Asset Pricing with Fiscal Uncertainty

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Abstract

This paper studies theoretically and empirically the effect of fiscal policy uncertainty on asset prices. We first propose a general equilibrium model endowed with a public and private sector. The latter is populated by heterogeneous agents who learn about the impact of the public sector’s fiscal policy on firms. Agents disagree about the effectiveness of the government’s fiscal policy and their disagreement generates fiscal uncertainty risk that is priced in equilibrium. We then estimate a novel model-implied measure of fiscal disagreement from a large cross-section of forecasts on future budget deficits. Using this proxy, we find that firms with lower loadings on fiscal disagreement outperform firms with high loadings by 6.58% annually. The implied market price of fiscal uncertainty risk is negative, highly statistically significant and cannot be explained by other standard risk factors. Finally, a calibrated version of the model matches well the empirical findings recovered in the data.

Keywords: Fiscal uncertainty, learning, disagreement, stock returns

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Over the past couple of years, investors have faced larger than usual uncertainty about government policies. For example, the uncertainty surrounding the expiration of the Bush tax cuts, the fiscal cliff and the government shutdown in 2013 are often cited as having had a negative impact on real quantities and asset prices. While the recent literature has studied the effect of fiscal uncertainty on macroeconomic quantities or the aggregate stock market, we show in this paper both theoretically and empirically that fiscal uncertainty is an important determinant of the cross-section of equity returns.

Motivated by work that links government uncertainty to asset prices (see, e.g., Pástor and Veronesi (2012, 2013)), we explore how agents’ disagreement about the impact of a fiscal policy on fundamentals affects firms’ stock returns. To this end, we propose a parsimonious general equilibrium model where heterogeneous agents disagree on the extent to which public spending affects the aggregate output growth rate. In particular, we assume that the government implements a countercyclical fiscal rule that reacts to negative (positive) past output shocks by expanding (contracting) its spending target, which impacts expected output growth. In order to implement its fiscal rule, the government diverts a fraction of aggregate output from the private sector. Agents have different beliefs about the scope for public stimulus: agent a (b) believes that the government sets a larger (smaller) long-term goal for the output growth rate induced by public intervention. Since agents “agree to disagree” and never converge to a consensus fiscal rule, their beliefs about the future evolution of output and public spending diverge systematically. The likelihood ratio of these beliefs is a proxy for the uncertainty about the fiscal rule and fluctuations in fiscal uncertainty are priced in equilibrium.

To generate a cross-section of firms, we adopt the EBIT-based modeling framework of Leland (1994), Goldstein, Ju, and Leland (2001), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010), among others, where firms have a static capital structure consisting of equity and a console bond. After modelling firms’ exogenous random default event to replicate the properties of an endogenous default, we obtain equity and bond prices, and risk premia in closed form, modulo a simple inverse Fourier transform. Using a set of calibrated parameters, we inspect whether the predictions of our model are
We then test the theoretical predictions in the data. To this end, we first construct a novel measure of fiscal disagreement from a large cross-section of forecasts about next year’s Federal budget deficit and US GDP growth. Using a Kalman filtering approach, we then estimate for each individual forecaster her perceived fiscal action. Each month, we then define fiscal disagreement as the difference between those forecasters who believe that the government’s fiscal policy is effective and ineffective. To measure each firm’s sensitivity with respect to the fiscal uncertainty proxy, we run rolling regressions from stock returns onto changes in our fiscal uncertainty proxy. Using data from 1994 to 2013, we show that loadings on changes in fiscal uncertainty vary a lot in the cross-section and negatively predict future stock returns. For example, when we sort stocks into different quintiles based on their fiscal uncertainty exposure, we find that low exposure stocks offer significantly higher returns than high exposure stocks. More concretely, the portfolio with the lowest exposure has an annual return of 13.49% whereas the highest exposure portfolio has a return of 6.99%. The spread is −6.5% per year and statistically highly significant. Interestingly, we find almost no relationship between fiscal exposure sorted portfolios and the leverage or size of a firm. Both the average leverage and the average firm size are almost constant across the different portfolios, implying that there is an almost flat relationship between exposure to fiscal uncertainty and leverage/firm size. We also re-confirm these findings using standard Fama and MacBeth (1973) regressions.

Our paper proceeds as follows. Section 1 sets up a general equilibrium featuring fiscal uncertainty and Section 2 presents the empirical analysis. Section 3 concludes. Proofs are deferred to the Appendix.

**Literature Review:** Our paper is most closely related to the literature studying the link between government policy and asset prices (see Pástor and Veronesi (2012, 2013)). In their model, expected firm profitability is affected by the prevailing government policy and this effect is unknown. Both the government and investors learn about the current policy’s impact in a Bayesian fashion. Two types of uncertainty affect stock returns: Impact and political uncertainty. The former origins from agents’ learning
process and is represented by the standard deviation of agents’ prior beliefs about the policy impact. The latter is associated with a political cost that is incurred whenever the government changes its policy. Different from their contribution, in our model fiscal policy uncertainty is generated by agents’ disagreement about the precise impact of a fiscal rule. Since we are interested in the implications of this uncertainty for the cross-section of returns, we explore how exposure to systematic risk and to the impact of public spending generates heterogeneous exposure to fiscal uncertainty. Moreover, fiscal uncertainty exposure is tightly linked to a firm’s fundamentals like leverage, its exposure to government spending, and the cyclicality of its earnings.

This is not the first paper that studies the effect of fiscal uncertainty on stock returns. For example, Croce, Kung, Nguyen, and Schmid (2012) examine the effects of fiscal policies in a production-based general equilibrium model in which taxation affects corporate decisions. When agents feature recursive preferences, the authors show that tax uncertainty contributes significantly to the market equity premium. Sialm (2006) finds that both term and equity premia are higher because they compensate investors’ for the risk that taxes change over time. Gomes, Michaelides, and Polkovnichenko (2012) study an incomplete market model with heterogeneous agents where government debt and equity are imperfect substitutes. Changes in tax rates and government debt affect asset prices. For example, an increase in public debt is shown to lead to both a higher risk-free rate and a lower equity premium. Different from these papers, we study a different type of fiscal uncertainty which stems from agents’ disagreement. Moreover, we study the effect on the cross-section of equity returns rather than the market itself.

The effect of government spending and more generally political cycles on asset prices is studied in Belo, Gala, and Li (2013) who construct a novel measure of industry exposure to government spending. It is defined as the proportion of each industry’s total output that is purchased directly by the government sector, as well as indirectly through the chain of economic links across industries. The authors find that stock returns are predictable over political cycles and that during Democratic (Republican) presidencies firms with higher exposure to government spending experience higher (lower) stock returns. Da, Warachka, and Yun (2014) study how fiscal policies affect consumption volatility
on the US state level and document that volatility is lower in states that implement counter-cyclical fiscal policies and that stock returns of firms which are headquartered in these states have lower stock returns. The role of government spending uncertainty on private investments has been explored in for example Julio and Yook (2012) who find that firms’ investments drop during election periods. Our paper also contributes to the literature that studies the effect of government spending on firm’s capital structure. For example, Graham, Leary, and Roberts (forthcoming) find that government debt is strongly negatively correlated with corporate debt and investment, suggesting a crowding out effect from public spending. Large and less risky firms are more affected as their corporate debt is a closer substitute to Treasuries.

A large macroeconomic literature studies the relationship between fiscal uncertainty and the real economy. Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2013) study the effect of temporary increases in fiscal policy uncertainty on real quantities within a New Keynesian model. To this end, the authors estimate different fiscal rules and interpret the changes in the volatility of the innovations in the fiscal rules as fiscal policy uncertainty.

Finally, our paper is also related to the literature that studies the asset pricing implications of learning and heterogeneous agents. For example, Basak (2000, 2005) and Buraschi and Jiltsov (2006) study how equilibrium consumption sharing between optimistic and pessimistic agents affects equity and option prices. Dumas, Kurshev, and Uppal (2009) also allow for learning and focus on the implications for equity and optimal portfolio allocations while Buraschi, Trojani, and Vedolin (2013) study how disagreement affects credit spreads in the cross-section.

1 The Model

In this Section, we describe a general equilibrium model endowed with a public (the government) and private sector where the latter is populated by disagreeing investors. The model draws upon two strands of the literature (i) the political uncertainty literature (see e.g., Pástor and Veronesi (2012)) and (ii) the literature studying how investors’ disagreement affects asset prices (see e.g., Scheinkman and Xiong (2003)). In the spirit of
the latter, we assume that agents do not observe the precise impact of the government’s fiscal action on growth rates of fundamentals. They also disagree on the characteristics of this action, namely its long-term goal and cyclicality. We next discuss the economic environment, derive equilibrium quantities and characterize the impact of agents’ disagreement on asset prices.

1.1 Fundamentals

We consider a continuous-time endowment economy defined on an infinite time-horizon, and populated by two groups of agents: the “public sector”, or Government, and the “private sector”. The public expense for consumption and investment \( G_t \) is modeled exogenously as a time-varying fraction \( \tilde{g}_t \) of aggregate output \( C_t \): \( G_t = \tilde{g}_t C_t \). The purpose of public expenditure is to control the expected output growth rate by means of an action \( A_t \). In other words, the aggregate endowment evolves as follows:

\[
\frac{dC_t}{C_t} = \left( \mu_c + A_t \right) dt + \sigma_c dZ_t,
\]

where \( \mu_c \) and \( \sigma_c \) are constant parameters. By definition the government action impacts public expense. For analytical convenience, we model the dynamics of \( g_t = 1 - \tilde{g}_t \), the percentage of output accruing to the private sector net of public expense:

\[
\frac{dg_t}{g_t} = -\alpha A_t dt + \sigma_g dB_t.
\]

Intuitively, larger actions reduce the expected private share: if a fiscal expansionary multiplier effect is present, the impact of the action on output is larger than its impact on the budget, hence \( \alpha \) should be positive and smaller than one.\footnote{While the process \( g_t \) is not bounded above by one – contrary to the consumption shares processes in Menzly, Santos, and Veronesi (2004), for instance – they allow analytical tractability. We check that the parameter set used in the empirical section implies a very small probability that the process exceeds one.} However, macroeconomic studies disagree on the extent of fiscal multipliers, often arguing about a contractionary effect due to crowding-out, in which case \( \alpha > 1 \). We refer to \( A_t \) as a “fiscal policy rule”
and we assume that it is designed to reach a long-term target $\bar{A}$ – at an exponential rate $\lambda$ – and to partially offset past shocks to output growth:

$$A_t = \bar{A}(1 - e^{-\lambda t}) + \rho \int_0^t e^{-\lambda(t-s)} \sigma_A \left( \frac{dC_s}{C_s} - (\mu_c + A_s) ds \right) ds + \sqrt{1 - \rho^2} \int_0^t e^{-\lambda(t-s)} \sigma_A dW_s. \quad (3)$$

Note that the second component of $A_t$ in expression (3) is an exponentially weighted average of past unexpected output growth, scaled by a constant $\sigma_A$, and pre-multiplied by a correlation parameter $\rho$. $\rho$ captures the correlation between aggregate output and the fiscal action. The intuition is that current public expense can have an effect on output growth, however, this effect is not immediate but takes time to materialize. The residual component is driven by an independent innovation $dW_t$. Taking differentials in (3) leads to the following dynamics:

$$dA_t = \lambda(\bar{A} - A_t) dt + \sigma_A \left( \rho dZ_t + \sqrt{1 - \rho^2} dW_t \right). \quad (4)$$

The private sector is uncertain about the fiscal action because $A_t$ is unobservable, as well as the Brownian motions $Z_t$ and $B_t$. There are two types of representative private agents, $a$ and $b$, holding different beliefs about the data generating processes, possibly due to contrasting views about the effectiveness of public economic intervention. For example, the economic crisis of 2008 ignited a heated discussion about US fiscal policy with the Federal Funds rate close to zero. While some economists emphasized an increase in government spending (see, e.g., Krugman (2009)), others argued that the best response would be to reduce both taxes and spending (see, e.g., Cato Institute (2009)).

