A Real Options Analysis of Dual Labor Markets and the Single Labor Contract*

Pedro Gete† and Paolo Porchia‡

First Draft: August, 2011
This Draft: March, 2012

Abstract

We study the optimal hiring and firing decisions of a firm under two different firing costs regulations: 1) Dual labor markets characterized by high firing costs for workers with seniority above a threshold ("permanent workers") and by low costs for "temporary workers". 2) The Single Labor Contract, a policy proposal to make firing costs increasing in seniority at the job. Our contribution is to focus on the option value implied by both regulations. We show that in the Dual regulation the workers more likely to be fired are those close to become permanent because the firm tries to keep alive the option to fire at low cost. On the contrary, the Single Contract transfers that maximum firing to the new hires. Thus, fired workers are fired sooner under the Single Contract. We characterize three other results from comparing both regulations: 1) If both regulations have the same average firing cost for workers who become permanent, temporary workers are less likely to be fired in the Single Contract. 2) Moreover, this new regulation increases hiring and average employment duration. 3) It also reduces turnover among temporary workers, but at the expense of higher turnover among permanent workers who are more often replaced by temporary workers.

*We are grateful to Jim Albrecht and Marco Trombetta for their encouragement and useful advice. We also appreciate the helpful comments of Ignacio Garcia-Perez and seminar participants at IE Business School. Pedro Gete gratefully acknowledges partial funding from the Spanish Ministry of Science and Innovation Grant No. ECO2010-17625. Paolo Porchia gratefully acknowledges financial support from the Spanish Ministry of Science and Innovation under the Ramón y Cajal program (RYC-2010-07401).

†Georgetown University and IE Business School. Email: pg252@georgetown.edu

‡Corresponding author. IE Business School. Castellón de la Plana 8. 28006 Madrid, Spain. Tel.: +34 91 782 17 06. Fax: +34 91 745 47 62. Email: paolo.porchia@ie.edu

1 Introduction

With aggregate unemployment rates reaching double digits in many countries, labor market reforms are at the center of the economic policy debate. This is especially the case in southern European countries characterized by "dual labor markets". A concept that describes labor regulations with two main types of contracts: on one side, permanent contracts protected with high firing costs; on the other side, temporary contracts with low firing costs that must be upgraded to permanent when worker's seniority at the job reaches a certain threshold.[1] These countries are among those with higher youth unemployment rates (in 2010 about 25% in France and Italy, more than 40% in Spain), and at least half of their young workers have a temporary contract (Scarpetta et al. 2010).

Among the different policy proposals, one seems especially popular: unifying the Dual Labor regulations into a Single Contract that would have firing costs increasing in seniority at the job.[2] For example, Nicolas Sarkozy endorsed the idea during the French 2007 presidential election (Cheron 2007); in Spain it is in the electoral program of one of the major political parties (Expansion 2011); in Portugal implementing a version of it was imposed by the EU in the 2011 rescue package (Bentolila 2011).

In this paper we compare the Single Contract and the Dual Labor regulations in a model that explicitly takes into account the option value implied by the different firing cost regulations. Under both regulations, to fire before a certain seniority threshold T is similar to an American option that gives the right of firing at low costs.[3] We show how each regulation affects the value of this option and what are the consequences of these differences. This is the main contribution of the paper.

In the model there is a firm which can be either active or idle. Active firms employ a worker and make stochastic profits which can be positive or negative. They can fire their worker at any time and become idle by paying a firing cost.[4] If the firm is idle it does not employ any worker and its profits are zero. An idle firm can hire a worker by paying a hiring cost and become active (we assume no matching frictions and perfectly elastic labor supply, i.e. "workers are waiting at the gate").

[1] For example, before the 2010 reforms, in Spain firing costs for temporary contracts (job seniority smaller than 3 years) were the wage amount of 8 days of work per year of job seniority. Meanwhile for permanent contracts the costs were the wage amount of 45 days per year of job seniority. The 2010 reform reduced the firing costs to 12 for temporary workers and to 33 days for permanent.

[2] There are several proposals of Single Contract that differ in their details but share this common element: Blanchard and Tirole (2003) and Cahuc and Kramarz (2004) for France, Boeri and Garibaldi (2007) and Ichino et al. (2009) for Italy, and a manifesto signed by 100 academic economists (Andrés et al. 2009) for Spain.

[3] An American put option is a financial contract in which the buyer of the option has the right, but not the obligation, to sell an agreed financial instrument, to the seller of the option at any time during the life of the option for a certain price.

[4] We do not consider fixed term contracts that imply zero firing cost at the expiration of the contract.

We model the Dual regulation assuming that the firing cost is a constant if firing happens before worker's seniority reaches a threshold T, and a higher constant if firing happens after T. For the Single Contract we assume that firing costs start at some positive level and continuously increase with worker's seniority until seniority reaches T. After this threshold the firing cost is the same constant level than for permanent workers in the Dual Labor.

We show that the optimal firing rule is not only a function of worker's productivity, but also of both the time to expiration of the option of firing at low cost, and of the cost of exercising it. Firms with permanent workers do not have that option, thus their firing behavior only depends on the productivity of the worker. The Dual regulation and the Single Contract differ on the timing of the costs of exercising the option, what changes radically the firm's behavior.

In the Dual regulation the workers more likely to be fired are those close to become permanent because the firm tries to keep alive the option to fire at low cost. On the contrary, in the Single Contract the option does not have much value for those workers because their firing costs are close to those of permanent workers. In the Single Contract the maximum firing happens with new hires because firms anticipate that the option loses value as worker's seniority increases. Thus, fired workers are fired sooner under the Single Contract.

We also show that if the regulations share the same average firing cost at T, and the same protection for permanent workers, then the Single Contract increases hiring and reduces turnover among temporary workers, but at the expense of higher turnover among permanent workers who are more often replaced by temporary workers. These results happen because for any duration strictly shorter than T the Single Contract has lower average and cumulative firing costs. Thus, higher incentives to both hiring and firing. Overall, the Single Contract generates a higher average time employed.[5]

We did comparative statics on the main parameters of the model to check the robustness of the previous results, and to assess the sensitivity of the two regulations. We noticed that when firms become more impatient (higher discount factor) the Single Contract generates more firing of temporary workers than the Dual because the anticipation of future costs plays a higher role in the Single Contract. And for high levels of risk aversion the Single Contract provides less incentives to fire, especially transitory workers.

Our paper is related to two literatures:

1) The paper uses techniques from the literature of investment under uncertainty (Dixit and Pindyck 1994 is an early survey, Cetin and Zapatero 2010, Hugonnier and Morellec 2007 or Miao and Wang 2007, are, among others, recent examples). Bertola and Bentolila (1990) is closely related. They also study a continuous time partial equilibrium labor demand model. However, their firing and hiring costs are linear and do not imply any option value.

[5]Our results are qualitative. Our model is too stylized for a full quantitative analysis.

2) By the questions studied, our paper complements the search and matching literature that has studied Dual Labor markets (for example, Bentolila et al. 2010, Cahuc and Postel-Vinay 2002, Costain et al. 2011, Dolado et al. 2007 or Sala et al. 2010). And we contribute to the small but growing literature on the Single Contract (Costain et al. 2011, Garcia-Perez 2009, Garcia-Perez and Osuna 2011). Our value added is to focus on the option value implicit in the firing regulations. To the best of our knowledge, nobody in the literature has done this. We believe this is relevant because when the option value is taken into account the firing rule is no longer a constant productivity level because the firm tries to keep alive the option to fire at low cost.

We borrow our solution technique from Carr (1998), who used it to price American put options with finite maturity in a continuous time model with Brownian motions. The idea is to convert the problem into one of an infinite-maturity option with a stochastic termination time. In an online appendix we show that our results hold in a discrete time model solved via value function iteration.

The paper proceeds as follows. Sections 2 describes the model and Section 3 the solution method. Section 4 discusses the results. Section 5 performs comparative statics. Section 6 concludes. Proofs and details of the solution method are in an online Appendix.

2 Model

We analyze an infinitely-lived firm in a continuous-time setting. The firm can be in any of two states: 1) It can be active, employing a worker and receiving a stochastic stream of profits net of wage costs y_t; or 2) it can be idle, have no employee and receive zero net profits. Profits can take either positive or negative values as they evolve as an arithmetic Brownian motion:

$$dy_t = \mu \, dt + \sigma \, dB_t \tag{1}$$

where μ is the expected profit growth (in levels) and σ is the profit growth volatility. Both μ and σ are constant.

An active firm can fire the worker at any time but it must pay a firing cost $q(\tau)$ that depends on how long the worker has been employed in the firm (τ). We focus on two cost functions:

i) The Dual Labor market, where the cost of firing a worker is a step function with two levels: if the fired worker has seniority smaller than a threshold $\overline{T}$ then the firm has to pay cost $\underline{q}$. If the worker has seniority larger than $\overline{T}$ then the firing cost is higher ($\overline{q}$)

$$q(\tau_t) = q^D(\tau_t) = \begin{cases} \overline{q} & \text{if} \quad \tau_t \geq \overline{T} \\ \underline{q} & \text{if} \quad \tau_t < \overline{T} \end{cases} \text{ with } \overline{q} > \underline{q} > 0 \tag{2}$$

ii) The Single Contract, where firing costs start at some positive level (q_0) and increase linearly with slope μ_q as the worker remains employed. Once seniority attains a threshold T_S the firing cost becomes constant

$$q(\tau_t) = \begin{cases} q^S(\tau_t) = q_0 + \mu_q \tau_t & \text{if} \quad \tau_t \leq T_S \\ \overline{q}^S = q_0 + \mu_q T_S & \text{if} \quad \tau_t > T_S \end{cases} \tag{3}$$
$$\mu_q > 0, q_0 > 0$$

If the firm fires its worker it switches to the idle state where net profits are zero. Idle firms monitor potential profits (y_t) and can hire a worker at any time by paying a hiring cost (c). If they do so they start producing at the next instant. Thus, the first profit received by an idle firm that hires a worker at t is $y_{t+\varepsilon}$, for infinitesimal ε.

We assume that the firm has subjective discount rate δ and it is risk averse. We follow the recent financial literature on firm's capital structure (Bhamra et al. 2010 or Chen 2010, among others) and assume an exogenous stochastic discount factor unaffected by the firm's firing/hiring policy. The firm maximizes its value by discounting cash-flows with the stochastic discount factor implied by CARA utility over potential profits:

$$u(y_t) = -\frac{1}{\gamma} \exp\left(-\gamma y_t\right), \tag{4}$$

where γ is the coefficient of absolute risk aversion. This is equivalent to the problem of a risk-neutral firm whose discount rate is

$$r = \delta + \mu\gamma - \sigma^2 \frac{\gamma^2}{2} \tag{5}$$

and whose risk-adjusted expected profit variation is[6]

$$\mu^* = \mu - \gamma\sigma^2. \tag{6}$$

Since the firm can decide at each time whether to fire or not, profits before firing and hiring costs (π_t) can be written as

$$\pi_t = I_t y_t \tag{7}$$

where I_t is an indicator function that takes the value one if the firm has a worker at time t, and zero otherwise. Firing means $dI_t = -1$, while hiring implies $dI_t = 1$. Thus π_t evolves as:

$$d\pi_t = I_t[\mu dt + \sigma dB_t] + y_t dI_t \tag{8}$$

The firm's problem is to decide the optimal times at which to hire (if the firm is idle) or to fire (if it is active) to maximize its expectation of cumulative discounted cash-flows. We denote by η_i the time at which the firm takes those decisions.[7] And use the indicator function I_{η_i} that takes the value one when the firing/hiring decision is taken. Then, the problem of the firm under the risk neutral measure is

$$\begin{aligned} V &= \max_{\eta_i} E\left[\int_0^\infty e^{-rt}\pi_t dt\right] - \sum_{i=1}^\infty E\left[e^{-r\eta_i} I_{\eta_i} q(\tau_{\eta_i}) + (1 - I_{\eta_i})c\right] \\ s.t. \quad & d\pi_t = I_t[\mu^* dt + \sigma dB_t] + y_t dI_t \\ s.t. \quad & dy_t = \mu^* dt + \sigma dB_t \\ s.t. \quad & q(\tau_t) = \begin{cases} \text{as in (3) for the Single Contract} \\ \text{as in (2) for the Dual Labor market} \end{cases} \end{aligned} \tag{9}$$

[6] These parameters are obtained by decomposing the stochastic discount factor

$$e^{-\delta t}\frac{u'(y_t)}{u'(y_0)} = e^{-\delta t - \gamma(y_t - y_0)} = e^{-\delta t - \gamma(\mu t + \sigma B_t)} = Z_{0,t}H_{0,t}$$

into the time discount factor $Z_{0,t} = e^{-r^* t}$, with $r^* = \delta + \mu\gamma - \frac{1}{2}\gamma^2\sigma^2$, and the risk-neutral density process $H_{0,t} = e^{-\frac{1}{2}\kappa^2 t - \kappa B_t}$, with market price of risk $\kappa = \gamma\sigma$. The Radon-Nikodym theorem and the Girsanov theorem imply that

$$E\left[\int_0^\infty Z_{0,t}H_{0,t}\pi_t dt\right] = E^*\left[\int_0^\infty Z_{0,t}\pi_t dt\right]$$

where E^* denotes expectation with respect to the probability measure under which $B_t^* = B_t + \kappa t$ is a standard Brownian motion (the risk-neutral probability measure). Substituting $B_t = B_t^* - \kappa t$ into the dynamics of profits π_t and y_t, we obtain the risk-neutral firm's value.

[7] Thus $i = 1, \ldots, \infty$. If the firm starts in the idle state, the firm is hiring when i is odd, and it is firing when i is even.

3 Solving the model

Both cost functions (2) and (3) imply that the value of the firm's option to fire depends on time, because firing is cheaper if it is done before the employment reaches $\overline{T}$ or T_S. To capture this feature of the option value we will solve the model using a randomizing approximation method proposed by Carr (1998) to price American put options with finite maturity. The idea is to convert the problem into one of an infinite-maturity option with a stochastic termination time.

To describe the method let's assume that $\overline{T} = T_S$ and denote it by T_S. Carr (1998) method partitions the employment time threshold T_S into n subintervals and it assumes that T_S is not a deterministic time but a stochastic time denoted by $\tilde{T}$. The random variable $\tilde{T}$ has mean T_S, and variance $Var\left(\tilde{T}\right)$ that converges to zero as $n \to \infty$. Thus, the deterministic case can be approximated with any accuracy by the stochastic case by increasing n.

