogeneity $\overline{ex}^i$ (over a 30-years horizon) for the 12-sectors economy in our empirical investigation. The dotted line corresponds to a symmetric network connectivity structure, while the solid line corresponds to an asymmetric network structure. Calibrated model parameters and sectors' exogeneity measures are reported in Table 2 and Table 3, respectively. Reported figures are conditional to no sector being in distress. The posterior probability of high economic state is fixed at its steady-state level.

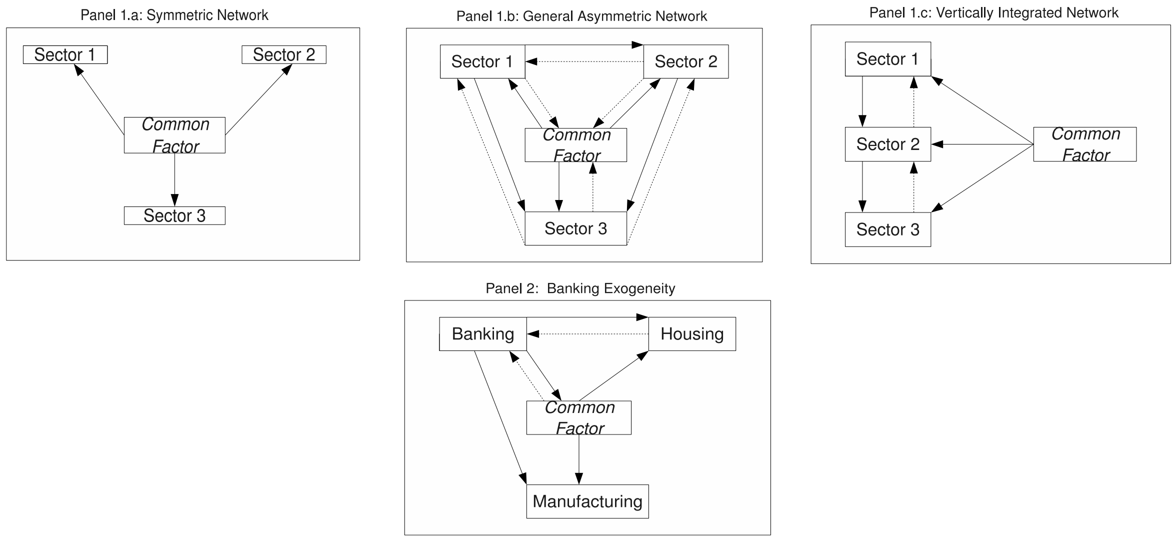


FIG. 9.– NETWORK STRUCTURES IN THE ORCHARD. In our paper sectors influence the regime switching probabilities of the latent common factor with their distress and recovery events, and their intensities of distress and recovery are in turn influenced by the common factor. In addition, distress and recovery intensities of some trees are directly influenced by events of other trees. In *Panel 1* we consider a 3-sectors economy and we report three examples of structures in the orchard, each one characterized by different levels of propagation of shocks to the cross-section. *Panel 1.a* is the *symmetric structure*. Sectors are subject to the influence of the exogenous common factor, but their dividend shocks do not have a feed-back on the latter, nor influence directly the intensities of remaining sector. The parameterization which gives rise to this case is: $\overline{\lambda} = const.$, $\underline{\lambda} = const.$, $\overline{\eta} = const.$, $\underline{\eta} = const.$, $k_u = const$, $u = h, l$. *Panel 1.b* reports the general case of an *asymmetric structure*, where all connecting layers are activated. Dotted arrows and solid arrows are meant to underline that the impact of the $i-$th sector's shocks (events) on the characteristics of the common factor or of the $j-$th sector is in general different from the impact of the common factor's or Sector $j-$th's shocks on Sector i. Parameters that generate this structure are: $\overline{\lambda} = f_h(x_t^1, x_t^2, x_t^3)$, $\underline{\lambda} = f_l(x_t^1, x_t^2, x_t^3)$, $\overline{\eta} = g_h(x_t^1, x_t^2, x_t^3)$, $\underline{\eta} = g_l(x_t^1, x_t^2, x_t^3)$, $k_u = a_u(x_t^1, x_t^2, x_t^3)$, $\quad u = h, l$, where f_u, g_u and a_u are positive functions. In *Panel 1.c* we report a specific example of asymmetric structure, called 'hierarchical structure', or *vertically integrated structure*. Sectors are subject to the systematic influence of the common factor, although do not affect its dynamics, and their events can only affect 'neighboring' sectors: $\overline{\lambda}^1 = f_h^1(x_t^2)$, $\underline{\lambda}^1 = f_l^1(x_t^2)$, $\overline{\lambda}^2 = f_h^2(x_t^1, x_t^3)$, $\underline{\lambda}^2 = f_l^2(x_t^1, x_t^3)$, $\overline{\lambda}^3 = f_h^3(x_t^2)$, $\underline{\lambda}^3 = f_l^3(x_t^2)$, $k_u = const.$, $u = h, l$. *Panel 2* reports a diagram for the stylized 3 sectors economy that is described and analyzed in Section VI, a structure that gives rise to *exogeneity for the Banking sector*, ad argued in the Section. This economy is a special case of the asymmetric structure in Panel 1.b.