As in Scheinkman and Xiong (2003) or Dumas, Kurshev, and Uppal (2009), this heterogeneity originates in subjective views about the model: in particular group $a$, supporting intervention, believes that the effect of public spending is countercyclical, in that the government aims at a balanced target that partially hedges past output shocks. Moreover, it assumes a large long-term growth goal. Group $b$ instead, supporting a laissez faire policy, does not believe that the action offsets past output shocks and undermines fiscally induced long-term growth. Denoting agents’ subjective parameters
by a subscript, we can summarize these views as follows: $\bar{A}_a > \bar{A}_b$, $\rho_a < 0$, $\rho_b = 0$. It is reasonable to assume that both cyclicality and long-term impact of the fiscal action are subject to disagreement. While net public spending propensity is observable, its effect on growth is an estimate subject to measurement error and influenced by prior beliefs about the political cycles. Both types of agents infer in a Bayesian fashion the policy rule $A_t$ from the available information set, which comprises output growth and private output share growth. Since they “agree to disagree” about the evolution of the rule, their posterior estimates $\hat{A}_i^t = E_i[A_t|\mathcal{F}_t^{c,g}]$, $i = a, b$ will diverge in general:

$$
\begin{align*}
\frac{d\hat{A}_i^t}{dt} &= \lambda(\bar{A}_i - \hat{A}_i^t)dt + \frac{\eta_i^t + \sigma_A \rho_i}{\sigma_c} \frac{d\hat{Z}_i^t}{dt} - \eta_i^t \frac{\alpha}{\sigma_g} d\hat{B}_i^t \quad (5)
\end{align*}
$$

These dynamics follow from Theorem 12.7 in Liptser and Shiryaev (2000). The processes

$$
\begin{align*}
\frac{d\hat{Z}_i^t}{dt} &= \frac{1}{\sigma_c} \left[ \frac{dC_t}{C_t} - (\mu_c + \hat{A}_i^t)dt \right] \quad (6) \\
\frac{d\hat{B}_i^t}{dt} &= \frac{1}{\sigma_g} \left[ \frac{dg_t}{g_t} + \alpha \hat{A}_i^t dt \right] \quad (7)
\end{align*}
$$

are standard Brownian motions under the subjective belief of agent $i$, and $E_i[\cdot]$ denotes expectation with respect to this probability measure. The distinct posterior variances $\eta_i^t = E_i[(A_t - \hat{A}_i^t)^2|\mathcal{F}_t^{c,g}]$ can be shown to converge asymptotically to the limit:

$$
\eta_i = \sqrt{\left(\lambda + \frac{\sigma_A \rho_i}{\sigma_c}\right)^2 + \sigma^2_A (1 - \rho_i^2) \left(\frac{1}{\sigma_c^2} + \frac{\alpha^2}{\sigma_g^2}\right) - \left(\lambda + \frac{\sigma_A}{\sigma_c} \rho_i\right) \left(\frac{1}{\sigma_c^2} + \frac{\alpha^2}{\sigma_g^2}\right)}.
$$


\(^3\)For instance, Belo, Gala, and Li (2013) find differential effects of government spending on firms during Democratic and Republican presidencies.
Following the incomplete information literature, we set $\eta_i^t = \eta_i$, assuming convergence has taken place. The subjective dynamics about the growth rates of output and private share are as follows:

\[
\frac{dC_t}{C_t} = (\mu_c + \hat{A}_t) dt + \sigma_c d\hat{Z}_t, \\
\frac{dg_t}{g_t} = -\alpha \hat{A}_t dt + \sigma_g d\hat{B}_t, \\
\] (9)

which implies the following restrictions about the innovations:

\[
d\hat{Z}_t^b = d\hat{Z}_t^a + \frac{\hat{A}_t^a - \hat{A}_t^b}{\sigma_c} dt, \quad d\hat{B}_t^b = d\hat{B}_t^a - \frac{\alpha (\hat{A}_t^a - \hat{A}_t^b)}{\sigma_g} dt. \\
\] (10)

We refer to $\phi_t = \hat{A}_t^a - \hat{A}_t^b$ as the disagreement between the inferences of the two types. The disagreement process is our proxy for fiscal uncertainty. Expression (10) together with Girsanov’s theorem suggest that the difference in belief can be conveniently summarized by the likelihood ratio process $\theta_t = \frac{dP^b}{dP^a} \bigg|_t$, with the following dynamics:

\[
\frac{d\theta_t}{\theta_t} = -\phi_t \left( \frac{1}{\sigma_c} d\hat{Z}_t^a - \frac{\alpha}{\sigma_g} d\hat{B}_t^a \right), \\
\] (11)

so that expectations about agent b’s subjective belief can be expressed in terms of group a’s: $E^b[X] = E^a[\theta_t X]$. By Itô’s lemma, the disagreement process $\phi_t$ evolves according to the following mean-reverting dynamics:

\[
d\phi_t = \omega(\overline{\phi} - \phi_t) dt - (\eta_a - \eta_b) \frac{\alpha}{\sigma_g} d\hat{B}_t^a + (\eta_a + \sigma_A \sigma_c \rho_a - \eta_b) \frac{1}{\sigma_c} d\hat{Z}_t^a. \\
\] (12)

where

\[
\omega = \lambda + \eta_b \left( \frac{\alpha^2}{\sigma_g^2} + \frac{1}{\sigma_c^2} \right) \quad \text{and} \quad \overline{\phi} = \frac{\lambda (\overline{A}_a - \overline{A}_b)}{\omega} .
\]

1.2 Characterization of Equilibrium

We assume that both agents have CRRA additive utility of inter-temporal consumption, with homogeneous parameter of relative risk aversion $\gamma$ and subjective discount rate $\delta$. Markets are complete, because we assume that agents can continuously trade at least one financial asset in addition to a locally risk-less bond in zero net supply – with interest
rate $r_t$ – and the stock market, a claim to aggregate output. There is a unique state-price density, denoted by $\xi^a_t$ ($\xi^b_t$) when represented relative to the subjective probability measure of agent $a$ ($b$). $\xi^a_t$ reads explicitly:  

$$ \xi^a_t = \exp \left( -\int_0^t \left( r_s + \frac{(\kappa^a_{s,Z})^2 + (\kappa^a_{s,B})^2}{2} \right) ds - \int_0^t \left( \kappa^a_{s,Z} d\hat{Z}^a_s + \kappa^a_{s,B} d\hat{B}^a_s \right) \right), $$

(13)

where $\kappa^a_{s,Z}$ and $\kappa^a_{s,B}$ are the market prices of risk as perceived by agent $a$. The following Proposition explicits the state-price density and its components:

**Proposition 1.** Relative to agent $a$’s belief, the state-price density reads explicitly:

$$ \xi^a_t = e^{-\delta t} (g_t C_t)^{-\gamma} \left( 1 + \theta^a_t \right)^\gamma, $$

(14)

while the equilibrium interest rate and market prices of risk are:

$$ r_t = \delta + (\bar{\mu} + (1 - \alpha) \widehat{A}^b_t) \gamma - \frac{1}{2} \gamma (\gamma + 1) (\sigma_c^2 + \sigma_g^2) $$

$$ + \frac{1}{2} \frac{\gamma - 1}{\gamma} w^a(\theta_t) w^b(\theta_t) \phi^2 \left( \frac{1}{\sigma_c^2} + \frac{\alpha^2}{\sigma_g^2} \right), $$

(15)

$$ \kappa^a_{t,Z} = \gamma \sigma_c + \frac{\phi}{\sigma_c} \sigma^a(\theta_t); \quad \kappa^b_{t,Z} = \gamma \sigma_c - \frac{\phi}{\sigma_c} \sigma^b(\theta_t) $$

$$ \kappa^a_{t,B} = \gamma \sigma_g - \frac{\phi \alpha}{\sigma_g} \sigma^a(\theta_t); \quad \kappa^b_{t,B} = \gamma \sigma_g + \frac{\phi \alpha}{\sigma_g} \sigma^b(\theta_t), $$

(16)

where $w^a(\theta_t)$ and $w^b(\theta_t) = 1 - w^a(\theta_t)$ are the equilibrium shares of private consumption of group $a$ and $b$, respectively:

$$ w^a(\theta_t) = \frac{c_t^a}{g_t C_t} = \frac{1}{1 + \theta^a_t}. $$

1.3 The price of the market portfolio

Without loss of generality, we consider group $a$’s belief as the reference one. Relative to the latter, the return of the market portfolio – claim to aggregate output – admits the following Ito representation:

$$ \frac{dS_t}{S_t} = \mu_t dt + \sigma_{Z,t} d\hat{Z}^a_t + \sigma_{B,t} d\hat{B}^a_t. $$

(19)

It is easy to see that $\xi^b_t = \xi^a_t / \theta_t$. See the proof of Proposition 1 in the Appendix.
The expected return and volatility coefficients are determined endogenously and reported, along with the equilibrium price, in the following Proposition.

**Proposition 2.** Given the equilibrium state-price density \( \xi^a_t \) reported in (14), the equilibrium price of the claim to aggregate consumption is:

\[
S^C_t = \frac{C_t}{(1 + \theta^a_t)^{\gamma}} \int_t^{\infty} e^{m_0(t, s) + L(t, s) + H(t, s) \hat{A}_t} \left[ \sum_{j=0}^{\infty} \left( \frac{\bar{\gamma}}{j} \right) \theta^a_t + \frac{2j - \gamma}{2} \right] \times \left( \int_{-\infty}^{\infty} e^{2\pi i z \log \theta^a_t} \tilde{T}(j, z, t, s, \phi_t) dz \right) ds \tag{20}
\]

where \( \bar{\gamma} \) denotes the smallest integer larger or equal to \( \gamma \), and expressions for functions \( m, L, H, \) and \( F_j, j = 0, 1, 2 \) are reported in the Appendix. If \( S(C_t, \hat{A}_t^a, \phi_t, \theta_t) \) denotes the RHS of (20), the stock return volatility components are:

\[
\begin{align*}
\sigma_{Z,t} &= \frac{\partial \log S}{\partial C_t} \sigma_c + \frac{\partial \log S}{\partial \hat{A}_t^a} \frac{\sigma_a \sigma_c \rho_a}{\sigma_c} - \frac{\partial \log S}{\partial \phi_t} \left( \eta_a + \sigma_A \sigma_c \rho_a - \eta_b \right) \frac{1}{\sigma_c} - \frac{\partial \log S}{\partial \theta_t} \frac{\phi_t}{\sigma_c} \\
\sigma_{B,t} &= -\frac{\partial \log S}{\partial \hat{A}_t^a} \frac{\alpha \eta_a}{\sigma_g} + \frac{\partial \log S}{\partial \theta_t} \frac{\alpha \phi_t}{\sigma_g} - \frac{\partial \log S}{\partial \phi_t} \frac{\alpha (\eta_a - \eta_b)}{\sigma_g} \tag{22}
\end{align*}
\]

The derivatives appearing in the expressions are detailed in the Appendix. The equilibrium risk premium of the market portfolio is then

\[
\mu_t - r_t = \sigma_{Z,t} \kappa_t^{a,Z} + \sigma_{B,t} \kappa_t^{a,B}, \tag{23}
\]

where the equilibrium market prices of risk \( \kappa_t^{a,Z} \) and \( \kappa_t^{a,B} \) are as in (17)-(18).

The stock price representation (20) is explicit modulo an inverse Fourier transform, which can easily be evaluated using Fast Fourier Transform algorithms.\(^5\) We refer to the Appendix for further details.

1.4 **The cross-section of equity returns and credit spreads**

We now populate our economy with a cross-section of \( N \) firms. To model their behavior, we adopt the EBIT-based approach introduced in Leland (1994) and extended in Goldstein, Ju, and Leland (2001), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010), among others. We assume that firms possess a static capital structure consisting

\(^5\)This Fourier Transform representation is novel and distinct from others appearing in the heterogeneous beliefs literature, such as in Dumas, Kurshev, and Uppal (2009).
of equity and a console bond, serviced by a perpetual and constant coupon payment stream \( q_i \), where \( i \) denotes a generic firm. Firms’ EBIT evolution, under the observation filtration and reference belief, is described by the following diffusion process:\(^6\)

\[
\frac{dE_i^t}{E_i^t} = \left( \mu_i + \beta_i \tilde{\lambda}_i \right) dt + \sigma_i \left( \rho_i d\tilde{Z}_i^a + \sqrt{1 - \rho_i^2} dW_i^a \right).
\]  

(24)

According to expression (24), expected earnings growth is affected by the fiscal action, where the sensitivity is governed by the firm-specific parameter \( \beta_i \). We think of \( \beta_i \) similar in spirit to the industry exposure to government spending variable in Belo, Gala, and Li (2013) who measure the effect of government spending on expected firm cash flows from national input and output accounts.

The systematic risk component of earnings is controlled by a correlation parameter \( \rho_i \), while the idiosyncratic shock \( W_i^t \) is observable. We also assume that agents do not learn the fiscal rule from the cross-section of earning growths. While focusing on macroeconomic signals alone to infer fundamentals could be attributed to some rational inattention, we emphasize that this assumption can be relaxed without altering qualitative results, but at the expense of parsimony.

Firms’ profits are taxed at rate \( \tau \), thus the purpose of issuing debt is to shield cash-flows from taxation. Debt is issued at par,\(^7\) and the proceeds are distributed to shareholders. If \( \tilde{k} \) denotes the firms’ pay-out ratio, dividends distributed to shareholders are then a fraction \( k = \tilde{k}(1 - \tau) \) of net earnings: \( D_i^t = k(E_i^t - q_i) \). Equity holders cannot increase the firm’s indebtedness, and can reduce it only by defaulting on coupon payment, whereby control is assumed by debt holders. These can either liquidate assets and recover a fraction \( 1 - \varphi_l \) of the abandonment value – the unleveraged firm value upon default – or opt for renegotiation, which yields a fractional recovery of \( 1 - \varphi_r \). Assuming \( \varphi_r < \varphi_l \), the incentive for renegotiation leads to a Nash bargaining game, where shareholders capture a portion \( p \) of the renegotiation surplus \( \varphi_l - \varphi_r \), where \( p \) denotes their bargaining power. We assume that shareholders optimally select the

\(^6\)Since, as customary in the literature, we model exogenously firms’ EBIT and aggregate output, the difference between the latter and cumulative EBIT is the remuneration of labor and other production factors: see e.g., Bhamra, Kuehn, and Strebulaev (2010).

\(^7\)That is, at its equilibrium market value implied by the coupon stream \( q_i \).
coupon rate \( q_t \) to maximize total firm value, although we do not solve explicitly for this optimal level. The traditional trade-off between the tax-benefits of debt and bankruptcy costs is apparent. The unleveraged firm value, denoted by \( V_i^U \), is the no-arbitrage value of the earnings stream after taxes:

\[
V_i^U = \mathbb{E}^a \left[ \int_t^\infty \frac{\xi_t^a}{\xi_t^a} E_t^i (1 - \tau) ds \left| \mathcal{F}_t^{c,g} \right. \right]. \tag{25}
\]

where \( \xi_t^a \) – expression (14) – is the equilibrium state-price density relative to the reference belief (group \( a \)'s). In the Appendix, we provide an explicit representation for \( V_i^U \).