We assume that the employment time (τ) starts in the first time interval and switches randomly to the next one when it receives a shock distributed as a continuous time Poisson process with hazard rate n/T_S. Thus, the average time expected in the first interval is $\frac{T_S}{n}$ and the variance $\left(\frac{T_S}{n}\right)^2$. The shocks at different intervals are $i.i.d.$ Thus, the average time to have received n shocks is $E\left(\tilde{T}\right) = T_S$, and the variance $Var\left(\tilde{T}\right)$ is $\frac{(T_S)^2}{n}$, which converges to zero as $n \to \infty$.

We denote by u a state variable that captures how many shocks have happened, or, in other words, in which interval is the employment time. We can write the firing cost $q(\tau, u)$ as a function of u since $q(\tau, u)$ gets into the flat shape of $\tau_t > T_S$ only after n shocks.

There are $n+1$ intervals (the first n before T_S, plus the one after T_S at which firing costs are constant). Thus, for example, if $n = 2$ then $u = 0, 1$ or 2. The variable u_t changes over time depending on the shocks, it evolves as a continuous-time markov chain with intensity $\frac{n}{T_S}$. For example, when $n = 2$ its intensity matrix is

$$\begin{bmatrix} -\frac{2}{T_S} & \frac{2}{T_S} & 0 \\ 0 & -\frac{2}{T_S} & \frac{2}{T_S} \\ 0 & 0 & 0 \end{bmatrix}$$

with the third state being an absorbing state.

We denote by $V(I_t = 1, y_t, q_t, u_t)$ the value function of a firm employing a worker ($I_t = 1$), receiving profits y_t, facing firing cost function q_t which depends on the employment duration, and on interval u_t. This firm must decide an optimal time η to fire. This optimal time can be infinite. If it fires, the firm will get the discounted continuation value of an idle firm

$V(I_t = 0, y_\eta, q_0, u_t = 0)$. Hence the active firm's value is

$$V(1, y_t, q_t, u) = \max_{\eta} E\left[\int_t^\eta e^{-rs} y_s ds - e^{-r\eta} q_\eta + e^{-r\eta} V(0, y_\eta, q_0, 0)\right] \tag{10}$$

The first term is expected cumulative discounted profits until the time of firing. The second term captures the firing costs of firing a worker of duration η. The third term is the continuation value.

The optimal η can be expressed as a minimum profit level that triggers firing once attained. We call this profit level the firing boundary, denoted as $\underline{y}(q, u)$, which depends on costs q (hence seniority at the job), and the state variable u which determines whether costs have switched to constant. For profit values above the boundary the firm prefers to keep the worker. For profits below the boundary the worker is fired and the firm goes idle. Firing occurs the first time the profit value y reaches the boundary. This can happen either for a profit shock, or for a jump in the state variable u, i.e. passage of time, is the firm has a higher incentive to fire as senirity at the job increases. Therefore the firm value will depend on the firing boundary for all values of u.

When the firm is idle profits are zero, but it can hire at any time η. Its value function is

$$V(0, y_t, q_0, 0) = \max_{\eta} E\left[e^{-r\eta} V(1, y_\eta, q_0, 0) - e^{-r\eta} c\right] \tag{11}$$

The first term is the discounted value upon hiring at time η and becoming an active firm. The second term captures the hiring costs discounted from the hiring time to the present.

There is a critical level of potential profits $\bar{y}$ that motivates the firm to hire, we call this the hiring boundary. It separates an inactivity region where low profits discourage the firm from hiring, from an activity region, where high profits induce the firm to hire. The hiring boundary depends on hiring costs, the evolution of the profits process, and on firing costs of the firm which just hired.

An online Appendix characterize the firing and hiring boundaries for both regulations and explain our numerical solution. Next section discusses their patterns.

4 Theoretical predictions

In this Section we analyze the qualitative predictions of the model. Our model is too stylized for a full quantitative analysis. Given the lack of closed form solutions we solve numerically a somewhat plausible parameterization. We checked that the patterns that we discuss are robust to different parameterizations. Moreover, in Section 5 we study how changes in the parameters affect the results.

4.1 Parameterization

Concerning the dynamics of profits (equation 1), we set the deterministic expected profit increase μ to 0.05 and the volatility σ to 0.14. If we measure profits in units of \$100 millions this corresponds to a firm experiencing \$5 million of expected annual profit increase, with a standard deviation of \$14 millions.[8] Concerning the preference parameters, we set the coefficient of absolute risk aversion γ to 3, and the subjective discount rate δ to 0.15. Section 5 does comparative statics on these parameters.

Concerning the firing costs, to focus on the differences between regulations due to different shapes of firing costs instead of different levels, we study the case when both regulations give the same protection to permanent workers

$$\overline{q} = \overline{q}^S \tag{12}$$

and this maximum protection is attained at the same seniority level

$$T_S = \overline{T} = T \tag{13}$$

Moreover, we assume that both regulations imply the same average firing cost for workers whose seniority is T, that is

$$\underline{q} = \frac{1}{T}\int_0^T q^S(\tau)d\tau \tag{14}$$

As we will discuss below, assumption (14) highlights an important feature of the Single Contract. Even if it is designed to have the same average cost as the Dual for workers that become permanent, its cumulative and average costs are necessarily lower for workers hired before T

$$\frac{1}{j}\int_0^j \underline{q}d\tau > \frac{1}{j}\int_0^j q^S(\tau)d\tau \qquad \forall\ j < T \tag{15}$$

Panel A of Figure 1, which plots the benchmark firing cost regulations, shows assumptions (12) – (15).

Insert Figure 1 about here

Concerning the firing cost parameters, we set them as multiples of the daily wage, which

[8] This volatility of earnings variation seems conservative for many industries. For example, in the auto sector, between 1947 and 2007, the average annual variation of real before tax profits was -389 millions (in U.S. dollars of 2005), while the standard deviation was much higher, \$7584 millions (Bureau of Economic Analysis, NIPA Tables 6.17 A,B,C,D).

we assume to be 0.05. This implies a monthly wage of around \$1500 for a worker generating an expected annual revenue of \$6800 to the firm, if we assume a profit rate of 25% of revenues, and wage costs of 2/3 of revenues (a rough approximation to the labor share in National Income).[9] We assumed $\overline{q} = 45$ wage days, and $T = 3$ given that one period in the model is one year. We set q_0, μ_q and $\underline{q}$ in order to meet assumptions $(12) - (15)$ with a non-negative q_0. The hiring cost (c) does not play an important role in the results, we set it to half of the smallest firing cost (the initial cost of the single contract). Table 1 summarizes the benchmark parameterization.

Insert Table 1 about here

4.2 Results

An active firm fires its worker when the profit level crosses the firing boundary from above. Hence, a higher firing boundary implies a higher incentive to fire. An idle firm hires a worker when the profit level crosses the hiring boundary from below. A lower hiring boundary implies a higher incentive to hire.

Panel B of Figure 1 reports the optimal firing and hiring boundaries under both types of regulations for the benchmark parameterization of Table 1. The regulations imply very different firing patterns, and also different hiring boundaries. First we discuss each regulation separately, then we compare them:

a) The Dual Labor: for $\tau \in [0, T]$ the firing boundary is increasing in seniority at the job, as seniority approaches T the firm demands more profits to keep the worker employed. Thus, most of the incentive to fire is concentrated at T. A pattern that is consistent with the empirical evidence and explained by the option value implicit in the Dual Labor. Firms like to have the option to fire at low cost, and they keep it alive by firing before T. Once the worker reaches T the option disappears.

   The slope of the firing boundary before T is increasing in the gap in firing costs $\left(\overline{q} - \underline{q}\right)$, and in how close seniority is of T. The first effect can be seen in Panel A of Figure 2, which plots the boundary for a lower value of $\underline{q}$ while keeping $\overline{q}$ constant. The higher the labor protection of the permanent worker relative to the temporary, the higher the value of keeping alive the option to fire at low cost. Moreover, a larger $\left(\overline{q} - \underline{q}\right)$ implies more hiring and more firing around T (the hiring and firing boundaries are closer). This higher turnover is a "churning effect", once temporary workers get close to T they are fired and (soon) replaced by new hires. The firm incurs firing and hiring cost to keep alive the

[9] We have $0.05 \approx \left(\frac{(6.8+\mu)}{0.25} * \frac{2}{3}\right) / 365$, i.e. a daily wage of \$50.

option to fire cheap.

Insert Figure 2 about here

Panel B of Figure 1 also shows that the firing boundary for permanent workers is flat and lower than for temporary workers. It is flat because now there is no option value, firing costs are constant. It is lower because permanent workers are protected by higher firing costs.

b) The Single Contract: the maximum of the firing boundary is at the start of employment ($\tau = 0$) and the firing boundary decreases in seniority. Two reasons explain these patterns: 1) At $\tau = 0$ firing costs are the cheapest. And lower firing costs encourage more firing. 2) Firing costs are increasing (up to T) creating an incentive to fire before costs become more expensive. The expected cost increase is maximal at $\tau = 0$, and it decreases progressively to zero as costs are closer to the maximum cost, i.e. as seniority gets closer to T. After T the firing boundary is flat and at its lowest level because costs are constant and at their maximum level.

Panel B of Figure 2 shows how the slope of the firing boundary depends on the slope of cost increase μ_q and on how close seniority is to T. It plots firing boundaries for larger Ts and smaller slopes μ_q of firing costs, while keeping unaltered the firing cost after T. We can see that both the intercept and the average slope of the firing boundary decrease as T becomes larger. The slower the transition to the highest firing costs the smaller the anticipation effect, and smaller the incentive to fire. The higher μ_q, the higher the initial incentive to anticipate firing and the faster the boundary decays as employment time goes by.

From the previous discussion we can draw two conclusions from comparing both regulations:

i) Relative to the Dual Labor, the Single Contract transfers most of the incentive to fire from the workers with seniority close to T to those just hired. The extent of this reshaping depends on the rate of cost increase μ_q in the Single Contract. Figure 3 plots a consequence of this reshaping: the average seniority of fired workers is lower in the Single Contract. This happens for both workers fired before (Panel A) and after T (Panel B). As it is intuitive, workers that started at a higher profit level have on average been employed more time when fired (it took more time for profits to cross the firing boundary).

Insert Figure 3 about here

ii) If the regulations share the same average firing cost at T (condition 14) and the same protection for permanent workers (condition 12), then the Single Contract generates more incentive to hire (lower hiring boundary) and higher turnover among permanent workers (the firing boundary for permanent worker is higher and its distance from the hiring boundary is smaller). Figure 4 confirms these results. Panel A shows that an unemployed worker has a higher probability of being hired under the Single Contract. Panel B shows that for different levels of firm profitability the Single Contract has a slightly higher probability of firing a permanent worker. Panel C shows that the Single Contract has lower probability of firing a transitory worker except for workers starting in very bad profit conditions. These results follow from condition (15), for any duration strictly shorter than T the Single Contract has lower average and cumulative firing costs. Thus, higher incentives to hire and fire. Figure 5 confirms that this is the explanation. Its Panel A proposes a cost structure violating condition (15) . And its Panel B shows that for this new cost structure the firing boundary of permanent workers is not anymore higher in the Single Contract. Moreover, now the Dual has lower hiring boundary.

Insert Figure 4 about here

Insert Figure 5 about here

Thus, an important message from Figures 3 and 4 is that during the temporary phase there is less chance of being fired in the Single Contract, but if the worker is fired it happens before than in the Dual Labor, when most of the firing happens at T. Overall, the higher likelihood of hiring and lower likelihood of firing transitory workers in the Single Contract generate a higher average time employed. As it is shown in Panel D of Figure 4.

5 Comparative Statics

In this Section we do two things: on one side to check the robustness of the results discussed in Section 4.2. On the other, to assess how changes in the parameters affect firm's firing behavior. We start with the subjective time-discount factor (δ). Panels A and B of Figure 6 plot the firing boundary as a function of δ at three different seniority levels ($\tau = 6$ is a permanent worker, $\tau = 2.5$ is worker close to become permanent, $\tau = 0.5$ is a worker hired recently). Two effects are at play. On one hand more impatient firms fire earlier, because they are less willing to tradeoff present losses for future profits. On the other hand, high δ implies that firing costs today are more expensive relative to future profits, hence an incentive to postpone firing. For $\tau = 0.5$ and $\tau = 2.5$ the first effect dominates and the boundary is monotonically increasing in δ for both regulations. However, for the workers with higher costs ($\tau = 6$) when δ is high enough the second effect dominates and more impatient firms fire later. Panel C plots the difference between the firing boundaries of the Dual and the Single as a function of δ for the same three seniority levels. The Single Contract is more sensitive than the Dual to changes in discount rates at the beginning of the employment relation. Higher δ makes the Single Contract to generate much more firing of temporary workers than the Dual. This is a consequence of condition (15) . Firms anticipate the average cost increase and when they are more impatient they ask for higher profits to keep the worker. The closer seniority is of T the smaller the anticipated cost increase, what favors the Single Contract.

Insert Figure 6 about here

Figure 7 plots the firing boundary for different values of expected risk neutral profit variation (μ^*) . Intuitively, in both regulations there is less firing when firms expect higher profits. When the deterministic drift is higher any bad profit shock will be more transitory. The shapes of the boundaries are not affected by μ^*. And Panel C shows that both regulations seem to react similarly to changes in this parameter.

Insert Figure 7 about here

Figure 8 plots the firing boundary for different values of risk neutral profit volatility (σ^*) . Panels A and B show that the shapes of the boundaries are not affected by σ^*

Insert Figure 8 about here

An increase of σ^* implies two opposite effects: 1*)* As in any standard option, given that payoffs are asymmetric (exercise in good times, wait in bad times) an increase of the risk-neutral volatility enhances the value of the option to fire and delays firing. *2)* Firing before T is a especial option, it is the option to fire at low cost. To keep this option alive the firm cannot let the employment duration last more than T. Thus, when higher volatility encourages the firm to keep this option alive, the firm fires sooner. Effect *1)* dominates for our parameterization and in Panels A and B, for both regulations, higher σ^* reduces firing. But Panel C, shows that effect *2)* is there, and it is important when comparing both regulations. Panel C plots the Dual Labor when the cost of firing a permanent worker (the cost gap $\overline{q} - \underline{q}$) in the Dual Labor is infinite, what makes the option to fire at low cost very valuable. We can see that for new hires effect *1)* is still prominent, but close to T an increase of volatility induces the firm to fire earlier. This is effect *2)* in play, more volatile firms fire sooner to keep alive the option to fire cheap.