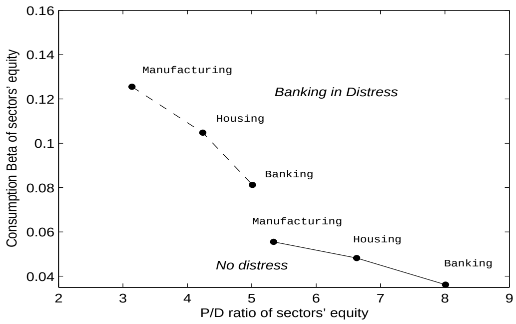
TABLE 7

PARAMETERIZATION FOR THE 3-SECTORS ECONOMY

	'HIGH' STATE	'LOW' STATE
Common factor	$k_h = 0.24(1 + 4 \times \mathbf{1}(x_t^1 = \underline{x}^1))$	$k_l = 1.31/(1 + 4 \times \mathbf{1}(x_t^1 = \underline{x}^1))$
Banking (1)	$\overline{\lambda}^1 = 0.02(1 + b_h \mathbf{1}(x_t^2 = \underline{x}^2))$	$\underline{\lambda}^1 = 0.2(1 + b_l \mathbf{1}(x_t^2 = \underline{x}^2))$
Housing (2)	$\overline{\lambda}^2 = 0.02(1 + a_h \mathbf{1}(x_t^1 = \underline{x}^1))$	$\underline{\lambda}^2 = 0.2(1 + a_l \mathbf{1}(x_t^1 = \underline{x}^1))$
Manufacturing (3)	$\overline{\lambda}^3 = 0.02(1 + a_h \mathbf{1}(x_t^1 = \underline{x}^1))$	$\underline{\lambda}^3 = 0.2(1 + a_l \mathbf{1}(x_t^1 = \underline{x}^1))$

NOTE.– Transition probabilities of the common state, and intensities of distress for each sector of the stylized economy depicted in Panel 2 of Figure 9. Intensities of recovery are constant, with $\overline{\eta} = \underline{\eta} = 0.25$ for all sectors. $\mathbf{1}(\cdot)$ denotes the indicator function of an event. a_u and b_u, $u = h, l$, are positive constants which modulate the direct contagion effects between sectors if the common factor is in the 'high' or in the 'low' state of the world.