It is in the interest of equity holders to choose a default policy that maximizes the equity value. The latter, denoted by \( V_t \), comprises the no-arbitrage valuation of the dividend stream until default and of the recovery value upon default. Formally, \( V_t \) is the value function of the following optimal stopping problem:

\[
V_t = \sup_{t_d} \mathbb{E}^a \left[ \int_t^{t_d} \frac{\xi_s^a}{\xi_t^a} D_s^i ds + \frac{\xi_t^a}{\xi_t^a} \left( p(\varphi_l - \varphi_r) V_i^U \right) \left| \mathcal{F}_t^{c,g} \right. \right]. \tag{26}
\]

The ideal default time, solution to (26), is typically identified in terms of a critical boundary, such that default is triggered the first time earnings fall below it. In model settings where earnings are the only state variable – such as Leland (1994) – this threshold is constant. In settings where earnings growth and volatility depend on a Markov chain modeling the business cycle – such as Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) – the default boundary is a piece-wise constant function of the state. In our setting, this characterization is problematic, since the default boundary is a continuous function of the (posterior) fiscal action, the disagreement process, and the agents’ likelihood ratio. The absence of closed-form solutions to (26) and the large number of state variables motivate us to introduce a simulation-based numerical method, inspired by the American-option pricing algorithm in Longstaff and Schwartz (2001). This method has the advantage of mitigating the curse of dimensionality to which both iterative (contraction mapping) and finite-elements numerical strategies are prone.\(^8\)

\(^8\)For example, Brandt, Goyal, Santa-Clara, and Stroud (2005) propose a solution method for optimal investment problems based on a similar reasoning.
The numerical method consists of approximating problem (26) with one where the default option is available to shareholders only for a long but finite time-horizon \([t,T]\) – \(t\) being the evaluation date – and at a finite number of dates.\(^9\) At time \(T\), the firm is perpetually bound to keep its current capital structure.\(^10\) If \(\Delta t_d\) denotes the time lapsed between defaultable dates, clearly the approximate equity value converges to its true counterpart as the decision frequency \((1/\Delta t_d)\) and horizon \((T - t)\) increases. The method iterates backwards through decision dates \(t_d(j), j = 1, \ldots, n_d\),\(^11\) to solve the following dynamic programming equation:

\[
V_{t_d(j)}^i = \max \left( p(\varphi_l - \varphi_r) V_{t_d(j)}^i(E^i, \tilde{A}^a, \phi, \theta), C_{t_d(j)}(E^i, \tilde{A}^a, \phi, \theta) \right)
\]

\[
C_{t_d(j)}(E^i, \tilde{A}^a, \phi, \theta) = \mathbb{E}^a \left[ \int_{t_d(j)}^{t_d(j+1)} \frac{\xi^a_s}{\xi^a_{t_d(j)}} D^i_s ds + \frac{\xi^a_{t_d(j+1)}}{\xi^a_{t_d(j)}} V_{t_d(j+1)}^i \right| F_{t_d(j)}^{c,g} \right] \quad (27)
\]

As in Longstaff and Schwartz (2001), the continuation value \(C_{t_d(j)}(E^i, \tilde{A}^a, \phi, \theta)\) is approximated by regressing simulated realizations of the argument in the expectation on an appropriate base of polynomials. The default boundary is defined as the smallest level of earnings such that default is prevented:

\[
E_{t_d(j)}^i(\tilde{A}^a, \phi, \theta) = \left\{ \inf_{E^i} : p(\varphi_l - \varphi_r) V_{t_d(j)}^i(E^i, \tilde{A}^a, \phi, \theta) \leq C_{t_d(j)}(E^i, \tilde{A}^a, \phi, \theta) \right\}. \quad (28)
\]

In the exercises to follow we consider the default boundary \(E_{t_d(0)}^i(\tilde{A}^a, \phi, \theta)\), at the beginning of the “defaultable” window, which is closest to the boundary implied by the infinite-horizon problem (26). As customary, we obtain the conditional equity risk premium \(\left( \mathbb{E}_{t_d(0)}^{a_d} [dV_{t_d(0)}^i/V_{t_d(0)}^i] - r_{t_d(0)} \right)\) by combining information on the equity return volatility and the market prices of risk. We defer more details to the Appendix. We are also interested in the equilibrium credit spread of a given firm, that is, the difference between the yield of defaultable and default-free debt.

---

\(^9\)This is akin to approximating an American option security with a Bermudan option.

\(^10\)The Appendix provides an explicit expression for the default-free leveraged equity value.

\(^11\)\(n_d\) denotes the total number of decision dates. The algorithm is initiated at time \(T\).
Taking into account its console nature, the yield is the rate that would price the bond correctly as a perpetuity. Thus firm’s i credit spread reads:

\[ cs_i^t = \frac{q_i}{B_i^t} - \frac{q_i}{B_i^t} \]  
\[ B_i^t = \mathbb{E}^a \left[ \int_t^{t^*} \frac{\xi_s}{\xi_t} q_i ds + \frac{\xi_t^a}{\xi_t^a} (1 - \varphi_r - p(\varphi_t - \varphi_r)) \mathcal{V}_t^i \bigg| \mathcal{F}_{t^*}^{c,g} \right], \]  
\[ B_i^t = \mathbb{E}^a \left[ \int_t^{\infty} \frac{\xi_s}{\xi_t} q_i ds \bigg| \mathcal{F}_{t}^{c,g} \right]. \]

In the expression for the no-arbitrage price of defaultable debt, \( B_i^t \), the optimal default time is \( t^*_d = \{ \inf_s : E_s^i < \overline{E}_s^i(\hat{A}^a, \phi, \theta) \} \), and the recovery value upon default is the renegotiation value net of equity holders’ share of the renegotiation surplus. We obtain (31) numerically exploiting the previous identification of the default boundary. The price of the default-free debt, \( B_i^t \), has the usual inverse Fourier transform representation that we have been using for non-defaultable claims, which is given in (A-45) Appendix.

## 2 Empirical analysis

In this section, we document how exposure to fiscal uncertainty affects stock returns in the cross section. To this end, we first construct a proxy of fiscal uncertainty using survey data on future government spending. We then sort stocks according to their exposure to this proxy and sort them into different binds. We find that firms with low (high) exposure display high (low) returns which implies a negative price of risk. We then use standard Fama and MacBeth (1973) regressions to assess the market price of fiscal uncertainty and find it to be significantly negative and large.

### 2.1 Data

**Stock Data.** Our data sample includes all common stocks (share code of 10 or 11) listed on the NYSE, AMEX, or Nasdaq which are available from CRSP. Firm-specific characteristics are retrieved from Compustat.
2.2 Proxy of Fiscal Uncertainty

To construct our proxy of fiscal uncertainty, we merge survey data from Consensus Economics (CE) with the following quarterly macroeconomic series obtained from St. Louis Fed’s FRED database and the BEA: US Government total expenditure, GDP level and growth. CE surveys are conducted monthly and consist of forecasts – made by financial institutions and firms – about different macroeconomic indicators both for the current and next year. We use forecasts for next year’s Federal budget deficit and US GDP growth. For these items, data is available since January 1994 and a cross-section of around 25 forecasts is available each month.\textsuperscript{12} We select the institutions/forecasters for whom at least 48 months of continuous responses are available.\textsuperscript{13} This leaves us with \( N = 36 \) individuals. Our aim is now to obtain an estimate of the latent fiscal action \( \hat{A}_t \) as perceived by each of these forecasters. To this end, we use a two-stage procedure based on Kalman filtering and Maximum-Likelihood estimation. In the first stage, we consider forecasts for next year’s Federal budget deficit and GDP growth. In particular, we postulate that \( b_t \), ratio between Federal budget deficit and GDP, evolves as follows relative to the econometrician information set:

\[
d b_t = \alpha b \hat{A}_t dt + \sigma b dW_t, \tag{33}
\]

For any date \( t \) for which next calendar year starts at date \( T \), the Appendix shows that the model-equivalents of the survey forecasts read:

\[
\hat{GDP}_{t,T} = D_0^i (T - t) + D_1^i (T - t) \hat{A}_t, \tag{34}
\]

\[
\left( \frac{\text{DEF}}{\text{GDP}} \right)_{t,T} = \alpha_b \left( D_0^i (T - t) - \bar{\mu} \right) + \alpha_b D_1^i (T - t) \hat{A}_t, \tag{35}
\]

where coefficients \( D_0^i (T - t) \) and \( D_1^i (T - t) \) are reported in the Appendix. We add Gaussian white noise measurement error to equations (A-48) and (A-51) and use them in a state-space model as measurement equations for the latent state \( \hat{A}_t \). We obtain forecaster-specific first stage parameter estimates by maximum likelihood, from the pre-

\textsuperscript{12}Consensus Economics surveys are available since 1990, but until 1994 replies are erratic at best.
\textsuperscript{13}We provide a more detailed description of the survey data and how we treat missing values in the Appendix.
Prediction errors of the Kalman filter estimates of $\hat{A}_t$. We emphasize that this procedure is not able to identify the correlation parameter $\rho_i$. To this purpose, the second stage estimation employs data on realized GDP growth and growth of Government expenditure as a fraction of GDP. We consider a state-space model where the measurement equations are a discretized version of the system (1)-(2), where we constraint common parameters to coincide with first stage’s estimates, and estimate the remaining ones, most importantly $\rho_i$, by maximum likelihood from Kalman filter prediction errors.

This procedure leaves us with i) a cross-section of forecaster-specific parameter estimates $(\hat{A}_i, \hat{\rho}_i)$ and ii) $N$ forecaster-specific time series of filter estimates for the fiscal action $A_t$, obtained from the first stage. Our strategy to classify the individuals into group $a$ and group $b$ is as follows: For each month $t$, we double sort the forecasters who responded to the survey with respect to parameter estimates $\hat{A}_i$ and $\hat{\rho}_i$. In accordance with the model’s assumptions, members of the high-$\hat{A}_i$/low-$\hat{\rho}_i$ quantile are posited to belong to group $a$, while those in the low-$\hat{A}_i$/high-$\hat{\rho}_i$ quantile belong to group $b$. We emphasize that the estimates of the correlation parameter $\rho_i$ are negative for almost all individuals, thus high-$\hat{\rho}_i$ values are indeed not statistically different from zero, as the model implies for group $b$. Finally, our proxy for fiscal uncertainty, the empirical counterpart of the disagreement process $\phi_t$, is:

$$\hat{\phi}_t = \frac{1}{N^a_t} \sum_{i \in A_t} \hat{A}_i - \frac{1}{N^b_t} \sum_{i \in B_t} \hat{A}_i,$$

In other words $\hat{\phi}_t$ is the difference of the equally-weighted averages of the fiscal action estimates in each group, where $A_t$ and $B_t$ are the two different bins in month $t$ and $N^j_t$, $j = a, b$ is the number of forecasters in each group.

[Insert Figure 1 here.]

Figure 1 plots fiscal disagreement together with two other commonly used proxies of uncertainty: the Baker, Bloom, and Davies (2013) economic policy index (upper panel) and the VIX (lower panel). We note two spikes, one in 2001 and another (larger one) before 2010. The former can be linked to the implementation of the Economic Growth and Tax Relief Reconciliation Act and the latter to the expiration thereof.
2.3 Portfolios sorted on fiscal uncertainty

To study the relationship between expected stock returns and fiscal uncertainty, we estimate beta loadings for each stock from using a two-factor model with the market excess return, $r_{x,m}$, and changes in fiscal uncertainty, $\Delta \phi_t$. At the end of each month, we run rolling regressions of the following form:

$$rx_{i,t} = \alpha_{i,t} + \beta_{i,t}^M r_{x,m} + \beta_{i,t}^f \Delta \phi_t + \epsilon_{i,t},$$

where $rx_{i,t}$ is the one-month excess return of stock $i$. The window size is chosen to be 36 months and we require each stock to have each last 24 non-missing returns out of the 36 months.

We then sort stocks into quintiles based on their loading, $\hat{\beta}^f$, and calculate portfolio returns for the subsequent period. Table 1 presents the results. Portfolio 1 contains the stocks with the lowest exposure to fiscal uncertainty (i.e., the stocks with negative betas), while portfolio 5 contains the stocks with the highest exposure (positive betas). We first note that high (low) exposure firms have lower (higher) returns. Low $\beta^f$ firms have an average annualized return of 13.49% and high $\beta^f$ stocks have a return of 6.99%. The returns are monotonically decreasing between portfolio 1 and portfolio 5. The spread between the high and low exposure firms, denoted by HML$^f$, is -6.5% and highly statistically significant (t-statistic of 3.01).

[Insert Tables 1 and 2 here.]

For each portfolio, we report in Table 2 alphas for different specifications. In particular, we report alphas for a one-factor CAPM (column 2), Fama and French (1993) three-factor model (column 3), and Carhart (1997) four-factor model (column 4). The last four columns report the factor loadings. The difference in average returns is mirrored in large differences in alphas. For example, the low exposure stocks, have a one-factor alpha which is 0.52% whereas the one-factor alpha for high exposure stocks is 0% per month. Including the size and book-to-market factors, the alpha for the low (high) exposure stocks is -0.30% (-0.18%). The bottom line presents the same quantities for the
HML\textsuperscript{f} portfolio which is long high exposure stocks and short low exposure stocks. We note that the alpha is statistically significant even when we control for the three Fama and French (1993) factors. To explore in more detail the time series behavior of the HML\textsuperscript{f} factor, we plot in Figure 2 (upper panel) the returns of the HML\textsuperscript{f} portfolio and in the lower panel we plot the cumulative returns of the low (pf1) and high (pf5) fiscal disagreement exposure stocks.

[Insert Table 3 and Figure 2 here.]

We first note that the difference between the low exposure (pf1) and high exposure (pf5) stocks is almost zero until the early year 2000. In 2001 the Economic Growth and Tax Relief Reconciliation Act (commonly referred to as one of the two Bush tax cuts) was passed. In general the act lowered tax rates and although these cuts were set to expire at the end of 2010. A second large divergence is observed around 2010 when the high exposure portfolio steeply increases vis-à-vis the low exposure portfolio. During this period, there was large uncertainty about whether the Bush-era tax cuts should be extended or not. In Table 3, we provide summary statistics of the HML\textsuperscript{f} strategy. We note that the fiscal trading strategy performs much better than other well-know strategy such as the market, size or book-to-market portfolio. Moreover, since the volatility is also comparably smaller, this results in an annual Sharpe ratio which is more than twice as large as for the market (0.73 versus 0.33). The low unconditional correlations between the different strategies imply that compensation for fiscal uncertainty is basically uncorrelated with other strategies.