Thus, the effects of σ^* on both regulations depend crucially on the seniority of the worker.

Figure 9 plots the firing boundary for different values of the risk aversion coefficient γ at two different seniority levels ($\tau = 6$ is a permanent worker, $\tau = 0.5$ is a worker hired recently).

Insert Figure 9 about here

Panels A and B show that for both seniority levels, both regulations display a non-monotonic pattern of the firing boundary with respect to an increase in risk aversion. This is explained by equations (5) and (6) . Higher risk-aversion lowers μ^* via equation (6) and, initially, increases the discount rate r of equation (5). As in Figures 6 and 7, both effects push for early firing. However, further increases of γ reduce r and induce the firm to fire less. Panel C reports the difference between the firing boundaries of the Dual Labor and the Single Contract. It shows that for high levels of risk aversion the Single Contract provides less incentives to fire, especially transitory workers.

6 Conclusions

In this paper we use a real options model to study firing and hiring under two different regulations: the Dual Labor market and the Single Contract. We focus on the option value implied by these regulations. We show that it implies that for temporary workers the optimal firing rule is a function of their seniority because the firm tries to keep alive the option to fire at low cost. Relative to Dual regulations, the Single Contract transfers most of the incentive to fire from workers close to become permanent to new hires. Thus, fired workers are fired sooner under the Single Contract. However, if both regulations have the same average firing cost for workers who become permanent, temporary workers are less likely to be fired in the Single Contract. Moreover, the Single Contract increases hiring and average employment duration. It also reduces turnover among temporary workers, but at the expense of higher turnover among permanent workers who are more often replaced by temporary workers. These result may be especially important in a model where workers can invest in human capital. Or in a model with search costs or other frictions related to turnover.

Our model focused on qualitative patterns and abstracted from several dimensions important in quantitative work, for example, differentials in wage and productivity between workers of different seniority, or general equilibrium effects.

References

Andrés, J., Bentolila, S. and Felgueroso, F.: 2009, "Propuesta para la reactivación laboral en España", *FEDEA (www.crisis09.es/propuesta)* .

Bentolila, J.: 05/10/2011, "La Reforma Laboral Avanza... en Portugal", *Nada es Gratis* .

Bentolila, S. and Bertola, G.: 1990, "Firing Costs and Labour Demand: How Bad is Eurosclerosis?", *The Review of Economic Studies* **57**(3), 381.

Bentolila, S., Cahuc, P., Dolado, J. and Le Barbanchon, T.: 2010, "Two-Tier Labor Markets in the Great Recession: France vs. Spain", *CEPR Discussion Papers* **8152**.

Bhamra, H., Kuehn, L. and Strebulaev, I.: 2010, "The Aggregate Dynamics of Capital Structure and Macroeconomic Risk", *Review of Financial Studies* **23**(12), 4187.

Blanchard, O. and Tirole, J.: 2004, "Contours of Employment Protection Reform", *Macroeconomic theory and economic policy: essays in honour of Jean-Paul Fitoussi* **59**, 48.

Boeri, T. and Garibaldi, P.: 2007, "Two Tier Reforms of Employment Protection: a Honeymoon Effect?", *The Economic Journal* **117**(521), F357–F385.

Cahuc, P. and Kramarz, F.: 2004, *"De la Précarité à la Mobilité: vers une Sécurité Sociale Professionnelle"*, La Documentation Française.

Cahuc, P. and Postel-Vinay, F.: 2002, "Temporary Jobs, Employment Protection and Labor Market Performance", *Labour Economics* **9**(1), 63–91.

Carr, P.: 1998, "Randomization and the American put", *Review of Financial Studies* **11**(3), 597.

Cetin, C. and Zapatero, F.: 2010, "Optimal Acquisition of a Partially Hedgeable House", *Marshall School of Business Working Paper No. FBE 25-09* .

Chen, H.: 2010, "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure", *The Journal of Finance* **65**(6), 2171–2212.

Cheron, A.: 2007, "Would a New Form of Employment Contract Provide Greater Security for French Workers? Critical Analysis and Proposals relating to Nicolas Sarkozy's Employment Contract Project".

Costain, J., Jimeno, J. and Thomas, C.: 2010, "Employment Fluctuations in a Dual Labor Market", *Banco de España, Working Paper* **1013**.

Dixit, A., Pindyck, R. and Davis, G.: 1994, *Investment under uncertainty*, Princeton University Press.

Dolado, J., Jansen, M. and Jimeno Serrano, J.: 2007, "A Positive Analysis of Targeted Employment Protection Legislation", *The BE Journal of Macroeconomics* **7**(1), 14.

Expansión: 07/19/2011, "El PP propone un contrato único y flexible anticrisis".

Garcia Pérez, J.: 2009, " Qué Efectos Tendría un Contrato único sobre la Protección del Empleo?", *Propuesta de Reactivación Laboral en España, FEDEA* .

Garcia Pérez, J. and Osuna, V.: 2011, "The Effects of Introducing a Single Employment Contract in Spain".

Hugonnier, J. and Morellec, E.: 2007, "Corporate Control and Real Investment in Incomplete Markets", *Journal of Economic Dynamics and Control* **31**(5), 1781–1800.

Ichino, P. et al.: 2009, *Disegno di Legge per la Transizione a un Regime di Flexicurity, n. 1481.*

Miao, J. and Wang, N.: 2007, "Investment, Consumption, and Hedging under Incomplete Markets", *Journal of Financial Economics* **86**(3), 608–642.

Sala, H., Silva, J. and Toledo, M.: 2010, "Flexibility at the Margin and Labor Market Volatility in OECD countries".

Scarpetta, S., Sonnet, A. and Manfredi, T.: 2010, "Rising Youth Unemployment During the Crisis: How to Prevent Negative Long-Term Consequences on a Generation?", *OECD Social, Employment and Migration Working Papers* .

Figures and Tables

Parameters of Profit Process	Preference Parameters
$\mu = 0.05, \quad \sigma = 0.14$	$\delta = 0.15, \quad \gamma = 3$

Firing and Hiring Cost Parameters
$T_S = \overline{T} = 3, \quad \mu_q = (40/3) * 0.05, \quad q_0 = 5 * 0.05, \quad \underline{q} = 25 * 0.05, \quad \overline{q} = 45 * 0.05, \quad c = 0.5q_0,$

Table 1. Benchmark Parameterization. This table shows the parameters used to numerically solve the model and construct the figures of Section 4.

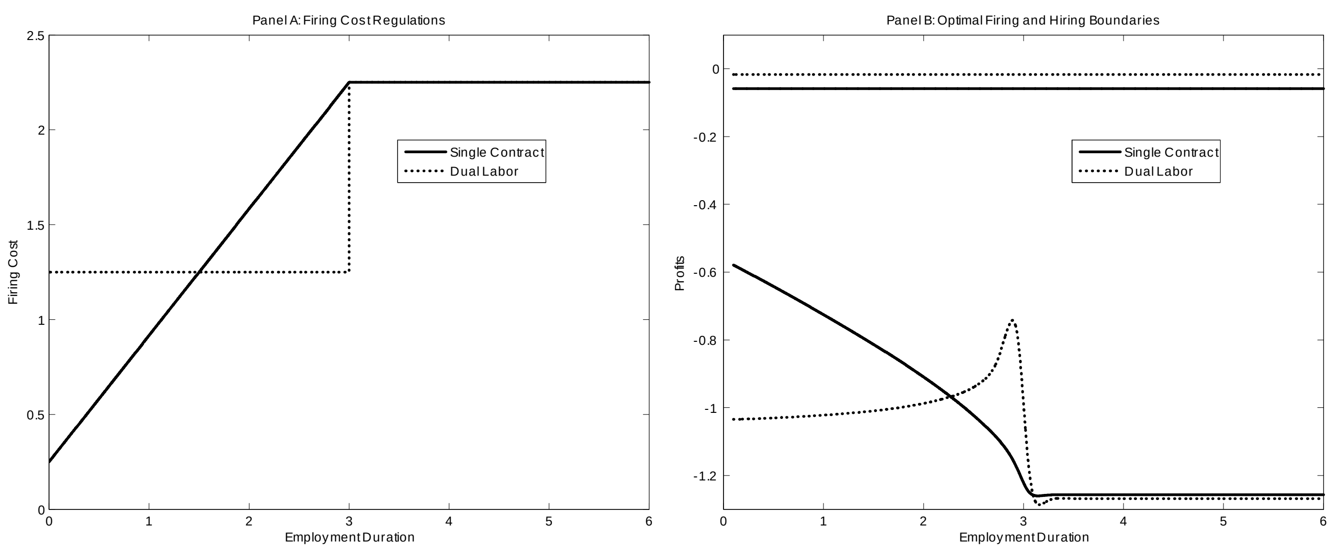

Figure 1. Firing Costs Regulations and Optimal Hiring and Firing Boundaries. Panel A plots the benchmark firing cost regulations of Table 1. Panel B plots the results for those parameterizations.

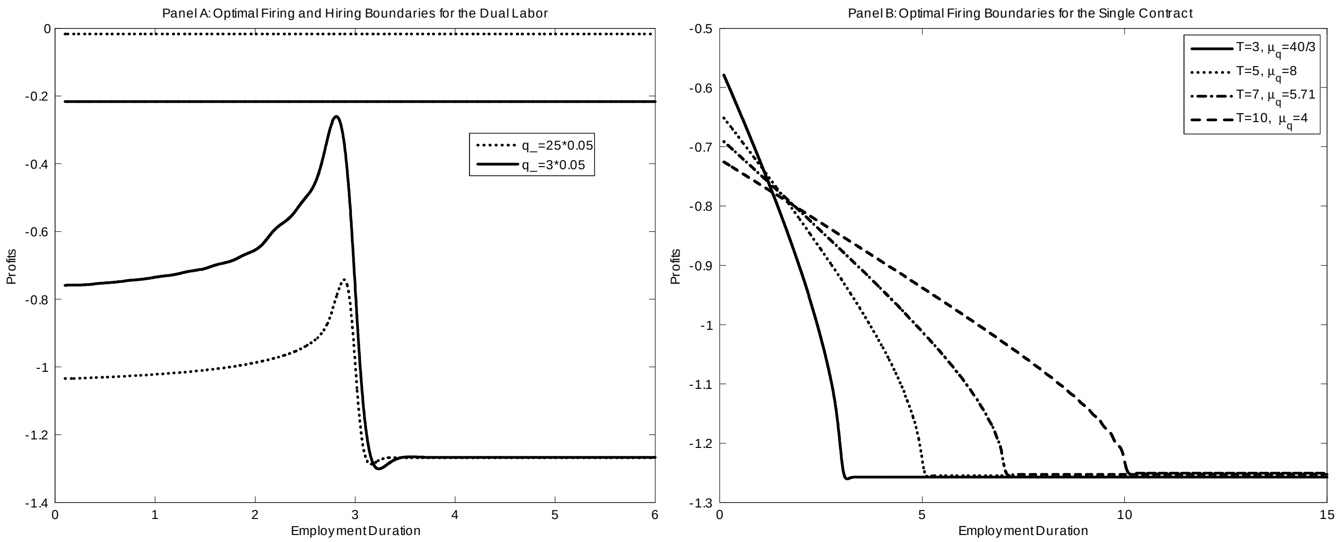

Figure 2. Comparative Statics on the Cost of Firing a Transitory Worker. Panel A shows the effects of increasing the gap in firing costs in the Dual regulation. Panel B plots the Single Contract for different slopes of firing costs that satisfy Conditions 12 and 14.

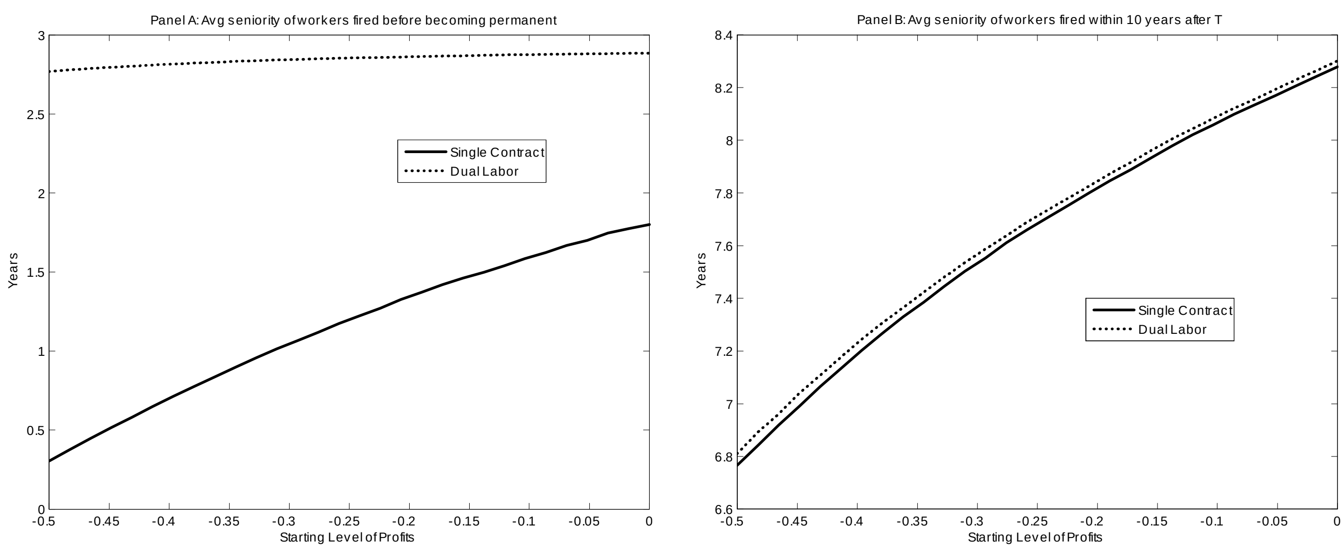

Figure 3. Expected Employment Times for Fired Workers. Panel A plots the average seniority of a worker fired before becoming permanent as a function of the starting profit level. Panel B plots the same thing for a permanent worker fired within 10 years of becoming permanent.