Panel 1



Panel 2

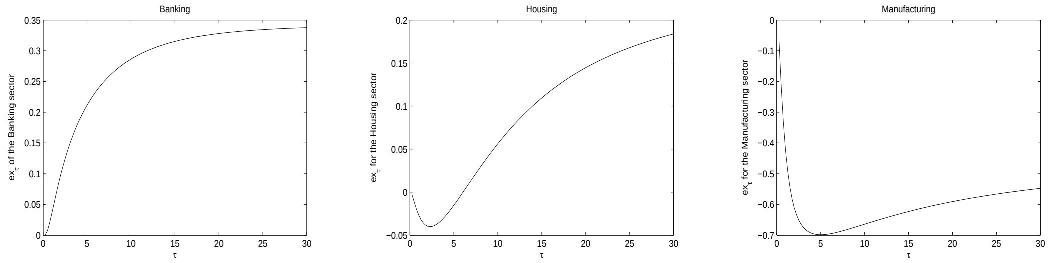
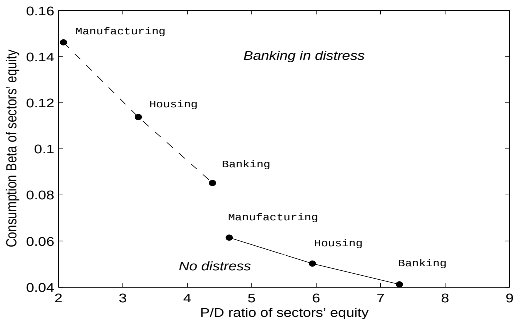


Fig. 10.– Consumption Betas, p/d ratios and sectors' Exogeneity for the Stylized Economy 'Banking Exogeneity'. *Panel 1.* Sector price-dividend ratios and consumption betas of the three-sectors economy of Panel 2 in Figure 9. The solid line connects figures conditional on no sector being in distress, while the dashed lines connects figures conditional on the Banking sector being in distress. Parameters are as in Table 7, with $a_h = a_l = 10$ and $b_h = b_l = 2.5$. *Panel 2* reports the corresponding measures of exogeneity defined in Proposition 3 for the Banking, Housing, and Manufacturing sectors, respectively, plotted as a function of time-horizon.

Panel 1



Panel 2

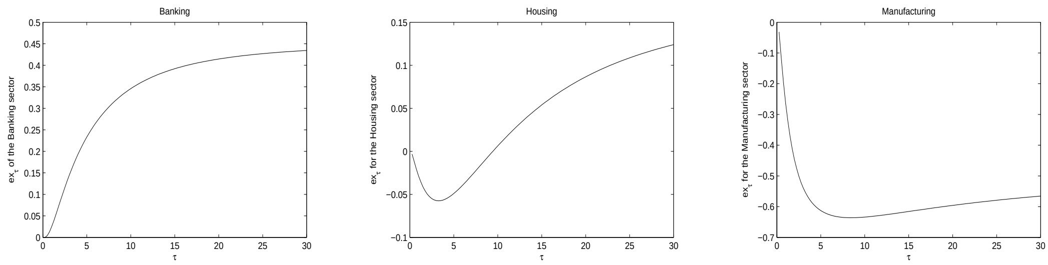


Fig. 11.– Consumption Betas, p/d ratios and sectors' Exogeneity for the Stylized Economy 'Banking Exogeneity': Higher Exogeneity Dispersion. *Panel 1.* Price-dividend ratios and consumption betas for each sector of the three-sectors economy depicted in Panel 2 of Figure 9, the 'Banking Exogeneity' network. The solid line connects figures conditional on no sector being in distress, while the dashed lines connects figures conditional on the Banking sector being in distress. Parameters are as in Table 7, with $a_h = 5$, $a_l = 15$, $b_h = 1.5$, $b_l = 3.5$. This parameterization implies that the Banking (Manufacturing) sector is more exogenous (endogenous) than in the economy of Figure 10. *Panel 2* reports the corresponding measures of exogeneity defined in Proposition 3 for the Banking, Housing, and Manufacturing sectors, respectively, plotted as a function of time-horizon.

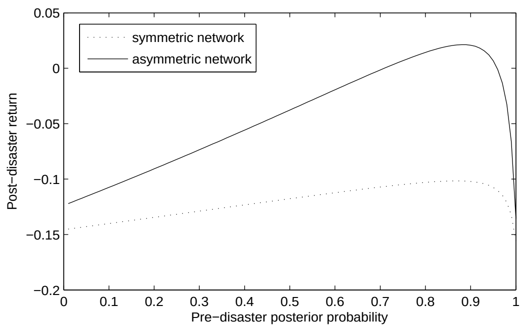


Fig. 12.– Returns Behavior and Learning in the Asymmetric Network. *Panel 1.* Return on the Housing sector in response to a distress of the Banking sector, as a function of the pre-update posterior probability of the 'high' state of the common factor. The characteristics of the sectors that populate the stylized economy are as in Panel 2 of Figure 9. Parameters are as in Table 7. The solid line reports returns arising in the asymmetric network – that is, with feed-back parameters $a_h = 5$, $a_l = 15$, $b_h = 1.5$, $b_l = 3.5$ –, while the dotted line reports returns arising in the symmetric network obtained deactivating all feed-back effects, that is, setting $a_h = a_l = b_h = b_l = 0$, and k_l and k_l constant.