2.4 The market price of fiscal uncertainty

The portfolio sorts indicated a significant negative relation between exposure to fiscal uncertainty and future returns. In the following, we run Fama and MacBeth (1973) regressions to explore in more detail the cross-sectional relationship between stock returns and our proxy of fiscal uncertainty.

As test assets we use the five portfolios formed by sorting stocks according to their exposure to the fiscal uncertainty factor.
Panel A in Table 4 reports the first stage regression results of regression the respective portfolio excess returns on the market excess returns $\text{mkt}_t$ and the fiscal uncertainty $\text{HML}^F$ factor. As expected given the sorting procedure, the coefficients on the fiscal uncertainty factor are monotonically increasing in the portfolios. Apart from the coefficient $\beta^F$ on portfolio 4, which is very close to zero, all coefficients are statistically strongly significant. The CAPM betas are close to—and often not significantly different from—one.

To estimate the factor prices $\lambda$ we follow the traditional two-stage procedure of Fama and MacBeth (1973) and regress the estimated betas on the average excess portfolio returns in the second stage regression. In line with the countercyclical nature of the fiscal uncertainty we find a negative market price of risk for $\text{HML}^F$ equal to minus 52 basis points per month (or $-6.26\%$ annualized). This is not statistically different from the average $\text{HML}^F$ return of minus 55 basis points per month. The Shanken (1992)-corrected standard errors are reported in brackets.

Figure 3 compares the performance of the simple CAPM with only the market excess return as a factor and a two-factor model that includes that fiscal uncertainty factor. Panel A plots the actual and predicted returns for our test portfolios using the CAPM whereas Panel B plots the same quantities for the extended model. The $R^2$ in the second stage regression for the CAPM is 90% whereas adding the fiscal uncertainty factor drives the second stage $R^2$ up to 99%.

In line with the previous portfolio sorting approach, we find that fiscal uncertainty risk is priced in the cross section of stock returns with a negative price.

3 Conclusion

In this paper, we study the implications of fiscal uncertainty on the cross-section of stock returns. We first propose a general equilibrium model where heterogeneous agents disagree on the extent to which public spending affects the aggregate output growth rate.
More precisely, agents disagree on the size of the fiscal multiplier: agent $a$ ($b$) believes that the government sets a larger (smaller) long-term goal for GDP growth rate induced by public intervention. This disagreement generates persistent divergence of their beliefs about the evolution of future state variables, and the likelihood ratio of these beliefs is a proxy for the uncertainty about the fiscal rule.

To model a cross-section of firms, we adopt a framework similar to Leland (1994) and provide closed-form solutions for equilibrium equity and debt prices and their risk premia. Using calibrated parameters, we show that fiscal disagreement is priced in equilibrium and receive a negative market price of risk.

To test our theory in the data, we construct a novel measure of fiscal uncertainty using a large cross section of survey data on future government budget deficits. In particular, using a filtering approach, we estimate forecaster by forecaster her/his perceived fiscal effectiveness. Fiscal disagreement is then defined as the difference between forecasters who believe that fiscal policy is effective and ineffective. Using this proxy, we find that exposure to fiscal uncertainty negatively and significantly predicts future stock returns. In particular, we find that firms with low exposure have higher returns than firms with lower exposure. A portfolio which is long high exposure firms and short low exposure returns produces an annualized return of $-6.5\%$, which is highly statistically significant and is not subsumed by other standard risk factors.
References


Table 1
Fiscal Uncertainty Portfolios Summary Statistics

This table reports summary statistics of portfolios sorted according to their exposure to fiscal uncertainty measured by $\beta_{i,t}^f$ in the following regression:

$$rx_{i,t} = \alpha_{i,t} + \beta_{i,t}^M rx_{m,t} + \beta_{i,t}^f \Delta\hat{\phi}_t + \epsilon_{i,t},$$

pf1 (pf5) is the portfolio with stocks with the lowest (highest) exposure and $\text{HML}^f$ is the portfolio which is long pf5 and short pf1. $\beta^f$ and $\beta_{post}^f$ are the pre- and post-sorting betas. Returns and standard deviations are annualized and expressed in percent. The data runs from February 1996 to December 2012.

<table>
<thead>
<tr>
<th></th>
<th>pf1</th>
<th>pf2</th>
<th>pf3</th>
<th>pf4</th>
<th>pf5</th>
<th>HML$^f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>13.97</td>
<td>11.65</td>
<td>9.47</td>
<td>9.31</td>
<td>7.39</td>
<td>-6.58</td>
</tr>
<tr>
<td>std. dev.</td>
<td>27.05</td>
<td>19.81</td>
<td>17.68</td>
<td>18.82</td>
<td>25.75</td>
<td>9.00</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.13)</td>
<td>(2.42)</td>
<td>(2.21)</td>
<td>(2.04)</td>
<td>(1.18)</td>
<td>(-3.02)</td>
</tr>
<tr>
<td>skewness</td>
<td>-0.17</td>
<td>-0.65</td>
<td>-0.68</td>
<td>-0.76</td>
<td>-0.24</td>
<td>-0.90</td>
</tr>
<tr>
<td>kurtosis</td>
<td>2.04</td>
<td>2.94</td>
<td>2.57</td>
<td>2.16</td>
<td>0.96</td>
<td>5.85</td>
</tr>
<tr>
<td>$\beta^f$</td>
<td>-0.472</td>
<td>-0.142</td>
<td>-0.008</td>
<td>0.123</td>
<td>0.442</td>
<td></td>
</tr>
<tr>
<td>$\beta_{post}^f$</td>
<td>-0.167</td>
<td>-0.044</td>
<td>-0.018</td>
<td>-0.0400</td>
<td>0.230</td>
<td></td>
</tr>
</tbody>
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Table 2
Fiscal Uncertainty Portfolio Alphas and Factor Loadings

This table reports average returns, together with the CAPM alpha from a regression that includes only the market, the three Fama and French factors (mkt, smb, hml) and the momentum factor (mom). The last four columns report the factor loadings from these factors onto the different portfolios. pf1 (pf5) is the portfolio of stocks with the lowest (highest) fiscal uncertainty beta, and $HML^f$ is a portfolio which is long pf5 and short pf1. The data runs from February 1996 to December 2012.

<table>
<thead>
<tr>
<th></th>
<th>return</th>
<th>CAPM</th>
<th>Alphas</th>
<th>4 factor</th>
<th>mkt</th>
<th>smb</th>
<th>hml</th>
<th>mom</th>
<th>4-factor loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td>pf1</td>
<td>13.97</td>
<td>0.52</td>
<td>0.30</td>
<td>0.30</td>
<td>1.16</td>
<td>0.99</td>
<td>0.14</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>pf2</td>
<td>11.65</td>
<td>0.47</td>
<td>0.23</td>
<td>0.22</td>
<td>0.96</td>
<td>0.67</td>
<td>0.34</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>pf3</td>
<td>9.47</td>
<td>0.33</td>
<td>0.12</td>
<td>0.12</td>
<td>0.90</td>
<td>0.52</td>
<td>0.35</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>pf4</td>
<td>9.31</td>
<td>0.29</td>
<td>0.07</td>
<td>0.07</td>
<td>0.96</td>
<td>0.56</td>
<td>0.35</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>pf5</td>
<td>7.39</td>
<td>0.00</td>
<td>-0.20</td>
<td>-0.18</td>
<td>1.16</td>
<td>0.85</td>
<td>0.14</td>
<td>-0.07</td>
<td></td>
</tr>
<tr>
<td>$HML^f$</td>
<td>-6.58</td>
<td>-0.52</td>
<td>-0.49</td>
<td>-0.48</td>
<td>0.00</td>
<td>0.14</td>
<td>0.00</td>
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<tr>
<td>t-stat</td>
<td>(-3.01)</td>
<td>(-3.15)</td>
<td>(-3.20)</td>
<td>(-3.20)</td>
<td>(0.02)</td>
<td>(-1.58)</td>
<td>(0.05)</td>
<td>(-1.63)</td>
<td></td>
</tr>
</tbody>
</table>
Table 3
Fiscal Uncertainty and Other Factors

This table reports mean, standard deviation (stdev) and the Sharpe ratio (SR) all in annualized terms together with the unconditional correlations between the HML$^f$ portfolio derived from sorting stock returns according to their fiscal uncertainty exposure, the three Fama and French factors (mrkt, smb, hml) and the momentum factor (mom). The data runs from February 1996 to December 2012.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>stdv</th>
<th>SR</th>
<th></th>
<th></th>
<th>correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>HML$^f$</td>
<td>-6.58</td>
<td>9.00</td>
<td>-0.73</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mrkt</td>
<td>5.46</td>
<td>16.53</td>
<td>0.33</td>
<td>-0.08</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>smb</td>
<td>3.23</td>
<td>12.70</td>
<td>0.25</td>
<td>-0.23</td>
<td>0.26</td>
<td>1.00</td>
</tr>
<tr>
<td>hml</td>
<td>3.30</td>
<td>12.07</td>
<td>0.27</td>
<td>0.08</td>
<td>-0.24</td>
<td>-0.36</td>
</tr>
<tr>
<td>mom</td>
<td>5.17</td>
<td>19.80</td>
<td>0.26</td>
<td>-0.19</td>
<td>0.19</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Test assets are the five portfolios sorted based on exposure to the fiscal uncertainty risk factor $\Delta\hat{\phi}$. HML$^f$ is the difference between the excess return on the high beta portfolio and the excess return on the low beta portfolio. Panel A reports factor betas and Newey and West (1987) standard errors (in parentheses) while Panel B reports the Fama and MacBeth (1973) factor prices and standard errors (in parentheses). Shanken (1992)-corrected standard errors are reported in brackets. Data is monthly and runs from February 1996 through December 2012.

### Panel A: Factor betas

<table>
<thead>
<tr>
<th>pf1</th>
<th>$\alpha$</th>
<th>mkt</th>
<th>HML$^f$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0.14</td>
<td>1.30</td>
<td>-0.73</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(23.78)</td>
<td>(-3.03)</td>
<td></td>
</tr>
<tr>
<td>pf2</td>
<td>0.27</td>
<td>1.02</td>
<td>-0.39</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>(1.10)</td>
<td>(19.26)</td>
<td>(-2.99)</td>
<td></td>
</tr>
<tr>
<td>pf3</td>
<td>0.25</td>
<td>0.94</td>
<td>-0.16</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>(1.15)</td>
<td>(21.17)</td>
<td>(-1.76)</td>
<td></td>
</tr>
<tr>
<td>pf4</td>
<td>0.32</td>
<td>1.00</td>
<td>0.06</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>(1.32)</td>
<td>(19.90)</td>
<td>(0.53)</td>
<td></td>
</tr>
<tr>
<td>pf5</td>
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<td>1.30</td>
<td>0.27</td>
<td>0.69</td>
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<tr>
<td></td>
<td>(0.44)</td>
<td>(23.78)</td>
<td>(1.13)</td>
<td></td>
</tr>
</tbody>
</table>

### Panel B: Factor prices

<table>
<thead>
<tr>
<th>mkt</th>
<th>HML$^f$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.68</td>
<td>-0.52</td>
<td>0.99</td>
</tr>
<tr>
<td>(1.75)</td>
<td>(-2.85)</td>
<td></td>
</tr>
<tr>
<td>[1.74]</td>
<td>[−2.85]</td>
<td></td>
</tr>
</tbody>
</table>
Table 5
Calibration: Parameter Values

This table presents structural parameter values for the calibration exercise. Parameters with an $i$ subscript refer to a US representative BBB-rated firm.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\gamma$</th>
<th>$\bar{\mu}$</th>
<th>$\sigma_c$</th>
<th>$\sigma_g$</th>
<th>$\lambda$</th>
<th>$\omega$</th>
<th>$b$</th>
<th>$\Phi^a$</th>
<th>$\Phi^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>5</td>
<td>0.00606</td>
<td>0.0491</td>
<td>0.0487</td>
<td>0.3995</td>
<td>2*0.3995</td>
<td>0.09</td>
<td>0.00574</td>
<td>0.00204</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$k$</td>
<td>$\mu_i$</td>
<td>$\beta_i$</td>
<td>$\rho_i$</td>
<td>$a_q$</td>
<td>$a_\beta$</td>
<td>$a_\rho$</td>
<td>$a_D$</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.5</td>
<td>$\bar{\mu}$</td>
<td>0.2474</td>
<td>1</td>
<td>0.1998</td>
<td>0.0491</td>
<td>0.5716</td>
<td>0.3938</td>
<td>-0.05887</td>
</tr>
</tbody>
</table>
5 Figures

Figure 1. Fiscal disagreement proxy

This figure plots fiscal disagreement estimated from survey data on future government budget together with the Baker, Bloom and Davies (2013) economic policy index and the VIX (lower panel). Data is monthly and runs from 1994 to 2012.
Figure 2. Portfolio returns

The upper panel plots monthly returns of a portfolio which is long stocks with high exposure to fiscal uncertainty (pf5) and short stocks with low exposure to fiscal uncertainty (pf1). The lower panel plots the cumulative returns from these portfolios. Data is monthly and runs from 1996 to 2012.
Figure 3. Model performance

The figure plots the actual annualized mean excess returns in percent versus the predicted excess returns for the five portfolios sorted based on exposure to the fiscal uncertainty factor. Panel A displays the results for the CAPM, i.e., by using only the market excess return (mkt) as a pricing factor while Panel B displays the model performance when the fiscal uncertainty factor HML$^f$ is included in the linear pricing model. Data is monthly and runs from February 1996 to December 2012.
APPENDIX A : Proofs and derivations

Proof of Proposition 1.