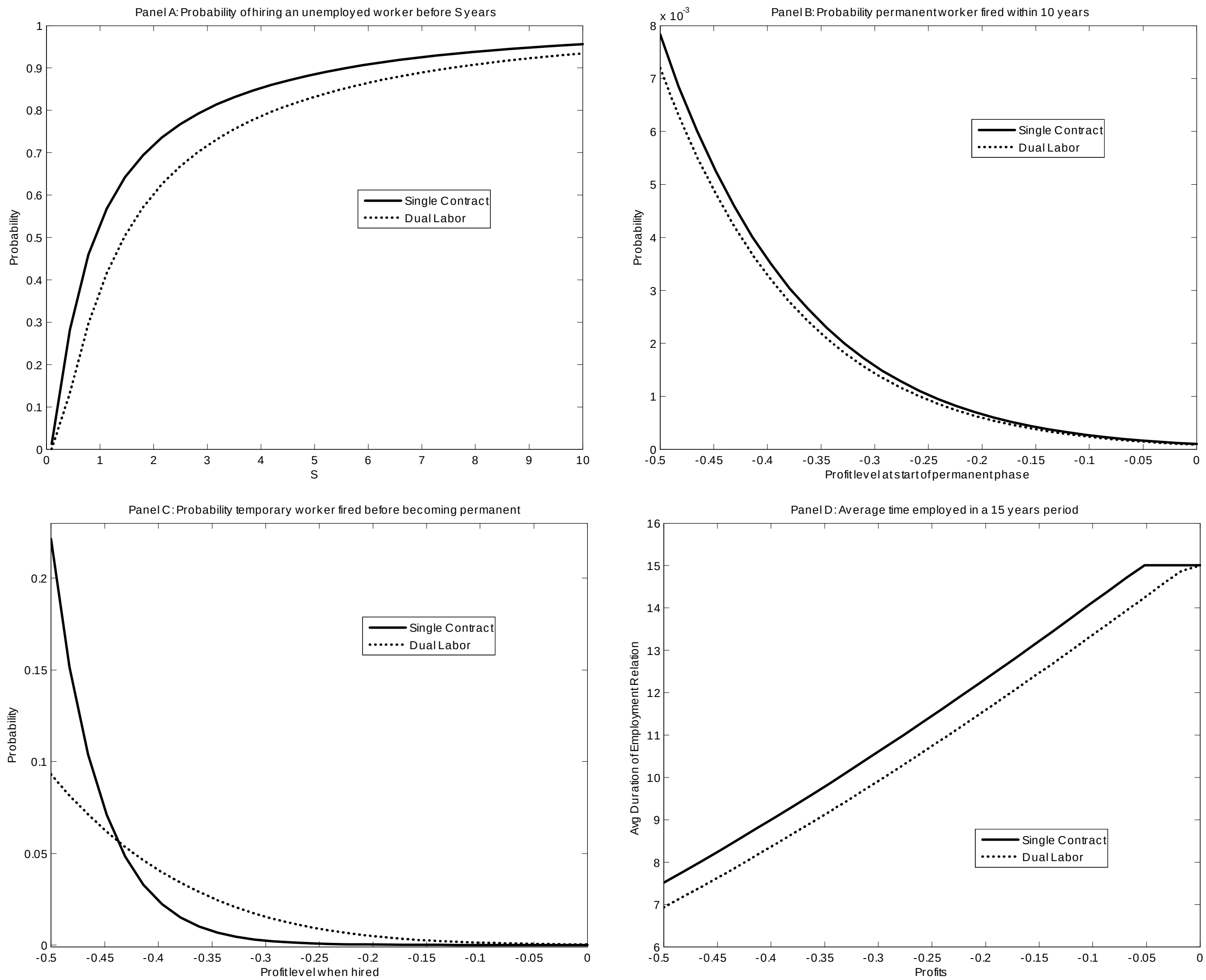

Figure 4. Firing and Hiring under Both Regulations. Panel A plots the probability that an unemployed is hired before a certain time S. Panel B plots the probability that a permanent worker is fired within 10 years as a function of the profit level at which she becomes permanent. Panel C redoes panel B but for a temporary worker fired before becoming permanent, and as a function of initial profit level. Panel D plots the average time that a worker starting at a certain profit level would remain employed in a 15 years period.

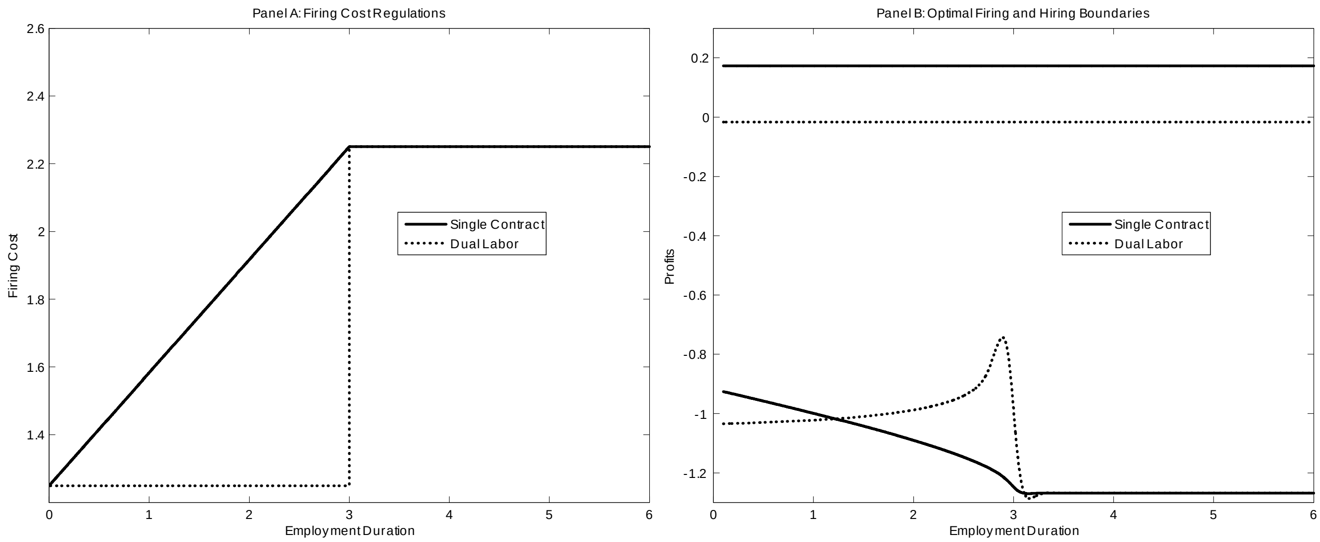

Figure 5. Alternative Single Contract and Optimal Hiring and Firing Boundaries. Panel A reports an alternative parameterization for the Single Contract that violates Condition (15), while the parameterization for the Dual remains the benchmark one. Panel B plots the hiring and firing boundaries for these two regulations.

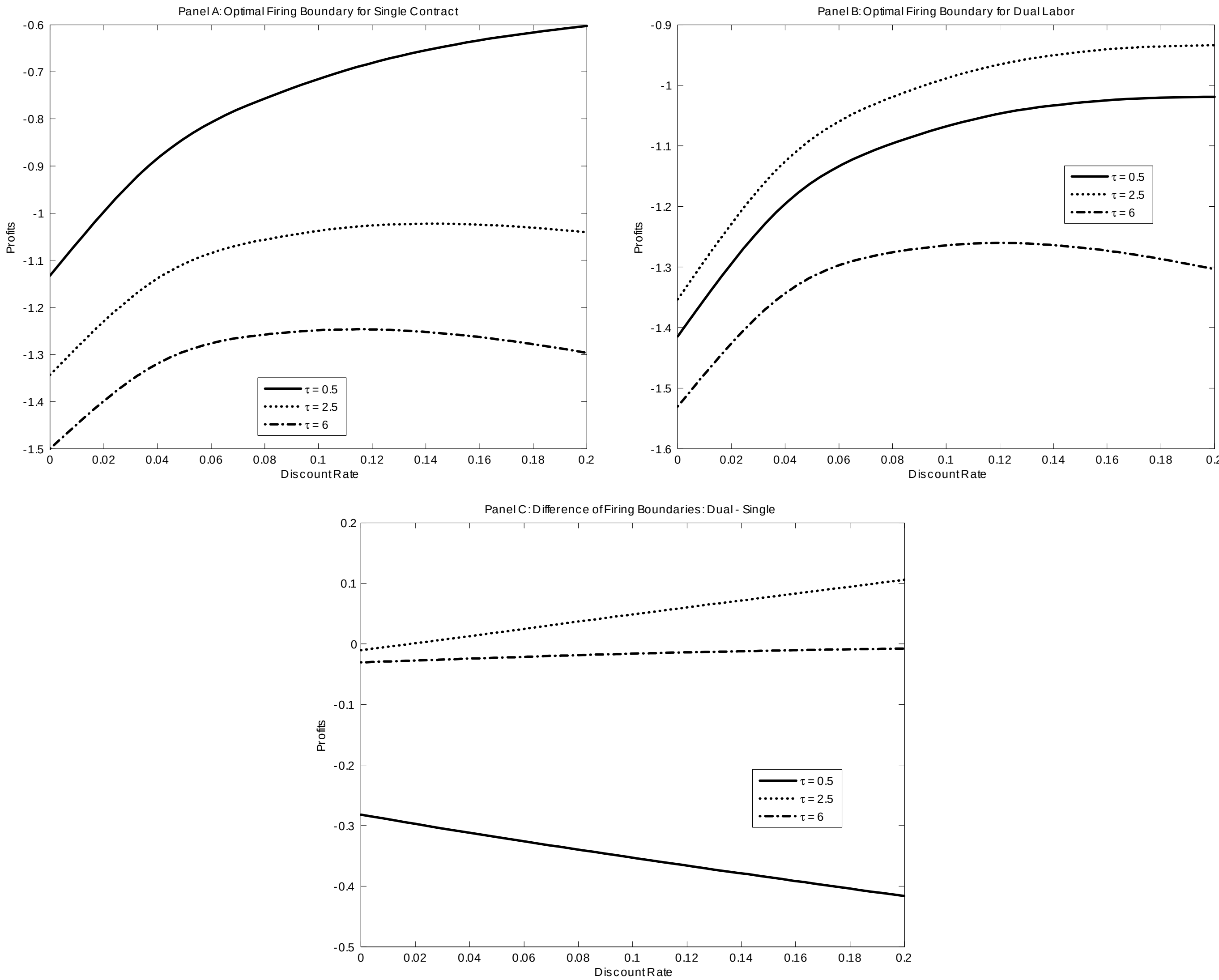

Figure 6. Comparative Statics: Subjective Discount Rate δ. Panel A plots the firing boundaries of the Single Contract at three different seniority levels ($\tau = 6$ is a permanent worker, $\tau = 2.5$ is a worker close to become permanent, $\tau = 0.5$ is a worker hired recently) for different values of δ. Panel B redoes Panel A but for the Dual Labor. Panel C compares the Dual and the Single Contract.

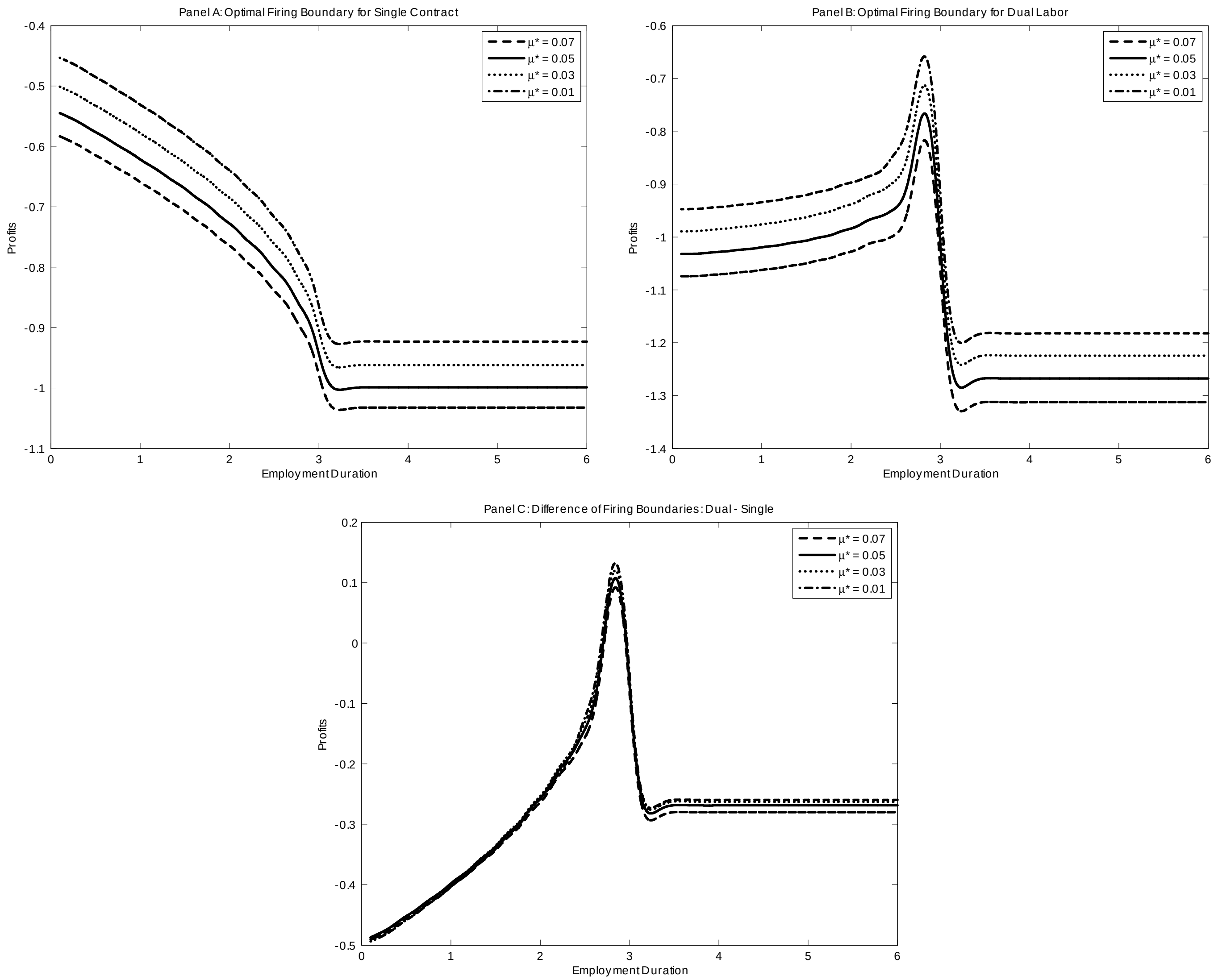

Figure 7. Comparative Statics: Risk-Neutral Expected Profit Variation μ^*. Panels A and B plot the firing boundaries of the Single Contract and the Dual Labor, respectively, for different values of μ^* as a function of seniority at the job. Panel C plots the difference between the Dual and the Single.