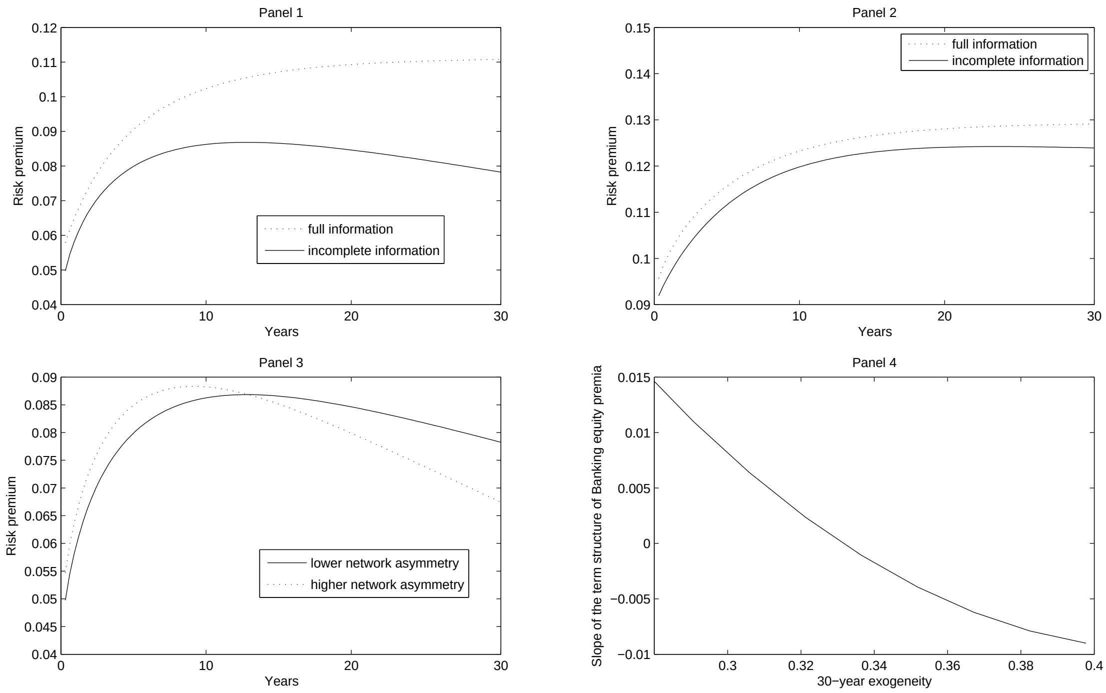


Fig. 13.– Term Structure of Risk Premia. Expected excess returns on the dividend strips for the Banking (*Panel 1*) and Manufacturing sectors (*Panel 2)*, in the 3-sectors economy of Section VI, sketched in Panel 2 of Figure 9 as 'Banking Exogeneity'. Parameters for each sector and the common factor are as in Table 7, with $a_h = a_l = 10$ and $b_h = b_l = 2.5$. Risk premia are plotted as a function of time to maturity of the dividend strip. The solid lines show risk premia in the incomplete information economy, while dotted lines show risk premia in the full-information economy. *Panel 3* reports the term structure of risk premia for the Banking sector when the economy is parameterized as in Table 7 with $a_h = a_l = 10$ and $b_h = b_l = 2.5$ (solid line), and with $a_h = 5$, $a_l = 15$, $b_h = 1.5$, $b_l = 3.5$ (dotted line). *Panel 4* reports the slope of the term structure of the Banking risk premium for different levels of the 30-years Banking exogeneity measure. The slope is defined as the 30-years equity premium minus the 6-month premium, while increasing levels of exogeneity have been obtained increasing a_l relative to a_h and increasing b_l relative to b_h.