Agents of groups \(a\) and \(b\) have time additive expected utility of the CRRA type, with the same RRA coefficient \(\gamma\), and subjective impatience rate \(\delta\). As customary in endowment economies, the agents are initially endowed with a fraction \(k^i\), \(i = a, b\) of the market portfolio, so that \(W^i_0 = k^i S_0\), where \(W^i_0\) is group’s \(i\) initial wealth and \(S_0\) the initial price of the market portfolio. By market completeness there is a unique state-price density, represented as \(\xi^i_t\) relatively to group \(i\)’s belief. The martingale approach of Cox and Huang (1988) implies that each agent solves the static consumption-investment problem:

\[
\sup_c E^i \left[ \int_0^\infty e^{-\delta s} \frac{c_s^{1-\gamma}}{1-\gamma} ds \bigg| F^c_0 \right] (A-1)
\]

\[
\text{s.t. } E^i \left[ \int_0^\infty \xi^i_s c_s ds \bigg| F^c_0 \right] \leq W^i_0 (A-2)
\]

Since \(X_0 = E^b[\xi^b_t X_t | F^c_0] = E^a[\theta_t \xi^b_t X_t | F^c_0] = E^a[\xi^a_t | F^c_0]\), for any \(F_t^c\) measurable contingent claim, we have \(\xi^a_t = \theta_t \xi^b_t\). Taking this into account, the first order conditions for problem (A-1)-(A-2) mandate that optimal individual consumption policies are \(c^a_t = (e^{\delta t} \vartheta^a \xi^a_t)^{-1/\gamma}\) and \(c^b_t = (e^{\delta t} \vartheta^b \xi^a_t/\theta_t)^{-1/\gamma}\), where \(\vartheta^i\) are the Lagrange multiplier such that the individual budget constraints (A-2) bind. Imposing \(\xi^a_0 = \vartheta^a\) and \(\theta_0 = \vartheta^a/\vartheta^b\), the aggregate resource constraint reads:

\[
(e^{\delta t} \xi^a_t)^{-1/\gamma} + (e^{\delta t} \vartheta^a / \theta_t)^{-1/\gamma} = g_t C_t,
\]

from which we retrieve the equilibrium state price-density (14). Applying Itô’s lemma to the last expression and to (13), and equating drift and diffusion components of the two dynamics, we obtain the equilibrium interest rate (15) and market prices of risk (17)-(18).

Proof of Proposition 2.

We are going to provide a characterization for the equilibrium price and risk premium of a generic security which guarantees a stream of dividends with rate \(X_s, s \in [0, \infty)\), with dynamics:

\[
\frac{dX_t}{X_t} = \left( \mu_X + \beta_X \tilde{A}_t \right) dt + \sigma_X \left( \rho_X \tilde{Z}^a_t + \sqrt{1 - \rho^2_X} dW_t \right) (A-3)
\]

The no-arbitrage pricing formula reads:

\[
S^X_t = E^a \left[ \int_t^\infty e^{-\delta (s-t)} \frac{X_s}{\xi^a_t} ds \bigg| F^c_t \right] (A-4)
\]

\[
= \frac{X_t}{(1 + \theta_t^{1/\gamma})} E^a \left[ \int_t^\infty \left( \frac{C_s g_s}{C_t g_t} \right)^{-\gamma} \left( 1 + \theta_s^{1/\gamma} \right)^{\gamma} \frac{X_s}{X_t} ds \bigg| F^c_t \right] (A-5)
\]
Note that

\[ e^{-\delta(s-t)} \left( \frac{C_{gs}}{C_{gt}} \right)^{\gamma} \frac{X_s}{X_t} = \exp \left( m_0(t, s) + (\beta_X - \gamma(1 - \alpha)) \int_t^s A_a^a du \right) \frac{\xi_s}{\xi_t} \quad (A-6) \]

\[ \frac{\xi_s}{\xi_t} = \exp \left( -\frac{1}{2} \left( \sigma_X^2 + \gamma^2 (\sigma_c^2 + \sigma_g^2) - 2\gamma \sigma_X \sigma_c \rho_X \right) (s-t) \right) \quad (A-7) \]

\[ + (\sigma_X \rho_X - \gamma \sigma_c) (\hat{Z}_t^a - \hat{Z}_s^a) - \gamma \sigma_g (\hat{B}_t^a - \hat{B}_s^a) + \sigma_X \sqrt{1 - \rho_X^2} (W_t - W_s) \]

\[ m_0(t, s) = \begin{cases} -\delta + \mu_X - \frac{\sigma_X^2}{2} - \gamma \left( \mu_c - \frac{\sigma_c^2}{2} \right) + \frac{\sigma_g^2}{2} (s-t) & \text{if } \xi_t \neq 0 \\ \frac{1}{2} \left( \sigma_X^2 + \gamma^2 (\sigma_c^2 + \sigma_g^2) - 2\gamma \sigma_X \sigma_c \rho_X \right) (s-t) & \text{if } \xi_t = 0 \end{cases} \quad (A-8) \]

\[ + 1 \left( \sigma_X^2 + \gamma^2 (\sigma_c^2 + \sigma_g^2) - 2\gamma \sigma_X \sigma_c \rho_X \right) (s-t) \quad (A-9) \]

We can use the stochastic exponential (A-7) for an absolutely continuous change of probability measure, with the dynamics of the state variable under the new measure dictated by Girsanov theorem. Thus

\[ S_t^X = \frac{X_t}{\left( 1 + \theta_t^1 \right)^{\gamma}} \mathbb{E}^{\xi_t} \left[ \int_t^\infty e^{m_0(t, s) + (\beta_X - \gamma(1 - \alpha)) \int_t^s A_a^a du} \left( 1 + \theta_s^1 \right)^{\gamma} ds \right| F_t^{\xi_t^a} \right] \quad (A-10) \]

\[ d\hat{A}_t^a = \lambda (\Lambda_a - \hat{A}_t^a) dt + \frac{\eta_a + \sigma_c \sigma_A \rho_C}{\sigma_c} (\sigma_X \rho_X - \gamma \sigma_c) dt + \eta_a \gamma \alpha dt + \frac{\eta_a^2 + \sigma_c \sigma_A \rho_i}{\sigma_c} d\tilde{Z}_t^{a,*} - \eta_a \frac{\alpha}{\sigma_g} d\tilde{B}_t^{a,*} \quad (A-11) \]

\[ \frac{d\theta_t}{\theta_t} = -\frac{\phi_t}{\sigma_c} (\sigma_X \rho_X - \gamma \sigma_c) dt + \gamma \alpha \phi_t dt - \phi_t \left( \frac{1}{\sigma_c} d\tilde{Z}_t^{a,*} - \frac{\alpha}{\sigma_g} d\tilde{B}_t^{a,*} \right) \quad (A-12) \]

\[ d\phi_t = \omega (\phi_t) dt + \gamma (\eta_a - \eta_b) \alpha dt + (\eta_a + \sigma_A \sigma_c \rho_a - \eta_b) \frac{\sigma_X \rho_X - \gamma \sigma_c}{\sigma_c} dt - \frac{(\eta_a - \eta_b) \alpha}{\sigma_g} d\tilde{B}_t^{a,*} + (\eta_a + \sigma_A \sigma_c \rho_a - \eta_b) \frac{1}{\sigma_c} d\tilde{Z}_t^{a,*} \quad (A-13) \]

Consider the function \( J(t, s) = \mathbb{E}^{\xi_t} \left[ e^{(\beta_X - \gamma(1 - \alpha)) \int_t^s A_a^a du} \right| F_t^{\xi_t^a} \]. By Feyman-Kac theorem, \( J \) solved the partial differential equation:

\[ J_t + J_{\hat{A}_t^a} \left[ \lambda (\Lambda_a - \hat{A}_t^a) + \frac{\eta_a + \sigma_c \sigma_A \rho_C}{\sigma_c} (\sigma_X \rho_X - \gamma \sigma_c) + \eta_a \alpha \gamma \right] \]

\[ + \frac{1}{2} J_{\hat{A}_t^a} \left[ \left( \frac{\eta_a + \sigma_c \sigma_A \rho_C}{\sigma_c} \right)^2 + \frac{\eta_a^2 \alpha^2}{\sigma_g^2} \right] + J (\beta_X - \gamma(1 - \alpha)) \hat{A}_t^a = 0 \quad (A-16) \]
with the terminal condition $J(s, s) = 1$. The solution reads

\[
J(t, s) = \exp(L(t, s) + H(t, s)\tilde{A}_t^s)
\]

(A-17)

\[
H(t, s) = \frac{(\beta_X - \gamma(1 - \alpha))}{\lambda} \left(1 - e^{-\lambda(s-t)}\right)
\]

(A-18)

\[
L(t, s) = \int_0^{s-t} \left[\lambda\tilde{A}_a + \frac{\eta_a + \sigma_c e\sigma_A\rho_a}{\sigma_c} (\sigma_X\rho_X - \gamma\sigma_c) + \eta_a a\gamma\right] H(0, u)du
\]

(A-19)

\[
+ \frac{1}{2} \int_0^{s-t} \left[\frac{(\eta_a + \sigma_c e\sigma_A\rho_a)^2}{\sigma_c^2} + \frac{\eta_a^2 a^2}{\sigma_g^2}\right] H(0, u)^2du
\]

(A-20)

By a change of numeraire technique analogous to the definition of the Forward-measure – Geman, El Karoui and Rochet (1995) – we can write:

\[
\mathbb{E}^{a,s}\left[e^{+(\beta_X - \gamma(1 - \alpha))\int_t^{s} \tilde{A}_u^s du} \left(1 + \frac{1}{s}\right)^\gamma \left| \mathcal{F}_t^{c,g}\right]\right] = J(t, s)\mathbb{E}^{a,**}(s)\left[\left(1 + \frac{1}{s}\right)^\gamma \left| \mathcal{F}_t^{c,g}\right]\right]
\]

We need the dynamics of the state variables $\theta_t$ and $\phi_t$ under the new measure, namely:

\[
\frac{d\theta_t}{\theta_t} = -\frac{\phi_t}{\sigma_c} (\sigma_X\rho_X - \gamma\sigma_c) dt + \gamma\phi_t dt - \frac{J_\tilde{A}_t^s(t, s)}{J(t, s)} \left(\frac{\eta_a + \sigma_c e\sigma_A\rho_a}{\sigma_c^2} + \frac{\eta_a \sigma_a^2}{\sigma_g^2}\right) \phi_t dt
\]

\[
- \phi_t \left(\frac{1}{\sigma_c} d\tilde{A}_t^{a,**}(s) - \frac{\alpha}{\sigma_g} d\tilde{B}_t^{a,**}(s)\right),
\]

(A-21)

\[
d\phi_t = \omega(\tilde{\phi} - \phi_t) dt + \gamma(\eta_a - \bar{\eta}_b) \alpha dt + (\eta_a + \sigma_A e\rho_a - \eta_b) \left(\frac{\sigma_X\rho_X - \gamma\sigma_c}{\sigma_c}\right) dt
\]

\[
+ \frac{J_\tilde{A}_t^s(t, s)}{J(t, s)} \left(\frac{(\eta_a + \sigma_c e\sigma_A\rho_a)(\eta_a + \sigma_c e\sigma_A\rho_a - \eta_b)}{\sigma_c^2} + \frac{\eta_a (\eta_a - \eta_b \alpha^2)}{\sigma_g^2}\right) dt
\]

\[
- (\eta_a - \eta_b) \alpha \frac{d\tilde{B}_t^{a,**}(s)}{\sigma_g} + (\eta_a + \sigma_A e\rho_a - \eta_b) \frac{1}{\sigma_c} d\tilde{Z}_t^{a,**}(s),
\]

(A-22)

where $\frac{J_\tilde{A}_t^s(t, s)}{J(t, s)} = H(t, s)$. We can then rewrite the security price as:

\[
S_t^X = \frac{X_t}{\left(1 + \frac{1}{s}\right)^\gamma} \int_t^{\infty} \mathbb{E}^{a,**}(s)\left[\left(1 + \frac{1}{s}\right)^\gamma \left| \mathcal{F}_t^{c,g}\right]\right] ds
\]

(A-23)

Now consider the expression $\left(1 + \frac{1}{s}\right)^\gamma$. We have:

\[
\left(1 + \frac{1}{s}\right)^\gamma = \frac{1}{\gamma} \left(\frac{1}{\gamma} \frac{1}{\gamma} + \frac{1}{s}\right)^\gamma = \frac{1}{\gamma} \left(2 \cosh \left(\frac{\nu_s}{2\gamma}\right)\right)^\gamma = \frac{1}{\gamma} \left(\frac{\nu_s - \frac{1}{2\gamma} + \frac{1}{2\gamma}}{2}\right)^\gamma
\]
where $\gamma$ denotes the smallest integer greater than $\gamma$,\footnote{Any integer larger than $\gamma$ would be suitable. The reason to multiply and divide by $(\theta_s^{-\frac{1}{2\gamma}} + \theta_s^{\frac{1}{2\gamma}})\gamma$ is that the Gamma function appearing in the Fourier transform is not defined for negative arguments.} and $\nu_s = \log \theta_s$. We use the following Fourier representation:

$$\frac{1}{(2 \cosh (\frac{\nu_s}{2\gamma}))^{\gamma-\gamma}} = \int_{-\infty}^{\infty} e^{2\pi i \nu_s z} T(z) dz$$

(A-24)

$$T(z) = \int_{-\infty}^{\infty} e^{-2\pi i \nu_s z} \left( \frac{1}{2 \cosh (\frac{\nu_s}{2\gamma})} \right)^{\gamma-\gamma} d\nu_s$$

$$= \frac{\Gamma \left( \frac{\gamma-\gamma}{2} - \gamma i z \right)}{\Gamma \left( \frac{\gamma-\gamma}{2} + \gamma i z \right)}$$

(A-25)

The explicit form (A-25) of the Fourier transform $T(z)$ is derived in Martin (2013). Since $\gamma$ is an integer, we can expand the term $(\theta_s^{-\frac{1}{2\gamma}} + \theta_s^{\frac{1}{2\gamma}})^\gamma$ using Newton binomial formula. Collecting all terms, we can write:

$$S_t^X = \frac{X_t}{(1 + \theta_t^\frac{1}{2})^\gamma} \int_t^{\infty} e^{m_0(t,s) + L(t,s) + H(t,s) \bar{z}^t} \left( \int_{-\infty}^{\infty} \sum_{j=0}^{\gamma} \gamma \theta_s^{j} \left[ \frac{\theta_s^{j}}{\gamma} \mathcal{F}^{c,g}_t \right] T(z) dz \right) ds$$