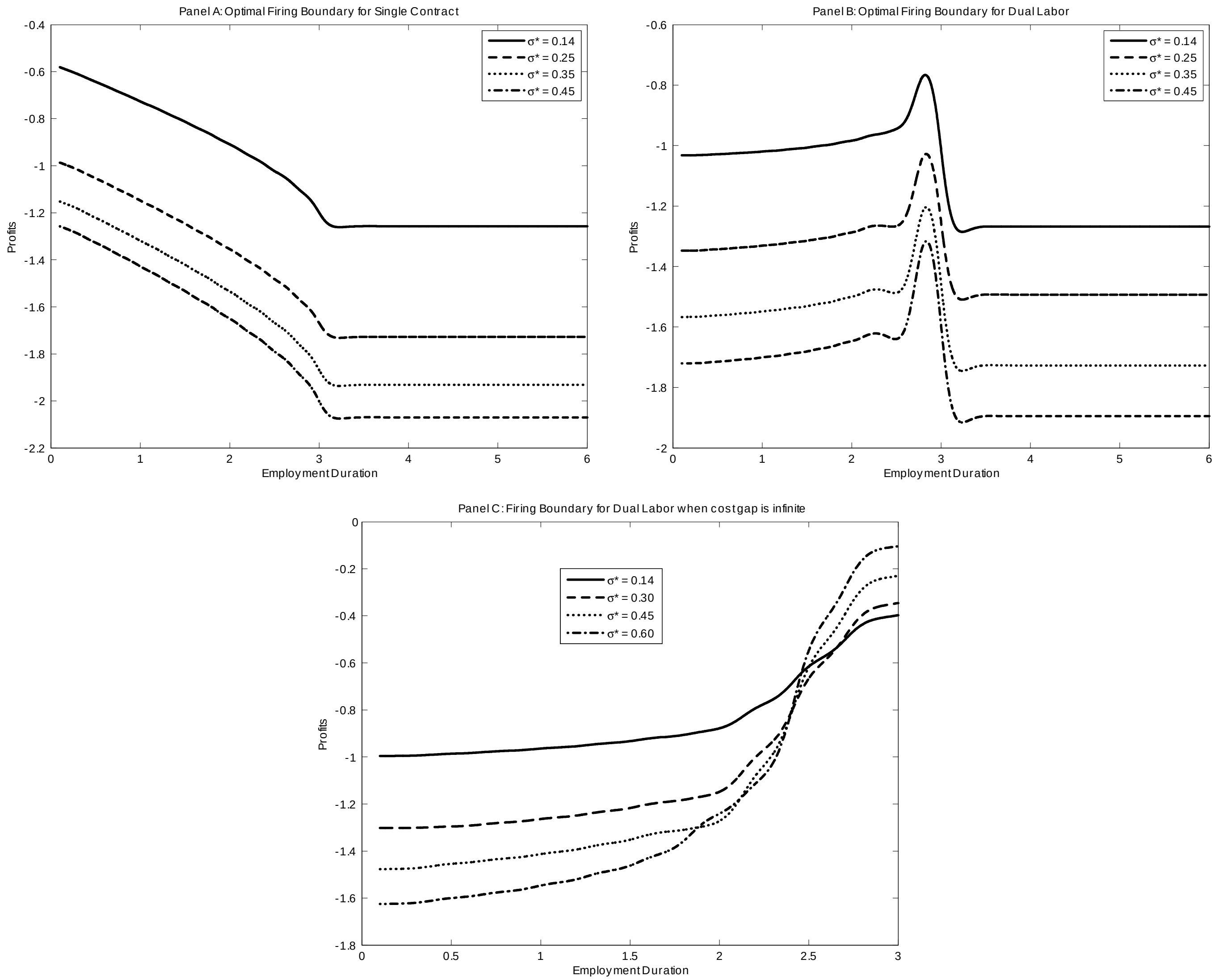

Figure 8. Comparative Statics: Risk-Neutral Volatility of Profit Variation σ^*. Panels A and B plot the firing boundaries of the Single Contract and of the Dual Labor for different values of σ^* as a function of seniority at the job. Panel C plots different firing boundaries of the Dual Labor for a parameterization with infinite costs of firing a permanent worker.

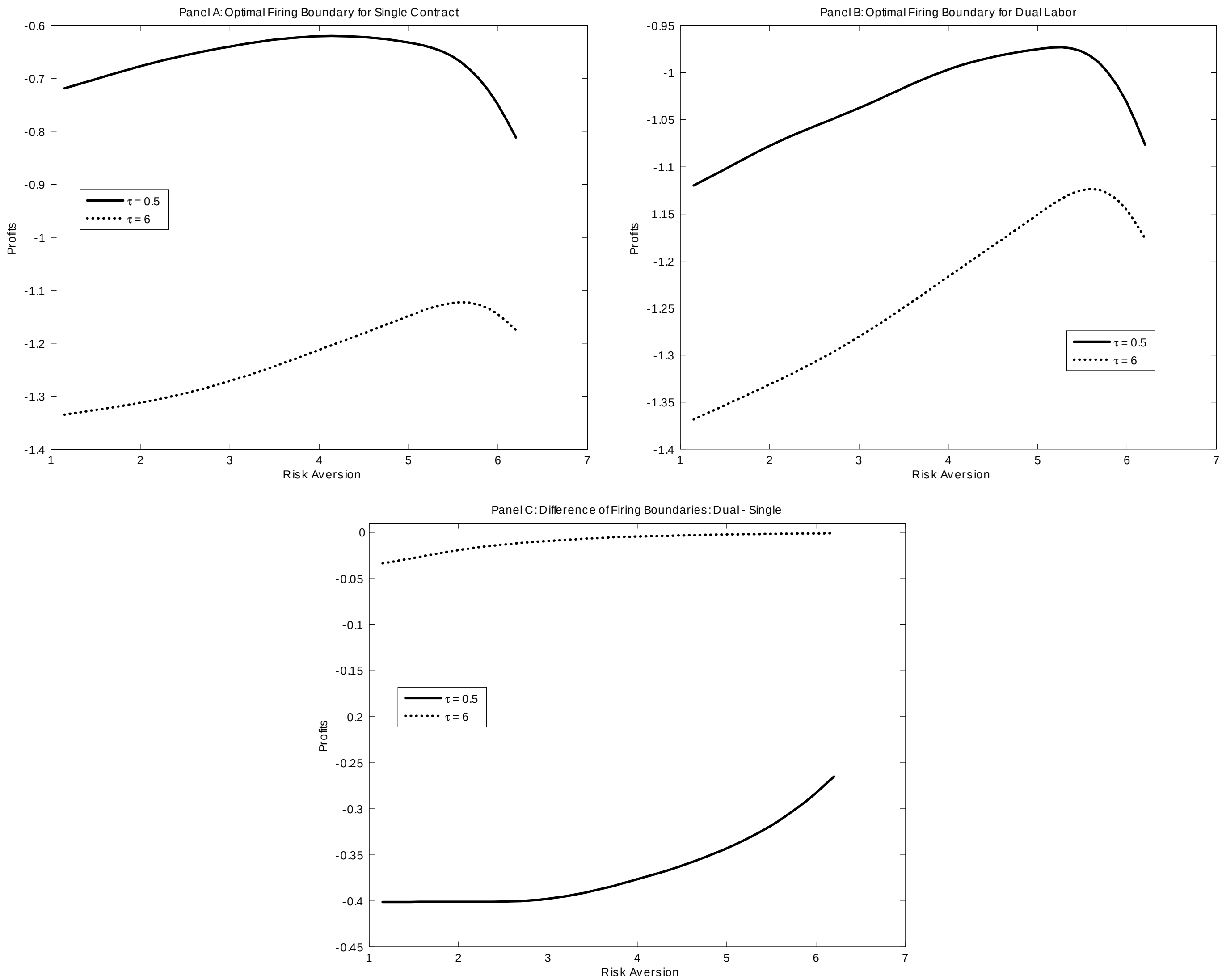

Figure 9. Comparative Statics: Risk-Aversion Parameter γ. Panel A and B plot the firing boundaries of the Single Contract and of the Dual Labor, respectively, with respect to the risk aversion parameter for two different seniority levels. Panel C plots the difference between both regulations.

ONLINE APPENDIX I (NOT FOR PUBLICATION): DISCRETE TIME VERSION

This appendix shows that the main qualitative results of the manuscript hold in a discrete-time formulation of the problem.

Equation (1) in the manuscript is replaced by[1]

$$y_{t+1} = y_t + \mu\,\Delta + \sigma\;\varepsilon_{t+1}\sqrt{\Delta} \tag{1}$$

where μ and σ are the constant yearly mean and standard deviation of profit variation, while Δ is the time step as a fraction of a year. Thus, the discrete time problem of the firm is

$$\begin{aligned} V_t &= \max_{\eta_i} E\left[\textstyle\sum_{t=1}^{\infty} e^{-rt}\pi_t dt\right] - \textstyle\sum_{i=1}^{\infty} E\left[e^{-r\eta_i} I_{\eta_i} q(\tau_{\eta_i}) + (1 - I_{\eta_i})c\right] \\ s.t. &\quad \pi_t = I_t\; y_t,\; y_{t+1} = y_t + \mu^*\,\Delta + \sigma\;\varepsilon_{t+1}\sqrt{\Delta} \\ s.t. &\quad q(\tau_t) = \begin{cases} \text{as the Single Contract in the manuscript} \\ \text{as the Dual Labor market in the manuscript} \end{cases} \end{aligned} \tag{2}$$

where all the notation is as in the manuscript.

1 Discrete Time Solution

Now we do not need to solve the model with the randomizing approximation of Carr (1998). Thus, the state variable u does not appear. As in the paper, we denote by $V(1, y, \tau)$ the firm's value conditional on being active and employing a worker.[2] $V(0, y)$ denotes the firm's value conditional on being idle, a function of profitability alone.

The Bellman equation of the active firm is:

$$V(1, y, \tau) = \max_I\; (1 - I)\left(y - q(\tau) + e^{-r\Delta}\mathbb{E}\left[V(0, y')\right]\right) + I\left(y + e^{-r\Delta}\mathbb{E}\left[V(1, y', \tau + \Delta)\right]\right) \tag{3}$$

where y' denotes next period profit and the firing cost $q(\tau)$ is either as in the Single Contract or the Dual Labor regulation.

The Bellman equation of the idle firm is:

$$V(0, y) = \max_I\; (1 - I)\left(e^{-r\Delta}\mathbb{E}\left[V(0, y')\right]\right) + I\left(e^{-r\Delta}\mathbb{E}\left[V(1, y', 0)\right] - c\right). \tag{4}$$

[1] To better compare results with those of the paper, we use a random walk, the discrete time analog of the process in the paper.

[2] We can equivalently use τ as a state variable instead of the firing cost q, because of their deterministic one to one mapping.

The firing boundary is the smallest profit level for which the firm continues active and does not fire:

$$\underline{y}(\tau) := \left\{ \inf \; y : \left(y - q(\tau) + e^{-r\Delta}\mathbb{E}\left[V(0,y')\right]\right) < \left(y + e^{-r\Delta}\mathbb{E}\left[V(1,y',\tau+\Delta)\right]\right) \right\}. \tag{5}$$

The hiring boundary is the highest profit level for which the idle firm continues idle and does not hire:

$$\overline{y} := \left\{ \sup \; y : e^{-r\Delta}\mathbb{E}\left[V(0,y')\right] > \left(e^{-r\Delta}\mathbb{E}\left[V(1,y',0)\right] - c \right) \right\}. \tag{6}$$

Thus the firm's value (2) can be characterized as:

$$V_t = \begin{cases} V(1,y,\tau) & \text{if} \quad I_{t-1} = 1 \text{ and } y_{t-1} > \underline{y}(\tau - \Delta) \\ V(0,y) & \text{if} \quad I_{t-1} = 1 \text{ and } y_{t-1} < \underline{y}(\tau - \Delta) \\ V(1,y,\tau) & \text{if} \quad I_{t-1} = 0 \text{ and } y_{t-1} > \overline{y} \\ V(0,y) & \text{if} \quad I_{t-1} = 0 \text{ and } y_{t-1} < \overline{y} \end{cases}$$

2 Discrete Time Results

We solve the model numerically following the benchmark parameterization of Table 1 in the manuscript. Figure 1Appendix reports the discrete time version of Figure 1B of the paper. We report firing and hiring boundaries corresponding to a weekly (Panel A), monthly (Panel B), and quarterly (Panel C) frequency. The employment rules are consistent with the qualitative patterns obtained in the continuous-time formulation.

Quantitative values are different because the closest we get of the continuous time limit is at weekly frequency ($\Delta = 1/52$). As the length of the time period increases from weekly to monthly or quarterly, the firm faces more risk because the firm's decision lasts for more time (i.e., less ability to time the employment situation). As a consequence the firing/hiring option is worth more and exercised more conservatively. Both firing and hiring are deferred. However, the Single Contract is much more affected as the length of the time period increases because its firing cost was continuously increasing.