(A-26)

where $\chi(j, z) = \frac{1}{2} + \frac{2j-\gamma}{2\gamma} + 2\pi iz$. Let

$$K(t, s, j, z) = \mathbb{E}^{a,**}(s) \left[ \theta_s^{j} \left[ \mathcal{F}^{c,g}_t \right] \right]$$

$$= \theta_t^{j} \mathbb{E}^{a,**}(s) \left[ \exp \left( -\int_t^s \phi_u \mu_\theta(u, s) \chi(j, z) du - \int_t^s \frac{1}{2} \phi_u^2 \left( \frac{1}{\sigma_c^2} + \frac{\alpha^2}{\sigma_g^2} \right) \chi(j, z) du - \int_t^s \chi(j, z) \phi_u \left( \frac{1}{\sigma_c} d\tilde{z}_u^{a,**}(s) - \frac{\alpha}{\sigma_g} d\tilde{B}_u^{a,**}(s) \right) \right] \right]$$

$$= \mathbb{E}^{a,**}(s) \left[ \exp \left( -\int_t^s \phi_u \mu_\theta(u, s) \chi(j, z) du - \int_t^s \frac{1}{2} \phi_u^2 \left( \frac{1}{\sigma_c^2} + \frac{\alpha^2}{\sigma_g^2} \right) \chi(j, z) du - \chi(j, z) \phi_u \left( \frac{1}{\sigma_c} d\tilde{z}_u^{a,**}(s) - \frac{\alpha}{\sigma_g} d\tilde{B}_u^{a,**}(s) \right) \right] \right]$$

(A-27)

where

$$\mu_\theta(t, s) = \frac{1}{\sigma_c} \left( \sigma_X \rho_X - \gamma \sigma_c \right) - \gamma \alpha + H(t, s) \left( \frac{\eta_a + \sigma_c \sigma_A \rho_a}{\sigma_c^2} + \frac{\eta_a \alpha^2}{\sigma_g^2} \right).$$
In (A-27) we have performed the usual change of probability measure using the stochastic exponential embedded in $\theta_s^{\chi(j,z)}$. The dynamics of $\phi_t$ under this new measure are given by:

$$d\phi_t = (\mu^0_\phi(t, s) + \mu^1_\phi(t, s)\phi_t)dt - (\eta_a - \eta_b)^\alpha\sigma_d\hat{\mathcal{A}}_{t}^{\alpha, \alpha}(s) + (\eta_a + \sigma_c\sigma_d\eta_a - \eta_b)\frac{1}{\sigma_c}d\hat{Z}_t^{\alpha, \alpha}(s)$$

$$\mu^0_\phi(t, s) = \omega\phi + \gamma(\eta_a - \eta_b)\alpha + (\eta_a + \sigma_c\sigma_d\eta_a - \eta_b)\frac{(\sigma_c\sigma_d\gamma - \gamma\sigma_c)}{\sigma_c}$$

$$+ H(t, s)\left(\frac{(\eta_a - \sigma_c\sigma_d\eta_a)(\eta_a + \sigma_c\sigma_d\eta_a - \eta_b)}{\sigma_c^2} + \eta_a(\eta_a - \eta_b)^2\right)$$

$$\mu^1_\phi(t, s) = -\omega - (\eta_a + \sigma_c\sigma_d\eta_a - \eta_b)\frac{\chi(j,z)}{\sigma_c^2} - \chi(j,z)(\eta_a - \eta_b)^2\sigma_g^2$$

(A-28)

By Feynman-Kac theorem, $K$ solves the partial differential equation

$$K_t + K_F (\mu^0_\phi(t, s) + \mu^1_\phi(t, s)\phi_t) + \frac{1}{2}K_{\phi\phi} \left[\frac{(\eta_a + \sigma_c\sigma_d\eta_a - \eta_b)^2}{\sigma_c^2} + \frac{(\eta_a - \eta_b)^2\alpha^2}{\sigma_g^2}\right]$$

$$- K \left(\phi_t\mu_\theta(t, s)\chi(j,z) + \frac{1}{2}\phi_t^2 \left(\frac{1}{\sigma_c^2} + \frac{\alpha^2}{\sigma_g^2}\right)(\chi(j,z) - \chi(j,z)^2)\right) = 0$$

(A-29)

with the terminal condition $K(s, s, j, z) = 1$. The solution of (A-29) is easily seen to be of the form

$$K(t, s, j, z) = \exp(F_0(t, s, j, z) + F_1(t, s, j, z)\phi_t + F_2(t, s, j, z)\phi_t^2)$$

(A-30)

with coefficients $F$ deterministic functions of time, solving the forward system of ODEs:

$$-\hat{F}_2 = 2F_2^2 \left[\frac{(\eta_a + \sigma_c\sigma_d\eta_a - \eta_b)^2}{\sigma_c^2} + \frac{(\eta_a - \eta_b)^2\alpha^2}{\sigma_g^2}\right] + 2F_2\mu^0_\phi(t, s)$$

$$- \frac{1}{2} \left(\frac{1}{\sigma_c^2} + \frac{\alpha^2}{\sigma_g^2}\right)(\chi(j,z) - \chi(j,z)^2)$$

(A-31)

$$-\hat{F}_1 = F_1\mu^0_\phi(t, s) + 2F_1F_2 \left[\frac{(\eta_a + \sigma_c\sigma_d\eta_a - \eta_b)^2}{\sigma_c^2} + \frac{(\eta_a - \eta_b)^2\alpha^2}{\sigma_g^2}\right] + 2F_2\mu^0_\phi(t, s)$$

$$- \mu_\theta(t, s)\chi(j,z)$$

(A-32)

$$-\hat{F}_0 = F_1\mu^0_\phi(t, s) + \left(F_2 + \frac{F_1^2}{2}\right) \left[\frac{(\eta_a + \sigma_c\sigma_d\eta_a - \eta_b)^2}{\sigma_c^2} + \frac{(\eta_a - \eta_b)^2\alpha^2}{\sigma_g^2}\right]$$

(A-33)

with $F_i(s, s, j, z) = 0$, $i = 0, 1, 2$. The first ODE is a standard Riccati one, thus it can be solved explicitly. For the second and third, though some explicit form could probably be given, we rely on a numerical ODE solver. To summarize:

$$S_t^X = \frac{X_t}{1 + \theta^j_t} \int_t^{\infty} e^{m_0(t,s)+L(t,s)+H(t,s)\hat{X}_t} \left[\sum_{j=0}^{\infty} \left(\frac{\gamma}{j}\right) \theta^j_t \frac{1}{2^j} \frac{\gamma^j}{2^j} \right] \times$$

$$\left(\int_{-\infty}^{\infty} e^{2\pi i z \log \theta_t} \tilde{T}(j, z, t, s, \phi_t)dz\right) ds$$

(A-34)

$$\tilde{T}(j, z, t, s, \phi_t) = T(z) \exp\left(F_0(t, s, j, z) + F_1(t, s, j, z)\phi_t + F_2(t, s, j, z)\phi_t^2\right)$$

(A-35)
The (inverse) Fourier transform \( \left( \int_{-\infty}^{\infty} e^{2\pi i z \log \theta_t} \tilde{T}(j, z, \phi_t) dz \right) \) can be implemented very efficiently using the FFT algorithm, rather than generic numerical integration. The IMSL Fortran Library manual (Math Library, Ch.6, Usage Notes) explains clearly how to approximate the continuous Fourier transform as a discrete one, and implement it with a FFT routine.

Given the price process \( S^X_t \), the conditional risk premium of the security is readily obtained from its definition in a diffusive market model: (Instantaneous Stock Return Volatility Vector) \( \cdot \) (Market Price of Risk Vector). The former is provided by Ito’s lemma applied to (A-34). If \( \tilde{S}(X_t, \tilde{A}^q_t, \phi_t, \theta_t) \) denotes the RHS of (A-34), the return volatility components are (discarding the unpriced Brownian component \( W \)):

\[
\sigma_{Z,t}^X = \frac{\partial \log S}{\partial X_t} X_t \sigma_X \rho_X + \frac{\partial \log S}{\partial \tilde{A}^q_t} \sigma_{A} \sigma_{\rho_A} - \frac{\partial \log S}{\partial \phi_t} (\eta_a + \sigma_{A} \sigma_{\rho_A} - \eta_b) \frac{1}{\sigma_c} - \frac{\partial \log S}{\partial \theta_t} \theta_t \frac{1}{\sigma_c}
\]

\[
\sigma_{B,t}^X = -\frac{\partial \log S}{\partial \tilde{A}^q_t} \sigma_{X} + \frac{\partial \log S}{\partial \theta_t} \theta_t \frac{1}{\sigma_c} - \frac{\partial \log S}{\partial \theta_t} \alpha (\eta_a - \eta_b).
\]  

(A-36)

with

\[
\frac{\partial \log S}{\partial X_t} = \frac{1}{S^X_t} \left( \frac{X_t}{1 + \theta_t^X} \right)^{\gamma} \int_t^\infty e^{m_0(t,s)+L(t,s)+H(t,s)\tilde{A}^q_t} \left[ \sum_{j=0}^\infty \left( \frac{\gamma}{j} \right) \theta_t^{1 + 2j + \frac{7}{2}} \times \right.
\]

\[
\times \left( \int_{-\infty}^\infty e^{2\pi i z \log \theta_t} \tilde{T}(j, z, t, s, \phi_t) dz \right) \right] ds
\]

(A-37)

\[
\frac{\partial \log S}{\partial \tilde{A}^q_t} = \frac{1}{S^X_t} \left( \frac{X_t}{1 + \theta_t^X} \right)^{\gamma} \int_t^\infty H(t,s)e^{m_0(t,s)+L(t,s)+H(t,s)\tilde{A}^q_t} \left[ \sum_{j=0}^\infty \left( \frac{\gamma}{j} \right) \theta_t^{1 + 2j + \frac{7}{2}} \times \right.
\]

\[
\times \left( \int_{-\infty}^\infty e^{2\pi i z \log \theta_t} \tilde{T}(j, z, t, s, \phi_t) dz \right) \right] ds
\]

(A-38)

\[
\frac{\partial \log S}{\partial \phi_t} = \frac{1}{S^X_t} \left( \frac{X_t}{1 + \theta_t^X} \right)^{\gamma} \int_t^\infty e^{m_0(t,s)+L(t,s)+H(t,s)\tilde{A}^q_t} \left[ \sum_{j=0}^\infty \left( \frac{\gamma}{j} \right) \theta_t^{1 + 2j + \frac{7}{2}} \times \right.
\]

\[
\times \left( \int_{-\infty}^\infty e^{2\pi i z \log \theta_t} \tilde{T}(j, z, t, s, \phi_t) dz \right) \right] ds
\]

(A-39)

\[
\frac{\partial \log S}{\partial \theta_t} = -\frac{1}{S^X_t} \left( \frac{X_t}{1 + \theta_t^X} \right)^{\gamma+1} \int_t^\infty e^{m_0(t,s)+L(t,s)+H(t,s)\tilde{A}^q_t} \left[ \sum_{j=0}^\infty \left( \frac{\gamma}{j} \right) \theta_t^{1 + 2j + \frac{7}{2}} \times \right.
\]

\[
\times \left( \int_{-\infty}^\infty e^{2\pi i z \log \theta_t} \tilde{T}(j, z, t, s, \phi_t) dz \right) \right] ds
\]

(A-40)

\[
\frac{1}{S^X_t} \left( \frac{X_t}{1 + \theta_t^X} \right)^{\gamma} \int_t^\infty e^{m_0(t,s)+L(t,s)+H(t,s)\tilde{A}^q_t} \left[ \sum_{j=0}^\infty \left( \frac{\gamma}{j} \right) \theta_t^{1 + 2j + \frac{7}{2}} \times \right.
\]

\[
\times \left( \int_{-\infty}^\infty e^{2\pi i z \log \theta_t} \left( \frac{1}{2} + \frac{2j + \frac{7}{2}}{2\gamma} + 2\pi i z \right) \tilde{T}(j, z, t, s, \phi_t) dz \right) \right] ds
\]

(A-41)
For the market portfolio case, to which Proposition 2 refers, the specific equilibrium quantities are obtained by setting, \( X_t = C_t, \mu_X = \mu_c, \beta_x = 1, \sigma_X = \sigma_c, \rho_X = 1 \).

The Unleveraged Firm Value.

The unleveraged firm value given in expression (25) is of the same type as the pricing functional (A-4) analyzed in the proof of Proposition 2. In particular, after setting \( X_t = E_t^i(1 - \tau), \mu_X = \mu_i, \beta_x = \beta_i, \sigma_X = \sigma_i, \rho_X = \rho_i \), we obtain

\[
V_t^i = \mathbb{E}^a \left[ \int_t^\infty \frac{E_{s}^a}{\xi_{s}^a} E_{s}^i(1 - \tau) ds \right] \mathcal{F}^{c,g}_t
\]

\[
= (1 - \tau) \left( \frac{E_t^i}{1 + \theta_t^{1/\gamma}} \right)^{\frac{1}{\gamma}} \int_t^\infty e^{ma(t,s)+L(t,s)+H(t,s)} \tilde{A}_t \left[ \sum_{j=0}^\infty \left( \frac{\pi}{j} \right)^{1/\gamma} \sum_{j=0}^\infty \left( \frac{\pi}{j} \right)^{1/\gamma} \times \left( \int_{-\infty}^\infty e^{2\pi iz \log \theta_t \tilde{T}(j, z, t, s, \phi_t)} dz \right) ds \right.
\]

\[
\tilde{T}(j, z, t, s, \phi_t) = T(z) \exp \left( F_0(t, s, j, z) + F_1(t, s, j, z) \phi_t + F_2(t, s, j, z) \phi_t^2 \right)
\]

All the relevant expressions are detailed in the proof of Proposition 2.

The Default-Free Leveraged Equity Value.

In our numerical strategy (to be detailed below), at the default option expiry time \((T)\) the firm is bound to keep its current capital structure, thus we need the value of leveraged equity for a default-free firm. Following the proof of Proposition 2, the latter is given by:

\[
\tilde{k}V_t^i - kB_t^i
\]

where we remind that \( \tilde{k} \) is the firm’s pay-out ratio, \( \tau \) is the corporate tax rate, \( k = \tilde{k}(1 - \tau) \), \( V_t^i \) is the unleveraged firm value given in (A-42). The value of default-free debt, \( B_t^i \), is:

\[
B_t^i = \mathbb{E}^a \left[ \int_t^\infty \frac{q_s^a}{\xi_s^a} q_t^a ds \right] \mathcal{F}^{c,g}_t
\]

\[
= \frac{q_t}{(1 + \theta_t^{1/\gamma})^{\frac{1}{\gamma}}} \int_t^\infty e^{ma(t,s)+L(t,s)+H(t,s)} \tilde{A}_t \left[ \sum_{j=0}^\infty \left( \frac{\pi}{j} \right)^{1/\gamma} \sum_{j=0}^\infty \left( \frac{\pi}{j} \right)^{1/\gamma} \times \left( \int_{-\infty}^\infty e^{2\pi iz \log \theta_t \tilde{T}(j, z, t, s, \phi_t)} dz \right) ds \right.
\]

\[
\tilde{T}(j, z, t, s, \phi_t) = T(z) \exp \left( F_0(t, s, j, z) + F_1(t, s, j, z) \phi_t + F_2(t, s, j, z) \phi_t^2 \right)
\]

All the relevant expressions are detailed in the proof of Proposition 2, after setting \( X_t = q_t, \mu_X = 0, \beta_x = 0, \sigma_X = 0, \rho_X = 0 \).

Details of the Construction of the Fiscal Uncertainty Proxy.

We refer to Section 2.2, where we illustrate the construction of our proxy for fiscal uncertainty.
According to expression (9), individual $i$ perceives GDP growth for the year starting at date $T$ as:

$$
\log(C_{T+1}) - \log(C_T) = \bar{\mu} + \int_T^{T+1} \hat{A}_s^i \, ds + \sigma_c \left( \hat{Z}_{T+1}^i - \hat{Z}_T^i \right)
$$

$$
d\hat{A}_s^i = \lambda (\hat{A}_s^i - \hat{\lambda}_s^i) \, dt + \frac{\eta_i + \sigma_c \rho_i}{\sigma_c} \, d\hat{Z}_s^i - \eta_i \frac{\alpha}{\sigma_g} \, d\hat{B}_s^i,
$$

(A-47)

so that, for any date $t \in [T-1, T)$, the model-equivalent of the next-year GDP growth survey forecast reads:

$$
\mathbb{E}^i \left[ \log(C_{T+1}) - \log(C_T) | \mathcal{F}_{t}^{c,g} \right] = \bar{\mu} + \int_T^{T+1} \mathbb{E}^i \left[ \hat{A}_s^i | \mathcal{F}_{s}^{c,g} \right] \, ds,
$$

$$
= D_0^i(T-t) + D_1^i(T-t)\hat{A}_t^i,
$$

(A-48)

with

$$
D_0^i(T-t) = \bar{\mu} + \hat{\lambda}_t^i - \left( e^{-\lambda(T-t)} - e^{-\lambda(T+1-t)} \right),
$$

(A-49)

$$
D_1^i(T-t) = \left( e^{-\lambda(T-t)} - e^{-\lambda(T+1-t)} \right) \frac{1}{\lambda},
$$

(A-50)

To obtain these expressions, it suffices to solve the SDE (A-47):

$$
\hat{A}_s^i = \hat{A}_s^i e^{-\lambda(s-t)} + \hat{\lambda}_t^i \left( 1 - e^{-\lambda(s-t)} \right) + \int_t^s e^{-\lambda(s-u)} \left( \frac{\eta_i + \sigma_c \rho_i}{\sigma_c} \, d\hat{Z}_u^i - \eta_i \frac{\alpha}{\sigma_g} \, d\hat{B}_u^i \right),
$$

then take expectations conditional on $\mathcal{F}_{t}^{c,g}$ on both sides (whereby the innovation term vanishes), and integrate over $s$ from $T$ to $T+1$. Similarly, for $b_t$, ratio between Federal budget deficit and GDP evolving as in (33), the prediction forecast reads

$$
\mathbb{E}^i \left[ b_{T+1} - b_T | \mathcal{F}_{t}^{c,g} \right] = \alpha_b \left( D_0^i(T-t) - \bar{\mu} \right) + \alpha_b D_1^i(T-t)\hat{A}_t^i.
$$

(A-51)

We add Gaussian white noise measurement error to equations (A-48) and (A-51) and use them as measurement equations in the following state-space model

$$
\widehat{\text{GDP}}_{t,T} = D_0^i(T-t) + D_1^i(T-t)\hat{A}_t^i + s_c \epsilon^c_t
$$

(A-52)

$$
\begin{pmatrix} \widehat{\text{DEF}}_{t,T} \\ \widehat{\text{GDP}}_{t,T} \end{pmatrix} = \begin{pmatrix} \alpha_b \left( D_0^i(T-t) - \bar{\mu} \right) + \alpha_b D_1^i(T-t)\hat{A}_t^i + s_b \epsilon^b_t \\
\alpha_b \left( D_0^i(T-t) - \bar{\mu} \right) + \alpha_b D_1^i(T-t)\hat{A}_t^i + s_b \epsilon^b_t \end{pmatrix}
$$

(A-53)

$$
\hat{A}_{t+1}^i = L_0 + L_1 \hat{A}_t^i + s_A \epsilon_{t+1},
$$

(A-54)

where the Gaussian white noise shocks ($\epsilon^c_t, \epsilon^b_t, \epsilon_t$) are mutually independent. The discrete-time dynamics (A-54) of the latent fiscal action $\hat{A}_t^i$, as perceived by the econometrician, are the

---

15Note that we are not yet classifying forecaster $i$ as member of group $a$ or $b$, therefore $\rho_i$ is unconstrained at this stage.
exact (in a distributional sense) monthly discretization of the continuous-time analog \((A-47)\). Coefficients \(L_0, L_1\), and \(s_A\) are given by:

\[
L_0 = A_i(1 - \exp(-\lambda \cdot \frac{1}{12})), \quad L_1 = \exp(-\lambda \cdot \frac{1}{12})
\]

\[
s_A = \sqrt{\frac{\sigma_A^2}{2\lambda} (1 - \exp(-2\lambda \cdot \frac{1}{12}))}, \quad \sigma_A^2 = \left(\frac{\eta_i}{\sigma_g} + \left(\frac{\eta_i + \sigma_c\sigma_A}{\sigma_c}\right)^2\right)
\]

The Kalman filtering of the latent fiscal action is standard, but we report it here for completeness. Let \(y_t = [\text{GDP}, (\text{DEF}/\text{GDP})]_t\), \(d_t = [D_0(T - t), \alpha_b (D_0(T - t) - \bar{p})]'\), \(Z_t = [D_1(T - t), \alpha_b D_1(T - t)]'\), and \(H = \text{diag}[\sigma_d^2, \sigma_y^2]\). The initial state of the latent variable is assumed to be a realization of its stationary distribution: \(\tilde{A}_0^i \sim N(a_0, a_1)\), \(a_0 = L_0/(1 - L_1)\), \(a_1 = s_A^2/(1 - L_1^2)\). We also denote by \(\tilde{\mu}_i = E[\tilde{A}_i^i|F_{t-1}^{c,g,f}]\), the estimate of the latent state under the observation filtration augmented by the analysts forecasts, and by \(P_t^i = E[(\tilde{A}_i^i - \hat{A}_{t-1}^i)^2|F_t^{c,g,f}]\) the mean-squared error of \(\tilde{A}_i^i\).

We have \(\hat{A}_0^i = a_0 \) and \(P_0^i = a_1\). Given the estimate \(\hat{A}_{t-1}^i\), the prediction of next periods state and the associated MSE are:

\[
\hat{A}_{t|t-1}^i = E[\hat{A}_i^i|F_{t-1}^{c,g,f}] = L_0 + L_1 \hat{A}_{t-1}^i
\]

\[
P_{t|t-1}^i = E[(\hat{A}_{t|t-1}^i - \hat{A}_{t-1}^i)^2|F_{t-1}^{c,g,f}] = L_1^2 P_{t-1}^i + s_A^2
\]

Consequently, the optimal prediction of the measurement \(y_t\) and associated MSE are

\[
y_{t|t-1} = E[y_{t|F_{t-1}^{c,g,f}}] = d_t + Z_t \hat{A}_{t|t-1}^i
\]

\[
F_t = E[(y - y_{t|t-1})(y - y_{t|t-1})'|F_{t-1}^{c,g,f}] = Z_t P_{t|t-1}^i Z_t' + H
\]

Indeed the joint distribution of \(\hat{A}_t^i\) and \(y_t\) conditional on \(F_{t-1}^{c,g,f}\) is multivariate Gaussian with means \(\hat{A}_{t|t-1}^i\) and \(y_{t|t-1}\), respectively, and variance-covariance matrix

\[
\begin{pmatrix}
P_{t|t-1}^i & P_{t|t-1}^i Z_t' \\
Z_t P_{t|t-1}^i & F_t
\end{pmatrix}
\]

Thus, once \(y_t\) is observed, one can use the closed-form for the conditional expectation \(E[\hat{A}_t^i|y_t]\) available for Gaussian random vectors, to obtain an updated estimate of the latent state and of the corresponding MSE:

\[
\hat{A}_t^i = \hat{A}_{t|t-1}^i + P_{t|t-1}^i Z_t' F_t^{-1}(y_t - y_{t|t-1})
\]

\[
P_t^i = P_{t|t-1}^i - P_{t|t-1}^i Z_t' F_t^{-1} Z_t P_{t|t-1}^i
\]

Whenever an observation is missing, that is, the forecaster/institution did not respond to the survey, we simply skip the updating step, hence we set \(\hat{A}_t^i = \hat{A}_{t|t-1}^i\) and \(P_t^i = P_{t|t-1}^i\).

We obtain forecaster-specific first stage parameter estimates \(\hat{\theta}_i^i = (\hat{\mu}, \hat{\lambda}, \hat{A}_i, \alpha_b, s_A)\) by maximizing the log likelihood of the prediction errors of the measurements \(y_t\):

\[
-\frac{N_i}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^{N_i} \log(|F_t|) - \frac{1}{2} \sum_{t=1}^{N_i} (y_t - y_{t|t-1})' F_t^{-1} (y_t - y_{t|t-1})
\]
where \( N_i \) is the sample size available for each forecaster.

This procedure is clearly not able to identify the correlation parameter \( \rho_i \). To this purpose, the second stage estimation employs data on realized GDP growth and growth of Government expenditure as a fraction of GDP. We consider a state-space model where the measurement equations are a discretized version of the system \( (1)-(2) \). The dynamics of the latent state \( A_t^i \) to consider are the true ones in \( (4) \), rather than dynamics \( (A-47) \) under the forecaster information filtration, because the signals employed at this stage measure realized quantities rather than forecasts. To summarize, the state-space system reads:

\[
\log(C_{t+1}) - \log(C_t) = (\bar{\mu} - \frac{\sigma^2}{2} + \tilde{A}_t^i) \Delta t + \sigma_c \sqrt{\Delta t} \epsilon_t^{c+1}
\]

(A-61)

\[
\log(\tilde{g}_{t+1}) - \log(\tilde{g}_t) = \alpha \tilde{A}_t^i \Delta t - \sigma_g \sqrt{\Delta t} \epsilon_t^{g+1}
\]

(A-62)

\[
A_{t+1} = \frac{\tilde{A}_t^i (1 - \exp(-\lambda \Delta t)) + \exp(-\lambda \Delta t) A_t}{\sqrt{\frac{\sigma^2}{2\lambda} (1 - \exp(-2\lambda \Delta t))} \rho_i \epsilon_t^{c+1} + \sqrt{1 - \rho_i^2 \epsilon_t^{c+1}} \sqrt{\Delta t}},
\]

(A-63)

where \( \Delta t = 1/4 \), all Gaussian shocks \( \epsilon \) are mutually independent, and, as in the first stage, we have used the (distributionally) exact discretization of an Ornstein-Uhlenbeck process. We exogenously set \( \alpha = 1 \), following contrasting estimates of fiscal multipliers in the macroeconomics literature. We constrain parameters \( \lambda \) and \( \tilde{A}_t^i \) to coincide with their first-stage estimates, and we impose that:

\[
\left( \frac{\eta \alpha}{\sigma_g} \right)^2 + \left( \frac{\eta_c + \sigma_c \rho_i}{\sigma_c} \right)^2 = \hat{\sigma}_s^2,
\]

where \( \hat{\sigma}_s^2 \) is also a first-stage estimate. We estimate the free parameters, most importantly \( \rho_i \), by maximum likelihood from Kalman filter prediction errors. The procedure is identical to the first stage, with the exception of the covariance between \( A_t \) and \( y_t \) conditional on \( F_{t-1}^{c,g,f} \), which has an additional term, due to the covariance between the latent fiscal action and GDP growth, namely:

\[
\text{cov}[A_t, y_t | F_{t-1}^{c,g,f}] = Z_t P_{t|t-1} + \left( \sigma_c \sqrt{\frac{\sigma^2}{2\lambda} (1 - \exp(-2\lambda \Delta t))} \rho \right)
\]

After properly adjusting the updating equations for the modified covariance, the rest of the procedure is unchanged.