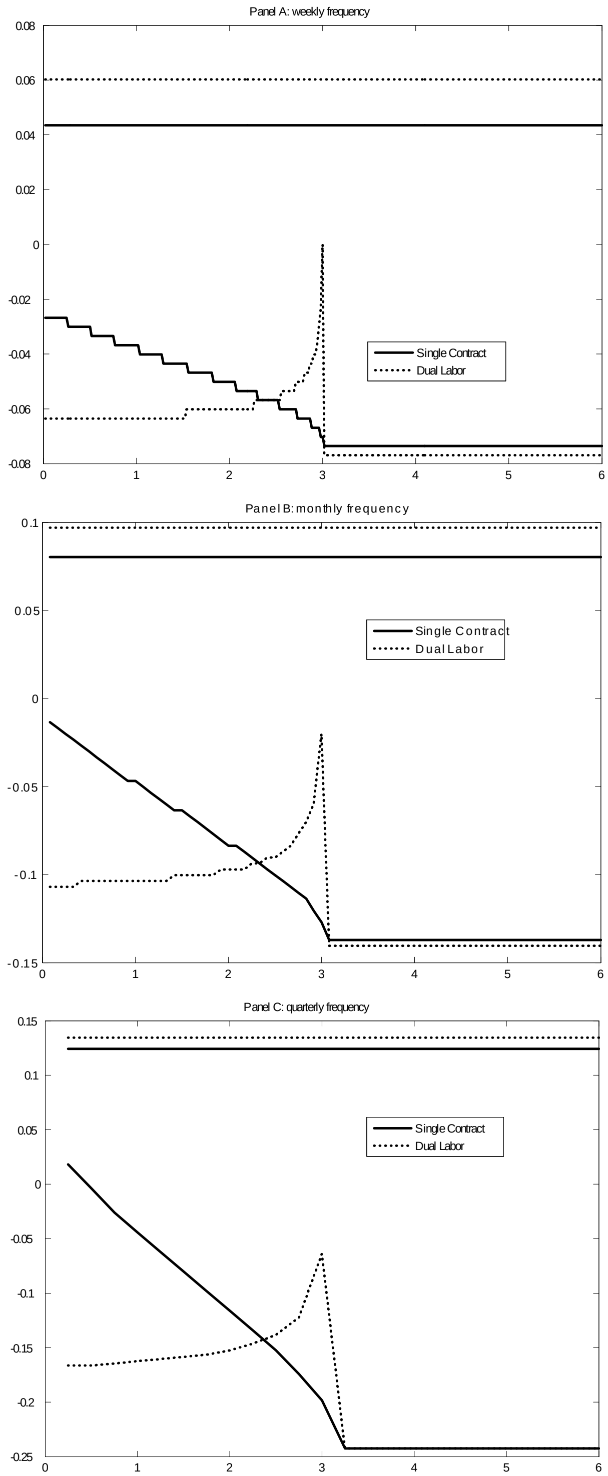

Figure 1Appendix. Firing and Hiring Boundaries. Panel A shows the firing and hiring boundary of the Single Contract (solid line) and the Dual Labor (dotted line) corresponding to a weekly time step ($\Delta = 1/52$). Panels B and C report the boundaries at monthly ($\Delta = 1/12$) and quarterly frequency ($\Delta = 1/4$).

3 Numerical Method.

We discretize the state-space by setting an equispaced grid of $n = 300$ realizations for profitability y, with lower bound $y_1 = -0.5$ and upper bound $y_n = 0.5$. We consider an equispaced grid of employment time τ ranging from 0 to $T = 3$ years. This is the duration of the temporary phase in both Single Contract and Dual Labor regulation in the baseline parameterization of Table 1. How dense is the grid depends on the time step Δ. For instance, at monthly frequency the grid for τ comprises $36 + 1$ elements.[3]

We jointly obtain the value function conditional on being active, $V(1, y, \tau)$, and idle, $V(0, y)$, by means of the following iteration, which uses Monte Carlo simulation and linear interpolation to compute expectations of next period value functions We iterate

$$V^a(1, y_i, \tau_j) = \max\left[\left(y_i - q(\tau_j) + e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(0, y'_s)\right), \left(y_i + e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(1, y'_s, \tau_j + \Delta)\right)\right],$$

$$V^a(0, y_i) = \max\left[\left(e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(0, y'_s)\right), \left(e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(1, y'_s, 0) - c\right)\right], \tag{7}$$

until the convergence criterion

$$\max\left[\max_{i,j}\frac{|V^a(1, y_i, \tau_j) - V^{a-1}(1, y_i, \tau_j)|}{V^{a-1}(1, y_i, \tau_j)}, \max_i \frac{|V^a(0, y_i,) - V^{a-1}(1, y_i)|}{V^{a-1}(1, y_i)}\right] < 0.001$$

is satisfied. Where y'_s denotes the s-th of n_s simulated realizations of next period profits, obtained from the current level y_i using (1), with risk-neutral parameters.

The value function corresponding to a given realization y'_s, $V^{a-1}(1, y'_s, \tau_j + \Delta)$, is obtained by linear interpolation (or extrapolation for off bounds values) from grid values:

$$V^{a-1}(1, y'_s, \tau_j + \Delta) = V^{a-1}(1, y^u_s, \tau_j + \Delta)\frac{y'_s - y^d_s}{y^u_s - y^d_s} + V^{a-1}(1, y^d_s, \tau_j + \Delta)\left(1 - \frac{y'_s - y^d_s}{y^u_s - y^d_s}\right),$$

where y^u_s and y^d_s are the greater and smaller neighbors on the grid.

The iteration is started from the last employment time value, $T + \Delta$, one period after the end of the temporary phase, where the value function does not depend on employment time any more (and the firing boundary becomes a constant) because firing costs are constant afterwards.

[3] With monthly frequency, 36 is the end of the temporary phase, we need one more period to obtain the implications of the permanent phase.

Hence the iteration becomes:

$$V^a(1, y_i, T+\Delta) = \max\left[\left(y_i - q(\tau_j) + e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(0, y_s')\right), \left(y_i + e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(1, y_s', T+\Delta)\right)\right],$$

$$V^a(0, y_i) = \max\left[\left(e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(0, y_s')\right), \left(e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(1, y_s', 0) - c\right)\right].$$

The algorithm then proceeds backwards as in (7). At the last iteration before convergence, we record firing and hiring boundaries as

$$\underline{y}(\tau_j) : \left\{\min\ y_i : \left(y_i - q(\tau_j) + e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(0, y_s')\right) < \left(y_i + e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(1, y_s', \tau_j+\Delta)\right)\right\},$$

$$\overline{y} : \left\{\max\ y_i : \left(e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(0, y_s')\right) > \left(e^{-r\Delta}\frac{1}{n_s}\sum_{s=1}^{n_s} V^{a-1}(1, y_s', 0) - c\right)\right\}.$$

ONLINE APPENDIX II (NOT FOR PUBLICATION): SOLUTION BOUNDARIES

1 Characterization of solution boundaries

To simplify notation, in this appendix we denote the risk neutral drift μ^*, defined in equation (6) of the manuscript, simply by μ.

Proposition 1: The Single Contract

We follow the line of reasoning of Jobert and Rogers (2006).**?** We assume that the firing boundary is monotonically increasing in time until the permanent phase, so that $\underline{y}(q, v_1) < \underline{y}(q, v_2)$ if $v_2 > v_1$. This allows us to handle the most general case and discuss jointly the Single Contract and the Dual Labor. If the opposite monotonicity holds, as it turns out for the Single Contract, the modifications needed will become clear from the context. We let Y_u denote a discrete variable that takes value v if profitability y is between the firing boundaries $\underline{y}(q, v)$ and $\underline{y}(q, v-1)$. If $Y = v$ when time to permanent phase is $u = v - 1$, and the time lapse causes a jump of u to v, profitability will be immediately in the firing region. The need to take into account the possibility of firing because of the simple passage of time, and not only shocks to profitability, implies that Y_u is a state variable. Of course immediate firing by time lapse cannot take place if the firing boundary decreases as time goes by, thus Y_u is not a state variable in this case.

The value functions and the optimal hiring and firing boundaries are determined by the Hamilton Bellman Jacobi (HBJ) equations, the value matching and the smooth pasting conditions. The firm's value function in the idle state depends on firing costs only through the initial firing cost q_0, hence it does not depend on the state variables u and Y_u.

We denote by V the vector of firm's value functions in all possible states of u (time to permanent phase), I (idle or active), and position Y_u. For position to be relevant the firm must be active, which implies that the number of possible position states depends on u. In particular, if $u = v$, profitability y cannot be smaller than $\underline{y}(q, v)$ for the firm to be active, so that there are $N(v) = n+1-v$ possible position states. Since there are $n+1$ time states u for the active firm, and an inactive state (the last entry), the vector V comprises $(n+1)(n+2)/2+1$ conditional value functions. It reads explicitly $V = [V(1, y, q, 0, Y_0) \ldots, V(1, y, q, u, Y_u), \ldots V(1, y, q, n, Y_n), V(0, y)]'$, $Y_u = 1, 2, \ldots, N(u)$ and $u = 0, 1, \ldots, n$, in this order. The following expression reports the system of HJB equations that hold when profitability y is in the continuation regions.

$$\mu V_y^1 + \frac{\sigma^2}{2} V_{yy}^1 + \Gamma V^1 + \Gamma_q V_q^1 + y\bar{1} = 0, \qquad \begin{cases} I = 1 : y \in (\underline{y}(q,u), \infty), \ u = 0, 1, \ldots, n \\ I = 0 : y \in (-\infty, \overline{y}) \end{cases} \tag{1}$$

The first $(n+1)(n+2)/2$ equations in (1) are the ordinary differential equations that the value function of the active firm satisfies when the expected marginal firm value arising from keeping the employee exceeds the marginal value induced by firing the employee and switching to inactive. The last equation in (1) is satisfied by the inactive firm when the expected marginal firm value arising from staying idle exceeds the marginal value induced by hiring the employee and switching to active. The last equation is autonomous, because the firm cannot change state just because time elapses, but only if the profit is high enough. Γ is a square is a Markov transition matrix of dimension $(n+1)(n+2)/2+1$, which governs the transition of the value function when employment time lapses and u switches to the next state. It is built according to the transition possibilities of V. The row corresponding to $V(1, y, q, u, Y_u)$ will have $-r-k$ on the main diagonal, then all zeros except k in the last column if $Y_u = u+1$ (because immediate firing occurs, leading to the idle firm value), or k in the column corresponding to $V(1, y, q, u+1, Y_u)$ otherwise. Γ_q =diag$[\mu_q, \mu_q, \ldots, \mu_q, 0, 0]$. Note that the second to last entry, corresponding to the active firm in the permanent phase, does not depend on μ_q and has only one possible value for Y_u. $\bar{1}$ denotes a vector of ones except zero in the last entry.

The value matching condition (2) says that for the active firm ($I = 1$) in the proximity of the boundary ($Y_u = u + 1$), immediate firing is convenient in the cost regime u at the critical profit $\underline{y}(q,u)$, if the active firm value function at $\underline{y}(q,u)$ coincides with the inactive firm value function once firing costs are subtracted from profits, hence the value function is continuous at $\underline{y}(q,u)$

$$V(1, \underline{y}(q,u), q, u, u+1) = V(0, \underline{y}(q,u)) - q, \quad u = 0, 1, \ldots, n \tag{2}$$

The same must be true for the hiring action: when the firm is inactive its value at the hiring boundary $\overline{y}$ coincides with the active firm value net of hiring costs:

$$V(1, \overline{y}, q, 0, n+1) - c = V(0, \overline{y}) \tag{3}$$

Since the firing boundaries are selected optimally, the marginal value of choosing a slightly different boundary must be equal when the firm passes to a different position state, including from active to idle. Indeed idleness can just be seen as an additional position state. Hence the smooth pasting condition (4) imposes continuity of the derivative of the value function with respect to y at all boundaries:

$$\frac{\partial}{\partial y}V(1, \underline{y}(q, v-1), q, u, v) = \frac{\partial}{\partial y}V(1, \underline{y}(q, v-1), q, u, v-1), \ v = u+2, \ldots, n \qquad u = 0, 1, \ldots, n \tag{4}$$

$$\frac{\partial}{\partial y}V(1, \underline{y}(q, v-1), q, u, v) = \frac{\partial}{\partial y}V(0, \underline{y}(q, v-1)), \quad v = u+1, \quad u = 0, 1, \ldots, n \tag{5}$$

We will guess that the solution for V takes the following form and we will check that our guess is correct:

$$V = e^{yF+q\Lambda}\,\nu + \gamma y + \delta \tag{6}$$

$$\delta = -\Gamma^{-1}\gamma\mu = \frac{\mu}{r}\gamma \tag{7}$$

$$\gamma = -\Gamma^{-1}\overline{1} = \frac{1}{r}\overline{1} \tag{8}$$

where the $(n+1)(n+2)/2+1-$ dimensional square matrix F solves the following matrix equation.

$$\mu F + \frac{\sigma^2}{2}FF + \Gamma + \Gamma_q\Lambda = 0 \tag{9}$$

The exponential in (6) is a matrix exponential.[1] Note that the last HJB equation in (1) is an autonomous equation of the form

$$\mu V_y^0 + \frac{\sigma^2}{2}V_{yy}^0 - rV^0 = 0. \tag{10}$$

(10) has a solution of the form $d\exp(c\ y)$, for some constant d and constant $c = (-\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$. The last term of V is the value function in the inactive state: when $y \to -\infty$ it never hires a worker and its value converges to that of a perpetually idle firm, 0, so that c must be positive. To impose this property, we partition F as

$$F = \begin{bmatrix} \overline{F} \\ \mathbf{0}_{(n+1)(n+2)/2} \quad f_2 \end{bmatrix} \tag{11}$$

[1] Given an $n \times n$ real matrix X, its matrix exponential $\exp(X)$ is defined in terms of the power series representation:

$$\exp(X) = \sum_{i=0}^{\infty}\frac{1}{i!}X^i$$

An important property of the matrix exponential: $\frac{d\exp(yX)}{dy} = X\exp(yX)$, where y is a scalar variable. We have assumed that $\frac{\partial e^{yF+q\Lambda}}{\partial q} = \Lambda e^{yF+q\Lambda}$. While this is not generally the case, it is true in the present context because $\Lambda = rF$.

with $f_2 = (-\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$ and $\overline{F}$ a $[(n+1)(n+2)/2] \times [(n+1)(n+2)/2+1]$ matrix. Substituting (11) into (9) we realize that $\overline{F}$ solves

$$\mu\overline{F} + \frac{\sigma^2}{2}\overline{F} = \begin{bmatrix} \overline{F} \\ \mathbf{0}_{(n+1)(n+2)/2} \quad f_2 \end{bmatrix} + \overline{\Gamma} + \overline{\Gamma}_q\Lambda = 0 \tag{12}$$

$\overline{\Gamma}$ is Γ without the last two rows, and similarly $\overline{\Gamma}_q$. The selected solution for the matrix $\overline{F}$ is such that F has all its eigenvalues negative except the last one because when $y \to \infty$ the firm never fires and its value converges to the present value of a perpetual stream of profits: $\gamma y + \delta$. When the firm is active, the continuation region depends on the state of the factor u. In particular, when $u = 0, 1, \ldots, n-1$, the firing boundary is linear in firing costs (hence in employment time) and it stays fixed to a constant value when there is a transition to $u = n$, which approximates the occurrence of the deterministic time T_u.

$$\underline{y}(q,u) = \begin{cases} aq + b^0 & \text{if} \quad u = 0 \\ aq + b^1 & \text{if} \quad u = 1 \\ \cdots & \cdots \quad \cdots \\ \underline{y} = a\overline{q}^U + b^n & \text{if} \quad u = n \end{cases} \tag{13}$$

We express the constant firing boundary in the constant cost phase as $a\overline{q}^U + b^n$ to emphasize that if the random cost switching time coincided with T_u there would be no discontinuity between the boundaries, and $\underline{y}(q, n-1)$ would converge to $\underline{y}(q,n)$ as $\tau \to T_u$. Because of the approximation error, instead, we have $b^{n-1} \neq b^n$. When n is large, we have $b^{n-1} \approx b^n$ and $\underline{y}(q, n-1) \to \underline{y}(q,n)$ as $\tau \to T_u$. If the firm is inactive, the hiring boundary $\overline{y}$ depends on costs only through the constant initial value q_0 and it is constant. Taking into account (6), the value matching and smooth pasting conditions above read

$$
\begin{aligned}
&e_1'(e^{F\underline{y}(q,0)+q\Lambda}\nu+\gamma\underline{y}(q,0)+\delta)+q=e_{(n+1)(n+2)/2+1}'e^{F\underline{y}(q,0)+q\Lambda}\nu\\
&e_{n+2}'(e^{F\underline{y}(q,1)+q\Lambda}\nu+\gamma\underline{y}(q,1)+\delta)+q=e_{(n+1)(n+2)/2+1}'e^{F\underline{y}(q,1)+q\Lambda}\nu\\
&e_{2n+2}'(e^{F\underline{y}(q,2)+q\Lambda}\nu+\gamma\underline{y}(q,2)+\delta)+q=e_{(n+1)(n+2)/2+1}'e^{F\underline{y}(q,2)+q\Lambda}\nu\\
&\vdots\\
&e_{(n+1)(n+2)/2}'(e^{F\underline{y}(q,n)+q\Lambda}\nu+\gamma\underline{y}(q,n)+\delta)+q=e_{(n+1)(n+2)/2+1}'e^{F\underline{y}(q,n)+q\Lambda}\nu\\
&e_{n+1}'(e^{F\overline{y}+q\Lambda}+\gamma\overline{y}+\delta)-c=\ e_{(n+1)(n+2)/2+1}'e^{F\overline{y}+q\Lambda}\nu\\
&e_1'(Fe^{F\underline{y}(q,0)+q\Lambda}\nu+\gamma)=e_{(n+1)(n+2)/2+1}'Fe^{F\underline{y}(q,0)+q\Lambda}\nu\\
&e_2'(Fe^{F\underline{y}(q,1)+q\Lambda}\nu+\gamma)=e_1'(Fe^{F\underline{y}(q,1)+q\Lambda}\nu+\gamma)\\
&e_3'(Fe^{F\underline{y}(q,2)+q\Lambda}\nu+\gamma)=e_2'(Fe^{F\underline{y}(q,2)+q\Lambda}\nu+\gamma)\\
&\vdots\\
&e_{n+1}'(Fe^{F\underline{y}(q,n)+q\Lambda}\nu+\gamma)=e_n'(Fe^{F\underline{y}(q,n)+q\Lambda}\nu+\gamma)\\
&e_{n+2}'(Fe^{F\underline{y}(q,1)+q\Lambda}\nu+\gamma)=e_{(n+1)(n+2)/2+1}'Fe^{F\underline{y}(q,1)+q\Lambda}\nu\\
&\vdots\\
&e_{n+1}'(e^{F\overline{y}+q\Lambda}+\gamma)=\ e_{(n+1)(n+2)/2+1}'e^{F\overline{y}+q\Lambda}\nu
\end{aligned}
\tag{14}
$$

where $e_1=[1,0,0,\cdots]$, $e_2=[0,1,0,\cdots]$ and so on. Substituting (13) into (14), setting $\Lambda=-aF$ and $a=-\frac{1}{\gamma}$, we obtain the following system of equations, that determines constants b^0, b^1,…,b^n, $\overline{y}$, and the $(n+1)(n+2)/2+1-$dimensional constant vector ν:

$$
\begin{aligned}
&e_1'\left[e^{Fb^0}\nu+\gamma b^0+\delta\right]=e_{(n+1)(n+2)/2+1}'e^{Fb^0}\nu\\
&e_{n+2}'(e^{Fb^1}\nu+\gamma b^1+\delta)=e_{(n+1)(n+2)/2+1}'e^{Fb^1}\nu\\
&e_{2n+2}'(e^{Fb^2}\nu+\gamma b^2+\delta)=e_{(n+1)(n+2)/2+1}'e^{Fb^2}\nu\\
&\vdots\\
&e_{(n+1)(n+2)/2}'(e^{Fb^n}\nu+\gamma b^n+\delta)=e_{(n+1)(n+2)/2+1}'e^{Fb^n}\nu\\
&e_{n+1}'(e^{\overline{y}F-\frac{1}{\gamma}Fq_0}+\gamma\overline{y}+\delta)-c=\ e_{(n+1)(n+2)/2+1}'e^{F\overline{y}-\frac{1}{\gamma}Fq_0}\nu\\
&e_1'(Fe^{Fb^0}\nu+\gamma)=e_{(n+1)(n+2)/2+1}'Fe^{Fb^0}\nu\\
&e_2'(Fe^{Fb^1}\nu+\gamma)=e_1'(Fe^{Fb^1}\nu+\gamma)\\
&e_3'(Fe^{Fb^2}\nu+\gamma)=e_2'(Fe^{Fb^2}\nu+\gamma)\\
&\vdots\\
&e_{n+1}'(Fe^{Fb^n}\nu+\gamma)=e_n'(Fe^{Fb^n}\nu+\gamma)\\
&e_{n+2}'(Fe^{Fb^1}\nu+\gamma)=e_{(n+1)(n+2)/2+1}'Fe^{Fb^1}\nu\\
&\\
&e_{n+1}'(Fe^{\overline{y}F-\frac{1}{\gamma}Fq_0}+\gamma)=\ e_{(n+1)(n+2)/2+1}'Fe^{F\overline{y}-\frac{1}{\gamma}Fq_0}\nu
\end{aligned}
\tag{15}
$$

We have proven the following Proposition:

Proposition 1 *An active firm fires its worker and switches to idle when the profit of the investment opportunity, y, crosses from above the firing boundary $\underline{y}(q,u)$, which takes the piece-wise linear form*

$$\underline{y}(q,u) = \begin{cases} aq + b^0 & \text{if} \quad u = 0 \\ aq + b^1 & \text{if} \quad u = 1 \\ \dots & \\ aq + b^{n-1} & \text{if} \quad u = n-1 \\ \overline{y} = a\overline{q}^U + b^n & \text{if} \quad u = n \end{cases} \tag{16}$$

with $a = -1/\gamma$. If current contract duration is $\overline{\tau} \in \left[\frac{(v)T}{n}, \frac{(v+1)T}{n}\right)$, $v \leq n$, then $u = v$. If y is below the boundary then immediate firing occurs. When current profitability y is between $\underline{y}(q,v)$ and $\underline{y}(q,v-1)$, with $v = u+1, \cdots, n$, and the state of the employment phase is u, the firm values is

$$V(1,y,q,u,v) = \overline{e}'(e^{yF+q\Lambda}\nu + \gamma y + \delta) \tag{17}$$

where $\overline{e}$ is a column vector with 1 in the $[u(u+1)/2 + v - u]-th$ position and zero otherwise. F is the matrix in (11), and $\delta = \frac{\mu}{r}\gamma$, $\gamma = \frac{1}{r}$.

When the firm is not employing a worker, it hires one when the profit of the investment opportunity, y, crosses from below the boundary $\overline{y}$. If y is above the boundary then immediate hiring occurs. The firm's value reads

$$V(0,y) = e'_{(n+1)(n+2)/2+1}(e^{yF+q\Lambda}\nu + \gamma y + \delta) \tag{18}$$

The hiring boundary $\overline{y}$, the constants $(b^0, \dots, b^{n-1}, b^n)$, and constant vector ν solve the system of equations (15).

Proposition 2: The Dual Labor

We apply the same reasonong of the Single Contract case, taking into account that we lose one state variable (q), hence the firing boundary is constant in each employment time state u, and the vector of firm values becomes $V = [V(1,y,0,Y_0)\dots, V(1,y,u,Y_u), \dots V(1,y,n,Y_n), V(0,y)]'$, $Y_u = 1,2,\dots,N(u)$ and $u = 0,1,\dots,n$, in this order. We state the following Proposition, whose derivation is an obvious adaptation of the derivation of Proposition 1.

Proposition 2 *An active firm fires its worker and switches to idle when the profit of the investment opportunity, y, crosses from above the firing boundary $\underline{y}(u)$, which takes $(n+1)$ distinct values depending on the value of the firing cost factor u, for $u = 0,1,\dots,n$. If current* *contract duration is* $\overline{\tau} \in \left[\frac{(v)T}{n}, \frac{(v+1)T}{n}\right)$*,* $v \leq n$*, then* $u_t = v$*. If* y *is below the boundary then immediate firing occurs. When current profitability* y *is between* $\underline{y}(v)$ *and* $\underline{y}(v-1)$*, with* $v = u+1, \cdots, n$*, and the state of the employment phase is* u*, the firm values is*

$$V(1, y, u, v) = \overline{e}'(e^{yF}\,\nu + \gamma y + \delta) \tag{19}$$

where $\overline{e}$ *is a column vector with 1 in the* $[u(u+1)/2 + v - u]-th$ *position and zero otherwise.* $\exp(\cdot)$ *above is a matrix exponential,*

$$F = \begin{bmatrix} & \overline{F} & \\ \mathbf{0}_{(n+1)(n+2)/2} & & f_2 \end{bmatrix} \tag{20}$$

with $f_2 = (-\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$ *and* $\overline{F}$ *a* $[(n+1)(n+2)/2] \times [(n+1)(n+2)/2 + 1]$ *matrix solving*

$$\mu\overline{F} + \frac{\sigma^2}{2}\overline{F} = \begin{bmatrix} & \overline{F} & \\ \mathbf{0}_{(n+1)(n+2)/2} & & f_2 \end{bmatrix} + \overline{\Gamma} = 0 \tag{21}$$

Moreover $\delta = \frac{\mu\gamma}{r}$*,* $\gamma = \frac{1}{r}$*.*

When the firm is not employing a worker, it hires one when the profit of the investment opportunity, y*, crosses from below the boundary* $\overline{y}$*. If* y *is above the boundary then immediate hiring occurs. The firm's value reads*

$$V(0, y) = e'_{(n+1)(n+2)/2+1}(e^{yF}\,\nu + \gamma y + \delta)$$

The hiring boundary $\overline{y}$*, the firing boundaries* $\underline{y}(u)$*, and the constant vector* ν *solve the system* *of equations:*

$$
\begin{aligned}
&e_1'\left[e^{F\underline{y}(0)}\nu + \gamma\underline{y}(0) + \delta\right] + \underline{q} = e'_{(n+1)(n+2)/2+1}e^{F\underline{y}(0)}\nu \\
&e'_{n+2}(e^{F\underline{y}(1)}\nu + \gamma\underline{y}(1) + \delta) + \underline{q} = e'_{(n+1)(n+2)/2+1}e^{F\underline{y}(1)}\nu \\
&e'_{2n+2}(e^{F\underline{y}(2)}\nu + \gamma\underline{y}(2) + \delta) = e'_{(n+1)(n+2)/2+1}e^{F\underline{y}(2)}\nu \\
&\vdots \\
&e'_{(n+1)(n+2)/2}(e^{F\underline{y}(n)}\nu + \gamma\underline{y}(n) + \delta) = e'_{(n+1)(n+2)/2+1}e^{F\underline{y}(n)}\nu \\
&e'_{n+1}(e^{\overline{y}F} + \gamma\overline{y} + \delta) - c = \ e'_{(n+1)(n+2)/2+1}e^{F\overline{y}}\nu \\
&e_1'(Fe^{F\underline{y}(0)}\nu + \gamma) = e'_{(n+1)(n+2)/2+1}Fe^{F\underline{y}(0)}\nu \\
&e_2'(Fe^{F\underline{y}(1)}\nu + \gamma) = e_1'(Fe^{F\underline{y}(1)}\nu + \gamma) \\
&e_3'(Fe^{F\underline{y}(2)}\nu + \gamma) = e_2'(Fe^{F\underline{y}(2)}\nu + \gamma) \\
&\vdots \\
&e'_{n+1}(Fe^{F\underline{y}(n)}\nu + \gamma) = e_n'(Fe^{F\underline{y}(n)}\nu + \gamma) \\
&e'_{n+2}(Fe^{F\underline{y}(1)}\nu + \gamma) = e'_{(n+1)(n+2)/2+1}Fe^{F\underline{y}(1)}\nu \\
&\\
&e'_{n+1}(Fe^{\overline{y}F} + \gamma) = \ e'_{(n+1)(n+2)/2+1}Fe^{F\overline{y}-}\nu
\end{aligned}
\tag{22}
$$

2 Numerical implementation

This Appendix explains how we constructed the figures reported in Sections 4 and 5.