To compute our fiscal uncertainty measure as detailed in the text, we use first stage Kalman filter estimates \( \tilde{A}_t^i \), because we are interested in extracting forecaster-specific estimates of the fiscal action, of which analyst forecasts provide a direct signal.

**Numerical Method for Firm’s Equity Value.**

Following the discussion in Section 1.4, firm \( i \)’s equity is the value function of the optimal stopping problem

\[
V_t = \sup_{t_d} \mathbb{E}^a \left[ \int_t^{t_d} \xi^q_s D_s^i ds + \xi^q_{t_d} \left( p(\varphi_l - \varphi_r)V_{t_d}^i \right) \bigg| \mathcal{F}_t^{c,g} \right].
\]

(A-65)

where \( V_{t_d}^i \) denotes the unleveraged firm value and \( t_d \) is any stopping time with respect to the filtration \( \mathcal{F}_t^{c,g} \). We use a simulation-based method adapted from the Longstaff and Schwartz
(2001) American option pricing algorithm. The method works by backward induction, solving recursively the relevant dynamic programming equation at a finite number of exercise (‘defaultable’ in our case) dates. Thus we fix an ‘expiration’ date \( T \), after which the option to default is no longer available, and the equity value of the leveraged firm coincides with its default-free counterpart given in (A-44). We also assume that the default time \( t_d \) in (A-65) is one of a finite sequence \( t_d(j), \ j = 1, \ldots, n_d \), with \( t_d(1) = t \) (evaluation time) and \( t_d(n_d) = T \). We denote by \( \Delta t_d \) the (constant) time interval between decision dates.

There are four relevant state variables for problem (A-65), collected in a vector \( Y_s = [E^i, A^i, \phi_s, \theta_s] \). We first perform a useful change of probability measure, similarly to the proof of Proposition 2, in order to rewrite (A-65) as:

\[
V_t = \sup_{t_d} \frac{1}{(1 + \theta_t^γ)} \mathbb{E}^{\alpha,*} \left[ \int_t^{t_d} e^{f_i} \left[ -\gamma(1-\alpha)\hat{A}^i_t - \gamma \pi + \frac{\gamma(\gamma+1)}{2} (\sigma_c^2+\rho_c^2) \right] du \left( 1 + \theta_t^\frac{1}{2} \right)^\gamma D_t^i ds \right. \\
\left. + e^{f_i} \left[ -\gamma(1-\alpha)\hat{A}^i_t - \gamma \pi + \frac{\gamma(\gamma+1)}{2} (\sigma_c^2+\rho_c^2) \right] du \left( 1 + \theta_t^\frac{1}{2} \right)^\gamma (p(\varphi_i - \varphi_T) V_{t_d}^i) \right] F_t^g. \tag{A-66}
\]

and the dynamics of the state variables under the new measure are:

\[
dE_t^i = (\mu_i + \beta_i \hat{A}^i_t - \gamma \sigma_c \rho_c dt + \sigma_i (\rho_c d\hat{Z}_t^a,s + \sqrt{1-\rho_c^2} dW_t) \tag{A-67}
\]

\[
d\hat{A}^i_t = \lambda(\hat{A}_t - \hat{A}_t^0) dt - \frac{\eta_a + \sigma_c \alpha \rho_a}{\sigma_c} \gamma \sigma_c dt + \eta_a \alpha \gamma dt + \eta_a \frac{\alpha}{\sigma_g} d\hat{B}_t^{a,*} \tag{A-68}
\]

\[
\frac{d\theta_t}{\theta_t} = \phi_t \gamma (1-\alpha) dt - \phi_t \left( \frac{1}{\sigma_c} d\hat{Z}_t^{a,*} - \frac{\alpha}{\sigma_g} d\hat{B}_t^{a,*} \right), \tag{A-69}
\]

\[
d\phi_t = \omega(\tilde{\phi} - \phi_t) dt + \gamma (\eta_a - \eta_b) \alpha dt - (\eta_a + \sigma_a \rho_a - \eta_b) \gamma dt - (\eta_a - \eta_b) \frac{\alpha}{\sigma_g} d\hat{B}_t^{a,*} + (\eta_a + \sigma_a \rho_a - \eta_b) \frac{1}{\sigma_c} d\hat{Z}_t^{a,*} \tag{A-70}
\]

- Conditional on the current realization \( Y_t \), \( N \) sample paths of the state vector are simulated on the horizon \([t, T]\), using (for instance) an Euler discretization scheme applied to SDEs (A-67)-(A-70). Obviously, the discretization frequency of the paths, \( 1/\Delta s \), is larger than the decision frequency \( 1/\Delta t_d \). We denote by \( Y_s(\omega_j), \ j = 1, \ldots, N \) the realization of \( Y \) at time \( s \) along the simulated path \( \omega_j \).

- At the final date \( t_d(n_d) = T \), the lack of default results in the firm turning into a default-free leveraged one, so that:

\[
V_T^i(\omega_j) = \max \left( p(\varphi_i - \varphi_T) V_T^j(Y_T(\omega_j)), k V_T^j(Y_T(\omega_j)) - k B_T^j(Y_T(\omega_j)) \right) \quad j = 1, \ldots, N. \tag{A-71}
\]

where we have used expression (A-44).

- At a generic default date \( t_d(i) \), \( i = 1, \ldots, n_d - 1 \), the dynamic programming principle implies that

\[
V_{t_d(i)}^j(\omega_j) = \max \left( p(\varphi_i - \varphi_T) V_{t_d(i)}^j(Y_{t_d(i)}(\omega_j)), C_{t_d(i)}(Y_{t_d(i)}(\omega_j)) \right) \quad j = 1, \ldots, N. \tag{A-72}
\]

\[\text{In practice, we use } \Delta s = 1/360.\]
where the continuation value along the path \( \omega_j \), \( C_{td(i)}(Y_{td(i)}(\omega_j)) \) is the expected discounted value of: \( i \) the cumulative dividend stream until next decision date, and \( ii \) the equity value next decision date:

\[
C_{td(i)}(Y_{td(i)}(\omega_j)) = \frac{1}{\left(1 + \theta_{td(i)}^\gamma \right)^{\gamma}} \mathbb{E}^{\gamma, \phi, \theta}[\int_{td(i)}^{td(i+1)} e^{\int_{td(i)}^{s} \left[-(1-\alpha)\tilde{A}_s - \gamma\pi + \frac{\gamma(\gamma+1)}{2}(\sigma_s^2 + \sigma_s^2)\right] ds} \times \left(1 + \theta_{td(i+1)}^\gamma \right)^{\gamma} \mathbb{E}^{\gamma, \phi, \theta}[V_{td(i+1)}(Y_{td(i)}(\omega_j))]
\]

(A-73)

Let \( y(t_d(i), t_d(i+1), \omega_j) \) denote the realization of the argument of the conditional expectation along the simulated path \( \omega_j \):

\[
y(t_d(i), t_d(i+1), \omega_j) = \frac{1}{\left(1 + \theta_{td(i)}^\gamma \omega_j \right)^{\gamma}} \mathbb{E}^{\gamma, \phi, \theta}[\int_{td(i)}^{td(i+1)} \left(1 + \theta_{td(i+1)}^\gamma \omega_j \right)^{\gamma} D_s^i(\omega_j) \times e^{\int_{td(i)}^{s} \left[-(1-\alpha)\tilde{A}_s - \gamma\pi + \frac{\gamma(\gamma+1)}{2}(\sigma_s^2 + \sigma_s^2)\right] ds} \times e^{\int_{td(i)}^{td(i+1)} \left[-(1-\alpha)\tilde{A}_s - \gamma\pi + \frac{\gamma(\gamma+1)}{2}(\sigma_s^2 + \sigma_s^2)\right] ds} \times \left(1 + \theta_{td(i+1)}^\gamma \omega_j \right)^{\gamma} V_{td(i+1)}(Y_{td(i)}(\omega_j))]
\]

(A-74)

As usual, the \((\omega_j)\) arguments denotes a realization along the simulated path \( \omega_j \). Next exercise-date equity value is known from the previous step. As in Longstaff and Schwartz (2001), we approximate the continuation value (A-73) by projecting \( y(t_d(i), t_d(i+1), \omega_j) \) on a suitable multidimensional polynomial basis.\(^{17}\) Namely, letting \( X(Y_s) \) denote the \( n_X \)-dimensional row vector with the individual summands of the polynomial basis evaluated at \( Y_s \), we consider the model:

\[
y(t_d(i), t_d(i+1), \omega_j) = X(Y_{td(i)}(\omega_j)) \cdot \beta(t_d(i)) + \epsilon_j \quad j = 1, \ldots, N
\]

(A-75)

for a \( n_X \)-dimensional column vector of coefficients \( \beta(t_d(i)) \), whose OLS estimator is:

\[
\beta^*(t_d(i)) = [X(Y_{td(i)}(\omega_j))'X(Y_{td(i)}(\omega_j))]^{-1}X(Y_{td(i)}(\omega_j))'y(t_d(i), t_d(i+1), \omega_j)
\]

(A-76)

where \( X(\cdot) \) is the \( N \times n_X \)-dimensional matrix obtained by stacking column-wise row vectors \( X(Y_{td(i)}(\omega_j)) \) for all simulated paths, and similarly \( y(t_d(i), t_d(i+1), \omega_j) \). Therefore:

\[
C_{td(i)}(Y_{td(i)}(\omega_j)) \approx X(Y_{td(i)}(\omega_j)) \cdot \beta^*(t_d(i))
\]

(A-77)

\(^{17}\)Following Longstaff and Schwartz (2001), a tensor product of \( n \)-degree (with \( n = 2 \) or \( 3 \)) Laguerre polynomials in the single state variables, \( E^i, \tilde{A}^i, \phi, \theta \) is a suitable choice.
Following Longstaff and Schwartz (2001), we implement the dynamic programming equation (A-72) as

\[ V_{t_d(i)}^i(\omega_j) = \begin{cases} 
    p(\varphi_l - \varphi_r)Y_{t_d(i)}^i(Y_{t_d(i)}(\omega_j)) & \text{if } p(\varphi_l - \varphi_r)Y_{t_d(i)}^i(Y_{t_d(i)}(\omega_j)) > X(Y_{t_d(i)}(\omega_j)) \cdot \beta^*(t_d(i)) \\
    y(t_d(i), t_d(i+1), \omega_j) & \text{if } p(\varphi_l - \varphi_r)Y_{t_d(i)}^i(Y_{t_d(i)}(\omega_j)) \leq X(Y_{t_d(i)}(\omega_j)) \cdot \beta^*(t_d(i))
\end{cases} \]  

(A-78)

rather than selecting fitted continuation values in the second alternative \((y(t_d(i), t_d(i+1), \omega_j))\) is given in (A-74).

Iterating this procedure backward, we obtain an estimate of the current firm \(i\)'s equity value:

\[ V_t^i \approx \frac{\sum_{j=1}^{N} V_{t_d(1)}^i(\omega_j)}{N} \]  

(A-79)

This estimator is known to be upward biased – see Glasserman (2004), Section 8.6. Alternatively, we can use the estimates \(\beta^*(t_d(i))\) \(i = 1, \ldots, n_d\) in the representation of the continuation values to form a default rule for a fresh set of simulated paths \(Y_s(\omega_j), s \in [t, T], j = 1, \ldots, N\), so that:

\[ V_t^i \approx \frac{\sum_{j=1}^{N} \tilde{y}(t, t_d(z_j), \omega_j)}{N} \]  

(A-80)

\[ \tilde{y}(t, t_d(z_j), \omega_j) = \frac{1}{\left(1 + \theta^\frac{1}{2}\right)^\gamma} \left[ \int_t^{t_d(z_j)} \left(1 + \theta_s(\omega_j)^\frac{1}{2}\right)^\gamma D_s^i(\omega_j) \times e^{f_t^s[-\gamma(1-\alpha)\bar{\omega}_s^\omega(\omega_j) - \gamma \bar{\omega} + \frac{\gamma(1+\gamma)}{2}(\sigma^2_s + \sigma^2_s^z)]} ds + \left(1 + \theta_{t_d(z_j)}(\omega_j)^\frac{1}{2}\right)^\gamma \times e^{f_t^{t_d(z_j)}[-\gamma(1-\alpha)\bar{\omega}_s^\omega(\omega_j) - \gamma \bar{\omega} + \frac{\gamma(1+\gamma)}{2}(\sigma^2_s + \sigma^2_s^z)]} du \right] (p(\varphi_l - \varphi_r) \times \)  

(A-81)

\[ \times V_{t_d(z_j)}^i(Y_{t_d(z_j)}(\omega_j))) \right) \]  

(A-82)

\[ z_j = \min \left[ n_d, \inf_i \left( p(\varphi_l - \varphi_r)Y_{t_d(i)}^i(Y_{t_d(i)}(\omega_j)) > X(Y_{t_d(i)}(\omega_j)) \cdot \beta^*(t_d(i)) \right) \right] \]

Estimator (A-80) is downward biased, but provides a better approximation of the true value according to the literature – see Glasserman (2004), Section 8.6. In order to better mimic the behavior of a firm with a perpetual option to default, we use the coefficients \(\beta^*(t_d(1))\) of the initial date to represent the continuation value at any date.

The price of corporate debt, (31), is estimated very similarly to (A-80), with \(q_i\) replacing \(D_t^i\) and \((1 - \varphi_r - p((\varphi_l - \varphi_r)))\) replacing \(p((\varphi_l - \varphi_r))\).

The equity risk premium of firm \(i\) is computed using the standard definition in a diffusive market context: (Instantaneous Stock Return Volatility Vector) \cdot (Market Price of Risk Vector). The equity return volatility is as in expression (A-36), where the sensitivities of the equity price to the initial conditions of the state variables need to be estimated. Using the previously estimated default rule \(\beta^*(t_d(i))\), we apply the path-wise delta method.
illustrated, for instance, in Glasserman (2004) Section 7.2. The procedure is standard, thus in the interest of space we do not report the details.$^{18}$

$^{18}$Available upon request.