Figure 1. The boundaries plotted in Panel B are characterized in Proposition 1 (Single Contract) and Proposition 2 (Dual Labor), and they have been obtained using $n = 35$ states for the state variable u in the Single Contract, and $n = 25$ in the Dual Labor. The parameters used are reported in Table 1. Each state of u has been mapped into employment duration: if current contract duration is $\overline{\tau} \in \left[\frac{vT}{n}, \frac{(v+1)T}{n}\right)$, $v \leq n$, then $u_t = v$. An interpolating curve with a modest degree of smoothing has been applied between the discrete realizations of the firing boundaries $\underline{y}(q,0), \underline{y}(q,1), \ldots, \underline{y}(q,35)$ for the Single Contract and those of the Dual Labor, $\underline{y}(0), \underline{y}(1), \ldots, \underline{y}(25)$.

Figure 2. Panel A plots the firing and the hiring boundaries for the Dual Labor when the temporary firing cost is $\underline{q} = 3 * 0.05$, everything else (including the number of points for the state variable u) is as in Panel B of Figure 1. Figure 2 also reports (as dotted lines) the benchmark hiring and firing boundaries of Figure 1, corresponding to $\underline{q} = 25 * 0.05$. Panel B plots the Single Contract firing boundary for different levels of T_S and μ_q, such that permanent costs are constant at $45 * 0.05$ and cumulative costs of the temporary phase are $25 * T_S$, namely $(T_S,\mu_q) = (3, 40/3), (5, 8), (7, 5.71), (10, 4)$. These boundaries have been obtained using $n = 10$ states for the state variable u. An interpolating curve with a modest degree of smoothing has been applied between the discrete realizations. The rest of the parameters is as in Table 1.

Figure 3. For the firing rules reported in Panel B of Figure 1, Panel A reports the expected employment duration of a just hired worker ($\tau = 0$) conditional on being fired during the temporary phase. It is obtained as follows for the Single Contract (the same reasoning applies to the Dual Labor): consider an active firm with current profits y_0, and with firing boundary $\underline{y}(q,\tau)$. As in Proposition 1, we have mapped the state variable u into employment duration τ_t by setting $u_t = v$ if current employment duration is $\overline{\tau} \in \left[\frac{vT}{n}, \frac{(v+1)T}{n}\right)$, $v \leq n$. We consider a worker at the beginning of the employment relation ($\tau = 0$) and consider the event of firing during the temporary phase, $\tau \in [0, T]$.

$$\theta = \inf \{t \in (0,T] : y_t \leq \underline{y}(q_t, \tau_t)\} \tag{23}$$

$P(\theta \leq T|y_0, \tau = 0)$ and $E[\theta|y_0, \theta \leq T, \tau = 0]$ denote, respectively, the probability that firing occurs before the end of the temporary phase (conditional on present profits and employment being at the starting values), and the expected firing time conditional on firing having occurred before the end of the temporary phase. These quantities can be computed from the density of the first passage time from zero of the Brownian Motion with deterministic drift $\widehat{y}_t$, where

$$\widehat{y}_t = y_t - \underline{y}(q_t, \tau_t) = (y_0 - \underline{y}(q_0, \tau_0)) - \sum^{i:t \leq \frac{(i+1)T}{n}} (b^i - b^{i-1}) + \int_0^t [\mu + r\mu_q] ds + \int_0^t \sigma dB_s \tag{24}$$

as discussed on Karatzas and Shreve (1991)

$$P(\theta \leq T|y_0, \tau = 0) = N(d_1(T)) + \exp\left(-2\frac{-\sum^{i:T \leq \frac{(i+1)T}{n}} (b^i - b^{i-1}) + \int_0^T [\mu + r\mu_q] ds}{\sigma^2 T}\right) N(d_2(T)) \tag{25}$$

$$E[\theta|y_0, \theta \leq T, \tau = 0] = \frac{E[\theta \mathbf{1}(\theta \leq T)|y_0, \tau = 0]}{P(\theta \leq T|y_0, \tau = 0)} = \frac{\int_0^T t \; dP(\theta \leq t|y_0, \tau = 0)}{P(\theta \leq T|y_0, \tau = 0)} \tag{26}$$

where

$$d_1(T) = \frac{-(y_0 - \underline{y}(q_0, \tau_0)) - \sum^{i:T \leq \frac{(i+1)T}{n}} (b^i - b^{i-1}) + \int_0^T [\mu + r\mu_q] ds}{\sigma\sqrt{T}} \tag{27}$$

and

$$d_2(T) = \frac{-(y_0 - \underline{y}(q_0, \tau_0)) + \sum^{i:T \leq \frac{(i+1)T}{n}} (b^i - b^{i-1}) + \int_0^T [\mu + r\mu_q] ds}{\sigma \sqrt{T}}, \tag{28}$$

and $dP(\theta \leq t | y_0, \tau = 0)$ is the probability density of the first passage time distribution.

We report quantities (25) and (26) computed by Monte-Carlo simulation:

$$P(\theta \leq T | \ y_0, \tau = 0) = \frac{1}{n} \sum_{j=1}^{n} 1(\theta^j \leq T) \tag{29}$$

$$E[\theta | \ y_0, \theta \leq T, \tau = 0] = \frac{\sum_{j=1}^{n} \mathbf{1}(\theta^j \leq T)\theta^j}{\sum_{j=1}^{n} \mathbf{1}(\theta^j \leq T)} \tag{30}$$

(30) is plotted in Panel A of Figure 3. Superscripts denote simulated realizations. θ^j denotes $\inf\{t \in (0, T] : y_t^j \leq \underline{y}(q_t^j, \tau_t^j)\}$ along the j−th simulated path of profits, if crossing of the firing boundary has occurred over this path. We simulate $n = 50000$ paths with 250 discretization steps per year.[2]

In Panel B of Figure 3 we consider a worker at the beginning of the permanent employment relation ($\tau = T$) and fired within the next 10 years, $\tau_t \in [T, T + 10y]$. We compute the conditional expected firing time (30) as before with the simplification that firing boundaries are constant in this phase.

Figure 4. For the firing rules reported in Panel B of Figure 1, Panel A reports the probability that an unemployed worker (initial level of profits $y_0 = -0.0826 < \overline{y}$) is hired before some time S, reported on the x axis, under each labor legislation. This probability, corresponds to (29) above, with T replaced by S and the firing boundary replaced by the constant hiring boundary. It is computed as detailed in the description of Figure 3. Since hiring boundaries are constant, the relative patterns would not change for different levels of initial profits. Panel C reports the probability of being fired during the temporary phase for a new hired worker ($\tau = 0$), conditional on different levels of the initial profits y_0. This probability, corresponds to (29) above, and it is computed as in Figure 3. Panel B reports the same probability for a worker

[2]We apply importance sampling in order to reduce the estimation error, using the property:

$$E[f(y_\theta)|y_0] = E^Q \left[e^{-\frac{1}{2}d^2\theta + dB_\theta^Q} f(y_\theta)|y_0 \right] \tag{31}$$

where under the probability measure Q, y evolves as:

$$dy_t = (\mu - \sigma d) \ dt + \sigma dB^Q$$

B^Q being a standard Brownian motion under Q. Simulating y under Q, with $d = 1.5$, and estimating the right hand side of (31) achieves a significant variance reduction in the estimation of (30), because of the higher number of firing events occurring with a smaller drift.

at the beginning of the permanent phase ($\tau = T_S = \overline{T}$) and the event of firing during the next 10 years. Panel D considers 15 years of future employment history for a worker who enters the job market at time 0, for several initial levels of profitability y_0. If the initial profitability is below the hiring boundary the worker starts unemployed. The panel reports the expected cumulative time spent employed out of the 15 years. This cumulative time is computed as follows: we simulate $n = 500000$ paths of 15 years of firm's profits with 250 discretization steps per year, conditional on a given initial profitability level y_0 and the firing and hiring boundary of each regulation. At each point in time, for each path, we record the employment status by the relative position of current profits with respect to the firing and hiring boundary. The approximate expected cumulative time spent employed is

$$E\left[\int_0^{15y} [\mathbf{1}(y_t > \underline{y}(q_t, \tau_t))\mathbf{1}(I_{t-1} = 1) + \mathbf{1}(y_t > \overline{y})\mathbf{1}(I_{\mathbf{t-1}} = 0)]\, ds \mid y_0\right]$$
$$= \frac{1}{n}\sum_{j=1}^{n} \frac{1}{250} \sum_{t=1}^{250*15} [1(y_t^j > \underline{y}(q_t^j, \tau_t^j))1(I_{t-1}^j = 1) + 1(y_t^j > \overline{y})1(I_{t-1}^j = 0)]$$

where I_t is an indicator function that has value 0 if the firm is not employing.

Figure 5. Panel A plots a Single Contract that has the same initial cost of the Dual Labor ($q_0 = \underline{q} = 25 * 0.05$) and cost appreciation rate $\mu_q = (20/3) * 0.05$, so that the permanent costs of the two legislations after $T_S = \overline{T} = 3$ years coincide ($45 * 0.05$). Panel B reports the firing and hiring boundaries obtained with the same methodology discussed for Panel B of Figure 1.

Figure 6. The figure plots the firing boundaries obtained by changing r^* through variations of δ. They have been obtained using $n = 10$ states for the state variable u and seven distinct values of the discount rate, $\delta = (0, 0.04, 0.08, 0.12, 0.15, 0.17, 0.2)$, and then interpolating between these values. Each state of u has been mapped into employment duration: if current contract duration is $\overline{\tau} \in \left[\frac{vT}{n}, \frac{(v+1)T}{n}\right)$, $v \leq n$, then $u_t = v$.

Figure 7 has been obtained as in Figure 6, but changing the risk neutral expected profit variation μ^* and keeping the rest of the parameters as in Table 1.

Figure 8 plots the firing boundaries of the Single Contract (Panel A) and of the Dual Labor (Panel B) obtained by changing the risk neutral volatility of profit variation σ^*, and keeping the rest of the parameters as in Table 1. Panel C reports the firing boundaries of the Dual Labor corresponding to different volatility levels (under the risk-neutral probability measure) when the option to fire expires at employment duration $\tau = \overline{T}$, that is, assuming permanent workers cannot be fired. It is the limiting case of our model when the permanent firing cost $\overline{q}$ is infinite, so that the firing boundary in the permanent phase is $\underline{y}(q_t, \tau) = -\infty$, $\tau \in (T_d, \infty)$. We adapted Proposition 2 to retrieve the firing and hiring policy for $\tau \in [0, T_d]$. Referring to the proof of Proposition 2, it is convenient to partition V^1 (the vector of the active firm values in each state of the factor u that determines the switch of the firing cost phase) as $V^1 = [\overline{V}(1,y,\underline{q}), V_n(1,y)]'$, where $\overline{V}(1,y,\underline{q}) = [V_0(1,y,\underline{q}), V_1(1,y,\underline{q}), \ldots, V_{n-1}(1,y,\underline{q})]'$. When $u = n$ the worker is permanent, and the firm's value is given by the expected discounted value of a constant flow of profits y_s, because we are assuming no firing. Thus, $V_n(1,y) = \frac{y}{r} + \frac{\mu}{r^2}$. In the idle state we denote the firm's value as $V^0 = V(0,y)$. As in Proposition 2, the firing and hiring boundaries are characterized by the Bellman equations solved by the continuation value of the firm in each state of the temporary phase, the value matching and the smooth-pasting conditions:

$$\mu\overline{V}_y + \frac{\sigma^2}{2}\overline{V}_{yy} + \Gamma\overline{V} + y\overline{1}_n + \begin{bmatrix} 0\overline{1}_{n-1} \\ k(\frac{y}{r} + \frac{\mu}{r^2}) \end{bmatrix} = 0 \quad y \in (\underline{y}(u), \infty),\ u = 0,1,\ldots,n-1 \tag{32}$$

$$\mu V^0_y + \frac{\sigma^2}{2}V^0_{yy} - rV^0 = 0 \quad y \in (-\infty, \overline{y}) \tag{33}$$

$$V_u(1,\underline{y}(u),\underline{q}) = V(0,\underline{y}(u)) - \underline{q}, \quad u = 0,1,\ldots,n-1 \tag{34}$$

$$\frac{\partial}{\partial y}V_u(1,\underline{y}(u),\underline{q}) = \frac{\partial}{\partial y}V(0,\underline{y}(u)), \quad u = 0,1,\ldots,n-1 \tag{35}$$

$$V_u(1,y,\underline{q}) = V(0,y) - \underline{q} \quad y \in (-\infty, \underline{y}(u)),\ u = 0,1,\ldots,n-1 \tag{36}$$

$$V(1,\overline{y},\underline{q}) - c = V(0,\overline{y}), \tag{37}$$

$$\frac{\partial}{\partial y}V(1,\overline{y},\underline{q}) = \frac{\partial}{\partial y}V(0,\overline{y}), \tag{38}$$

$$V(1,y,\underline{q}) - c = V(0,y) \quad y \in (\overline{y}, \infty) \tag{39}$$

where Γ is the following $n \times n-$dimensional matrix

$$\Gamma = \begin{bmatrix} -r-k & k & 0 & 0 & \ldots & \ldots & 0 \\ 0 & -r-k & k & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & \ldots & 0 & 0 & 0 & -r-k \end{bmatrix} \tag{40}$$

Solutions for $\overline{V}$ and V^0 take the following form:

$$\overline{V} = e^{yF}\nu + (\gamma y + \delta) \tag{41}$$

$$V^0 = e^{yf_2}\nu_0 \tag{42}$$

$$\delta = -\Gamma^{-1}\left(\gamma\mu + \begin{bmatrix} 0\overline{1}_{n-1} \\ k\frac{\mu}{r^2} \end{bmatrix}\right) \tag{43}$$

$$\gamma = -\Gamma^{-1}\begin{bmatrix} \overline{1}_{n-1} \\ 1+\frac{k}{r} \end{bmatrix} \tag{44}$$

$$F := \mu F + \frac{\sigma^2}{2} FF + \Gamma = 0 \tag{45}$$

$$f_2 := \mu f_2 + \frac{{\sigma_2}^2}{2} f_2^2 - r = 0 \tag{46}$$

The rest of the proof follows Proposition (2).

Figure 9 plots the firing boundaries for seven values of the risk aversion parameter $\gamma = (1.5, 2, 3, 4, 5, 6, 6.2)$. We used $n = 10$ states for the state variable u, and interpolated the boundaries between the γ values. All parameters are those of Table 1 except $\sigma = 0.15$ and $\delta = 0.17$.

References

Karatzas, I. and Shreve, S.: 1991, *Brownian motion and stochastic calculus*, Springer.